Supplementary for ”Uncertainty propagation from Sentinel-2A/B-derived velocity to glacier strain rates: a first-principles perspective”

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1 Logarithmic strain rate calculation details

This study extends Nye's five–point rosette concept to the remote–sensing grid (Nye, 1959), and its implementation strictly follows the logarithmic strain rate workflow proposed by Alley et al. (2018). The present text summarises the computation of logarithmic strain rates used herein, with the aim of clarifying the subsequent error–propagation derivation. Any further details omitted could be sought in the original publication (Alley et al., 2018).

Logarithmic strain rate is obtained by integrating particle trajectories within the ice body. The defining expression is:

$$\dot{\varepsilon}_l = \frac{1}{\Delta_m t} \ln \frac{L_f}{L_0}, \tag{1}$$

where $\Delta_m t$ is the duration of the strain rate measurement (independent of the temporal baseline Δt used in NCC process), L_0 denotes the original length of the parcel, and L_f is the parcel's final length after the elapsed temporal interval $\Delta_m t$.

In practice, when a field site is instrumented, one central stake is complemented by four additional stakes arranged in the four cardinal directions, with the interval to be l, i.e., the spatial resolution of the Satellite in this study. The five stakes define eight parcels oriented at 0°, 45°, 90° and 135°. The trajectory of each stake during $\Delta_m t$ is recorded, and the eight final lengths are extracted from the terminal positions. These lengths are then averaged pairwise within each original orientation, yielding four orientation–specific lengths from which four directional strain rates $\dot{\varepsilon}_\theta$ ($\theta = 0, 45, 90, 135$) are computed via equation (1).

To replicate this measurement with remote–sensing data, stake motion is simulated by means of an adaptive time–step Euler–Heun integrator. Prior to integration, the raw gridded two–dimensional velocity field is bilinearly interpolated to yield the continuous velocity function $\vec{u}(x, y)$. Each simulation proceeds in two stages. Given the stake position p_i after the i^{th} step, an initial time increment $\delta_m^{(i)} t$ is selected. The forward–Euler predictor for the next position is:

$$p_{i+1}^E = p_i + \vec{u}(p_i)\delta_m^{(i)} t, \tag{2}$$

and the Heun (improved Euler) corrector yields:

$$p_{i+1}^H = p_i + \frac{\vec{u}(p_i) + \vec{u}(p_{i+1}^E)}{2} \delta_m^{(i)} t. \tag{3}$$

The relative discrepancy between the two predictions is:

$$\delta_E^H = \frac{||p_{i+1}^E - p_{i+1}^H||}{||p_{i+1}^H||}. \tag{4}$$

Whenever δ_E^H exceeds a user–defined tolerance τ, the step size $\delta_m^{(i)} t$ is halved; otherwise it is doubled, yet never allowed to exceed the prescribed upper bound $\Delta_m t$. Integration ceases once the cumulative elapsed time reaches $\Delta_m t$.

After the four directional strain rates have been recovered, the Cartesian components $\dot{\varepsilon}_{xx}, \dot{\varepsilon}_{xy}, \dot{\varepsilon}_{yy}$ are obtained via least–squares inversion of the second–order tensor rotation relation, namely:

$$\boldsymbol{\varepsilon} = (\mathbf{A}^\top \mathbf{A})^{-1} \mathbf{A}^\top \dot{\boldsymbol{\varepsilon}}_{\boldsymbol{\theta}}, \tag{5}$$

with:

$$\mathbf{A} = \begin{bmatrix} \cos^2 0° & \sin^2 0° & 2\cos 0° \sin 0° \\ \cos^2 45° & \sin^2 45° & 2\cos 45° \sin 45° \\ \cos^2 90° & \sin^2 90° & 2\cos 90° \sin 90° \\ \cos^2 135° & \sin^2 135° & 2\cos 135° \sin 135° \end{bmatrix} \quad \text{and} \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \dot{\varepsilon}_{xx} \\ \dot{\varepsilon}_{xy} \\ \dot{\varepsilon}_{yy} \end{bmatrix}. \tag{6}$$

Numerically, in this study, we used $\Delta_m t = 1$ day, the initial $\delta_m^{(i)} t = 0.1$ day and $\tau = 1e - 6$.

2 Error expression derivation

NCC extracts flow velocity by determining the pixel displacement of grid effects. In this process, considering the limitation of sampling precision d (in units of pixels, characterizing the intrinsic nature of this physical quantity), for the true pixel displacement Δx, the measured displacement is:

$$\tilde{\Delta x} = d\left[\frac{\Delta x}{d}\right] + e \tag{7}$$

In this way, the discrepancy between the true and measured NCC displacement Δx, expressed in pixel units is Zhang et al. (2026):

$$e_{\Delta x} = \tilde{\Delta x} - \Delta x = e + \left(d\left[\frac{\Delta x}{d}\right] - \Delta x\right). \tag{8}$$

Consequently, the corresponding pixel–level random error e_v is obtained by dividing the pixel displacement error $e_\Delta x$ by the temporal baseline Δt. Throughout the subsequent discussion, both velocity and length are scaled in pixels. e is a random error introduced by other measurement processes, exhibiting spatial homogeneity and independence. Other forms of systematic errors are temporarily not considered.

The flow velocity v and pixel displacement Δx have the following relationship:

$$v = \frac{l}{\Delta t}\Delta x \tag{9}$$

where l is the spatial resolution, and Δt is the temporal baseline of the two images. Considering that for the same image pair, l and Δt are constant parameters, the flow velocity error therefore satisfies:

$$e_v = \frac{l}{\Delta t} e_{\Delta x} \tag{10}$$

2.1 Error expression for the nominal strain rate

According to the measurement formula7, The nominal strain rate derived by the finite difference scheme between two points can be written as:

$$\begin{aligned}\dot{\varepsilon}_n &= \frac{\delta\tilde{v}}{\delta l} = \frac{\tilde{v}_2 - \tilde{v}_1}{\delta l} \\ &= \frac{dl}{\delta l \Delta t}\left(\left[\frac{\frac{v_2}{l}\Delta t}{d}\right] - \left[\frac{\frac{v_1}{l}\Delta t}{d}\right] + \frac{(e_2 - e_1)}{d}\right) \\ &:= \dot{\varepsilon}_{n(a)} + \dot{\varepsilon}_{n(b)}.\end{aligned} \tag{11}$$

where v is the velocity, with tilde meaning "measured", and different subscript meaning different location. δl is the difference scale, which is the spatial resolution in this study. d is the capicity of distinguishing the pixel. Δt is the temporal baseline. e_1 and e_2 is the random measurement error at each point.

The first term simplifies to the true strain rate,

$$\dot{\varepsilon}_{n(a)} = \frac{d}{\delta l \Delta t}\frac{(v_2 - v_1)\Delta t}{d} = \frac{\delta v}{\delta l} \tag{12}$$

The link between the $\frac{\delta u}{\delta l}$ and the exact strain rate $\dot{\varepsilon}$ is revealed via a Taylor–series expansion:

$$\begin{aligned}\frac{\delta v}{\delta l} &\approx \frac{1}{\delta l}\left(\frac{\partial v}{\partial x_i}\delta l + \frac{1}{2}\frac{\partial^2 v}{\partial x_i^2}\delta l^2\right) \\ &= \dot{\varepsilon} + \frac{1}{2}\frac{\partial \varepsilon}{\partial x_i}\delta l \\ &= \dot{\varepsilon} + \frac{1}{2}\frac{\partial \varepsilon}{\partial x_i}(s+1)l,\end{aligned} \tag{13}$$

where v and x_i denotes the velocity in either direction and the length in gradient direction and s counts the pixel strides used in NCC extraction. Choosing the minimal δl (i.e., the Sentinel–2A/B pixel size, $\delta l = 10$ m $= l$) and eliminating strides minimises the discrepancy between $\frac{\delta u}{\delta l}$ and the true strain rate $\dot{\varepsilon}$, allowing us to treat it as a proxy for the latter when analysing the influence of the remote–sensing observations themselves.Ultimately, we have $\dot{\varepsilon}_{n(a)} \approx \dot{\varepsilon}$

The second term, a stochastic component, is expressed as:

$$\dot{\varepsilon}_{n(b)} = \frac{dl}{\delta l \Delta t}\left(\frac{\frac{v_1}{l}\Delta t}{d} - \frac{\frac{v_2}{l}\Delta t}{d} - \left(\left[\frac{\frac{v_1}{l}\Delta t}{d}\right] - \left[\frac{\frac{v_2}{l}\Delta t}{d}\right]\right) + \frac{e_2 - e_1}{d}\right) := \frac{dl}{\delta l \Delta t}\epsilon. \tag{14}$$

For sufficiently large Δt, the second term becomes negligible owing to the denominator, and the measured velocity gradient converges to the true value. Conversely, when the temporal baseline Δt is small, the first term may be masked by the dominance of the second because the temporal baseline, serving as the denominator of the second term, amplifies this term. We now specifically analyse the magnitude of the second term.

For the extracted ϵ, it can be decomposed into two components, $\epsilon = \epsilon_1 + \epsilon_2$:

$$\begin{cases} \epsilon_1 = \frac{\frac{v_1}{l}\Delta t}{d} - \frac{\frac{v_2}{l}\Delta t}{d} - \left(\left[\frac{\frac{v_1}{l}\Delta t}{d}\right] - \left[\frac{\frac{v_2}{l}\Delta t}{d}\right]\right) \\ \epsilon_2 = \frac{e_2 - e_1}{d} \end{cases} \tag{15}$$

According to the Taylor expansion, we have $v_1 \approx v_2 + \frac{\partial v}{\partial l}\delta l$. Defining $k_1 = \frac{v_1 \Delta t}{dl}, k_2 = \frac{\frac{\partial v}{\partial l}\delta l \Delta t}{dl}$, ϵ_1 can then be written in the following mathematical form:

$$\epsilon_1 = k_2 - [k_1 + k_2] + [k_1] \tag{16}$$

In this study, noting that in our research problem, $\frac{\partial u}{\partial l} = \dot{\varepsilon} \sim 0.01 - 0.001 \text{ d}^{-1}, \delta l = l = 10 \text{ m}, \Delta t \leq 32 \text{ d}, d \sim 0.2$, we have $k_2 \sim 10^{-2} - 10^1$. When k_2 is much smaller than 1, $\epsilon_1 \approx k_2 - [k_1] + [k_1] = k_2 \sim 0$, which in fact means that the limitation of sampling precision does not induce error, and the actual statistics of all orders of ϵ_1 are zero.

When the magnitude of k_2 reaches the order of 1, k_1 and k_2 form the superposition of two random variables with two degrees of freedom. According to the rounding function inequality, for any real number k, we have:

$$k - \frac{1}{2} \leq [k] < k + \frac{1}{2} \tag{17}$$

The error control inequality is thus obtained:

$$-\frac{1}{2} \leq [x] - x \leq \frac{1}{2}, \quad \forall x \in \mathbb{R} \tag{18}$$

For the expression with two degrees of freedom, we have:

$$|\epsilon_1| \leq 1 \tag{19}$$

When k_2 is sufficiently large, if we assume the velocity field is smooth, the generation of rounding discontinuity points will no longer depend on the spatial distribution characteristics of the velocity field, but will possess sufficient randomness, such that the distribution of rounding residuals also exhibits numerical randomity. Therefore, ϵ_1 can be approximately regarded as satisfying a uniform distribution. According to the statistical formulas of the uniform distribution, we have:

$$\mathbb{E}(\epsilon_1) = 0, \mathbb{E}(\epsilon_1^2) = \frac{1}{3} \tag{20}$$

Finally, if $k_2 \sim 1$, the situation becomes somewhat more complicated, because the generation of rounding discontinuity points involving k_2 will depend on the spatial characteristics of the velocity field and is no longer applicable to general statistical laws. However, if we perform statistics on the average of a sufficiently large amount of data, certain regularities can still be found, because the spatial variation of k_1 is sufficiently large. Fixing k_2 first, when k_1 is sufficiently large, the rounding discontinuity points of k_1 still possess randomness in their values. At this point, ϵ_1 constitutes a uniform distribution over the range $[\{k_2\} - 1, \{k_2\}]$, where $\{k_2\}$ denotes the fractional part of k_2, with a value range of $[0, 1)$. Its statistics are therefore:

$$\mathbb{E}(\epsilon_1) = \{k_2\} - \frac{1}{2} \tag{21}$$

Below, we estimate the expectation of the average for a large amount of data. When the data volume reaches a certain level, $\{k_2\}$ is sufficient to form a uniform distribution, and therefore we have:

$$\mathbb{E}(\bar{\epsilon_1}) = 0 \tag{22}$$

indicating that the actual result approaches the case of $k_2 \gg 1$, because of the law of large numbers: $\bar{\epsilon_1} \sim \mathbb{E}(\epsilon_1)$. At this point, the situation is equivalent to k_1 and k_2 taking random values simultaneously, and we have $\mathbb{E}(\bar{\epsilon_1}^2) = \frac{1}{3}$.

In actual computation, the order of magnitude of k_2 cannot be explicitly determined, and depending on its proximity to 1, it causes significant changes in the statistical distribution of ϵ. However, since

k_2 comprises three regimes: much smaller than 1, close to 1, and much larger than 1, according to the additivity of random variables, it is not difficult to obtain:

$$\mathbb{E}(\epsilon_1) = 0, \mathbb{E}(\epsilon_1^2) \leq \frac{1}{3} \tag{23}$$

Furthermore, performing an order-of-magnitude estimation for the parameters used in our tests, we find that under the conditions where the sampling error effect is most significant, $\Delta t \leq 10$ d. At this point, the probability of $k_2 \geq \frac{1}{2}$ is extremely small (requiring a strain rate greater than 0.01), making the case of $k_2 \ll 1$ occupy the vast majority of the sample population. Therefore, we can estimate $\frac{1}{3}$ as the upper bound of $\mathbb{E}(\epsilon_1^2)$ under the worst-case scenario, while under normal circumstances $\mathbb{E}(\epsilon_1^2)$ should be much smaller than $\frac{1}{3}$. We denote:

$$\mathbb{E}(\epsilon_1^2) = a_1 \tag{24}$$

For ϵ_2, since we have already assumed that e is spatially homogeneous, i.e., $\mathbb{E}(e) = 0$. Moreover, due to statistical independence, we readily have $\mathbb{E}(e\epsilon_1) = \mathbb{E}(e)\mathbb{E}(\epsilon_1) = 0$. Finally, we define:

$$\mathbb{E}(\epsilon_2^2) = a_2 \tag{25}$$

Since it has been discussed that for the average $\bar{\epsilon_1}$, the rounding discontinuity points induced by both k_1 and k_2 are random, ϵ itself also possesses independence. This ultimately allows the cross term between $\dot{\varepsilon}_{n(a)}$ and $\dot{\varepsilon}_{n(b)}$ to be separable when taking expectations, leading to a cross-term expectation of zero.

Back to the overall analysis of the strain rate error, when we take $\delta l = l$, according to proportional scaling, the second term $\dot{\varepsilon}_{n(b)}$ satisfies $\mathbb{E}(\dot{\varepsilon}_{n(b)}) = 0$ and $\mathbb{E}(\dot{\varepsilon}_{n(b)}^2) = a\big(d/\Delta t\big)^2$. Thus, the nominal strain rate is related to the temporal baseline via:

$$\mathbb{E}(\dot{\varepsilon}_n^2) = \mathbb{E}(\dot{\varepsilon}_{n(a)}^2) + \mathbb{E}(\dot{\varepsilon}_{n(b)}^2) + 2\mathbb{E}(\dot{\varepsilon}_{n(a)})\mathbb{E}(\dot{\varepsilon}_{n(b)}) = \mathbb{E}(\dot{\varepsilon}^2) + a\left(\frac{d}{\Delta t}\right)^2 \tag{26}$$

Correspondingly,

$$\mathbb{D}(\dot{\varepsilon}_n) = \mathbb{E}(\dot{\varepsilon}_n^2) - \mathbb{E}^2(\dot{\varepsilon}_n) = \mathbb{E}(\dot{\varepsilon}_n^2) - \mathbb{E}^2(\dot{\varepsilon}) = \mathbb{E}(\dot{\varepsilon}_n^2) - \mathbb{E}(\dot{\varepsilon}^2) = a\left(\frac{d}{\Delta t}\right)^2 \tag{27}$$

where the calibration factor $a = a_1 + a_2$ compensates for departures from strict normality and for the influence of the technically random error e. According to the error magnitude estimation above, $a_1 \ll \frac{1}{3}$.

It is worth noting that in actual statistical findings, it is observed that NCC does not necessarily extract the velocity field into a completely sawtooth-like stepwise distribution, but still retains a tendency towards the true velocity field. In other words, e tends to reduce the absolute difference between Δx and $\tilde{\Delta x}$, rather than amplifying a as described by the nominal expression $a = a_1 + a_2$. In this case, ϵ will no longer follow a uniform distribution, but rather a distribution with a higher centre and lower tails. This causes a to be further reduced on the basis of $a_1 \ll \frac{1}{3}$.

When $\delta l > l$, the derivation concerning the error variance presented above remains fundamentally valid. The partial difference lies in the fact that the value of a may increase slightly, because $k_2 = \frac{\frac{\partial v}{\partial l}\delta l \Delta t}{dl}$ will increase, causing the error term to cross rounding discontinuity points more easily. After proportional scaling, the error term becomes $\mathbb{D}(\dot{\varepsilon}_n) = a\left(\frac{dl}{\delta l \Delta t}\right)^2$. The main uncontrollable factor is that the true-value term may no longer converge to the true strain rate, because the higher-order terms in the Taylor expansion of the spatial difference of velocity may no longer be negligible. Nevertheless, if we temporarily accept the insignificance of the fractal structure of glacier strain rates, with the velocity field dominated by linear variation, then the optimal temporal baseline formula mentioned in the main text can be derived based on the order-of-magnitude comparison between the error term and the true term.

2.2 Error expression for the logarithmic strain rate

In the absence of interference from other forms of error, according to our error assumption, when Δx is much larger than d, the occurrence of discontinuity points hardly exhibits any spatial particularity and approaches a random distribution. At this point, e_v possesses spatial homogeneity in space. The derivations below only make use of this property of e_v. Therefore, these derivations are applicable not only to our error assumption, but also to all error forms exhibiting spatial homogeneity.

During the logarithmic strain rate measurement, the error e_v contaminates the velocity field used by the adaptive time–step Euler-Heun integrator and hence alters the final length L_f. Quantifying the resulting bias in L_f for an arbitrary velocity error is formidable, because, unlike the finite difference case where the error and the true velocity can be separated cleanly, the true velocity non–linearly controls the retrieval of the final length (Alley et al., 2018). Additional error sources include the interpolation errors arising from the discrete velocity field (Alley et al., 2018); their superposition precludes a direct a–priori estimate.

Fortunately, bounding the error in L_f is still possible under simplified conditions. We now assume a one–dimensional, linearly varying velocity field, with the velocity error e_v significantly smaller than the velocity. Such a field is reproduced exactly by interpolation, thereby eliminating interpolation error. In addition, sufficiently small velocity error makes the velocity sample points in each step not significantly affected by the velocity error, so that we can just allocate the exact velocity in each simulation. With an adaptive step–size strategy, the update of a point p_i in one improved–Euler step is:

$$p_{i+1} = p_i + \left(u^{(i)} + e_v^{(i)}\right)\delta_m^{(i)} t + \tau, \tag{28}$$

where $u^{(i)}$ and $e_v^{(i)}$ are the true velocity and the velocity error at the i–th evaluation, and $\delta_m^{(i)} t$ the optimal adaptive time step. The tolerance τ is prescribed to be far smaller than the pixel resolution of the NCC method (e.g., here $\tau = 10^{-6} \ll d$; its role is clarified below. After N adaptive steps spanning the measurement interval $\Delta_m t$, the final position is:

$$p_N = p_0 + \sum_{i=0}^{N-1} \left(u^{(i)} + e_v^{(i)}\right)\delta_m^{(i)} t + N\tau. \tag{29}$$

Hence the discrepancy between the true and measured end–points is:

$$e_p = \sum_{i=0}^{N-1} e_v^{(i)} \delta_m^{(i)} t + N\tau. \tag{30}$$

The second term $N\tau$ is analyzed first. Because τ and N are anti–correlated, tighter tolerances shrink $\delta_m^{(i)} t$ and thus enlarge N-a quantitative estimate is required. The improved Euler scheme is second–order accurate, so its local truncation error is third order (Hanna, 1988), i.e:

$$\tau = K\delta_m^3 t, \tag{31}$$

where K is a bounded constant for smooth velocity fields varying gently over the scale $u^{(i)}\Delta_m t$. Ignoring K gives the representative step size $\delta_m t = \tau^{1/3}$. The second term can therefore be approximated as:

$$N\tau = \frac{\Delta_m t}{\delta_m t}\tau = \Delta_m t\, \tau^{2/3}. \tag{32}$$

With $\tau \ll d$, this contribution can be orders of magnitude smaller than the e_v and is neglected.

The first term in (30) is now considered. In the worst case-rare but analytically tractable-the velocity errors $e_v^{(i)}$ share the same sign and attain the maximum magnitude $e_v^m = e + \frac{d}{2}$. Then:

$$e_p = e_v^m \sum_{i=0}^{N-1} \delta_m^{(i)} t = e_v^m \Delta_m t. \tag{33}$$

Since the final length L_f is the difference between two such end–points, the maximum length error attributable to this term is:

$$e_{L_f} = 2e_v^m \Delta_m t. \tag{34}$$

The measured logarithmic strain rate becomes:

$$\tilde{\dot{\varepsilon}} = \frac{1}{\Delta_m t} \ln \frac{\tilde{L_f}}{L_0} = \frac{1}{\Delta_m t} \ln\left(\frac{L_f}{L_0} + \frac{2e_v^m \Delta_m t}{L_0}\right). \tag{35}$$

For $e_v^m \Delta_m t \ll L_f$, a first–order Taylor expansion of the logarithm around $x_0 = L_f/L_0$ yields:

$$\tilde{\dot{\varepsilon}} \approx \frac{1}{\Delta_m t} \ln\left(\frac{L_f}{L_0}\right) + \frac{2e_v^m}{L_f} = \dot{\varepsilon} + \frac{2e_v^m}{L_f}. \tag{36}$$

By choosing appropriate L_0 and $\Delta_m t$, L_f is prevented from becoming either extremely large or small, so the magnitude of the strain rate error matches that of e_v, consistent with the finite difference result.

In the best case, the contributions $\sum e_v^{(i)} \delta_m^{(i)} t$ cancel perfectly, e.g., when $\delta_m^{(i)} t$ is identical for every step and $e_v^{(i)}$ appear in equal and opposite pairs. Then the first term vanishes and both the final length and the logarithmic strain rate are recovered exactly.

In summary, the error analysis for logarithmic strain rate reveals that in the worst case the result collapses to the usual finite difference error, whereas in the best case the true value is retrieved (neglecting the negligible tolerance term). From random–process theory, under ideal error statistics, i.e., e_v is centred, symmetric, and isotropic, exactly the assumptions made for NCC–derived errors, the expected value of the accumulated error term $\sum e_v^{(i)} \delta_m^{(i)} t$ is zero when the step size is constant. This conclusion holds in any dimension. Constancy of the step size is reasonable because, as shown above, the adaptive scheme keeps $\delta_m^{(i)} t$ clustered around the mean value dictated by the improved–Euler accuracy theory. These findings highlight the superiority of the logarithmic strain rate: it tends to yield the true value regardless of the magnitude of e_v, and even in the worst case it approximates the nominal (finite difference) result.

The extension to real two–dimensional velocity fields is straightforward. Replacing scalar velocities and errors with their vector counterparts does not alter the conclusions, because the pivotal result, namely, that the expectation of the accumulated velocity–error vector $\sum \vec{e}_v^{(i)} \delta_m^{(i)} t$ is approximately zero, remains valid. Moreover, provided the two–dimensional velocity field exhibits negligible non–linearity over the interpolation grid spacing (here 10 m, the NCC spatial resolution), the linearisation assumption should be justified. Finally, The above formula holds only on the premise that the flow velocity error is much smaller than the flow velocity itself. Obviously, when the measured flow velocity field deviates greatly from the actual flow velocity field distribution, this method will still fail.

3 Additivity of independent systematic errors

Building upon the two types of errors assumed above, NCC extraction is also subject to other forms of errors (e.g., orthorectification errors, atmospheric errors, etc.). Assuming that these errors are mutually independent with the errors discussed above, as well as among themselves, we now discuss the influence of these errors on the strain rate.

We assume that there is an additional error e_{other} (in the unit of pixel) in the flow velocity extraction, i.e.:

$$e'_{\Delta x} = e_{\Delta x} + e_{\text{other}} \tag{37}$$

First, we discuss the case where e_{other} exhibits spatial homogeneity. In this case, $\mathbb{E}(e_{\text{other}}) = 0$. Thus, combining with independence, the error of the nominal strain rate is:

$$\mathbb{E}(\dot{\varepsilon}_n) = 0, \mathbb{D}(\dot{\varepsilon}_n) = (a + 2\frac{d_{\text{other}}^2}{d^2}) \left(\frac{d}{\Delta t}\right)^2 \tag{38}$$

where d_{other} is a parameter related to the spatial difference distribution of the systematic error e_{other}, and its conservative estimate is:

$$d_{\text{orb}} = \max \left\{ \left| \frac{\delta e_{\text{other}}}{\delta l} \right| l \right\} \tag{39}$$

This indicates that the influence of additional systematic errors on the nominal strain rate is mainly reflected in the modulation of a. If the spatial variation of such systematic errors is relatively smooth, even if its absolute value is large, the impact on the result remains small.

For the logarithmic strain rate, since the derivations above only employed the condition of spatial homogeneity, this additional systematic error does not alter the logarithmic strain rate.

If other forms of errors exhibit spatial heterogeneity, then this influence is uncontrollable, and in particular will alter the calculated result of the logarithmic strain rate. An intuitive explanation is that a spatially heterogeneous systematic error with a sufficiently large differential amplitude is equivalent to a spatial structural change in the original velocity field; therefore, its strain rate itself will undergo unpredictable changes.

4 Detailed infomation for velocity product in Motivation

Table 1: Product Information Table

Abbreviation	Product Name	Reference
grimp_glacier_insar_vx	TSX_E66.50N_07Aug16_18Aug16_09-23-32_vx_v04.0.tif	Joughin et al. (2021)
grimp_glacier_optical_vx	OPT_E66.50N_2016-08_vx_v03.0.tif	Howat (2020)
grimp_icesheet_annually_vx	GL_vel_mosaic_Annual_01Dec15_30Nov16_vx_v05.0.tif	Joughin (2023c)
grimp_icesheet_quarterly_vx	GL_vel_mosaic_Quarterly_01Jun16_31Aug16_vx_v05.0.tif	Joughin (2023b)
grimp_icesheet_monthly_vx	GL_vel_mosaic_Monthly_01Aug16_31Aug16_vx_v05.0.tif	Joughin (2023a)
grimp_icesheet_weekly_vx	GL_vel_mosaic_s1cycle_30Jul16_10Aug16_vx_v02.0.tif	Joughin (2022)
its_live_landsat_vx	LC08L1TP232013201608190 1T1_LC08L1TP2320132016080301T1_32624_G0240V01_P087.nc	Gardner et al. (2025)
its_live_regional_annual_vx	NSIDC-0776_RGI05A_2016_V02.0.nc	Gardner et al. (2025)
its_live_regional_multiyear_vx	NSIDC-0776_RGI05A_2014-2022_V02.0.nc	Gardner et al. (2025)
cci_annual_vx	greenland_iv_250m_s1_20151001_20160930_v1_3.nc	Levinsen et al. (2015)
cci_helheim_series_vx	greenland_iv_250m_s1_20150610_20170321_helheim_gletsjer_v1_1.nc	Levinsen et al. (2015)
golive_vx	L8_233_013_016_2016_207_2016_223_v1.1.nc	Scambos et al. (2016)
Mouginot_annual_vx	vel_2016-07-01_2017-06-31.nc	Mouginot et al. (2019)
promice_v	IV_ROTR_20160728_20160819.nc	Solgaard and Kusk (2024)

Note: In GRIMP, each of the velocity products has two grid data sets, vx and vy. In the table, only the name of vx is displayed.

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