Machine-Precision Prediction of Low-Dimensional Chaotic Systems

Christof Schötz[1,2] and Niklas Boers[1,2,3]

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[2]Potsdam Institute for Climate Impact Research
[3]University of Exeter

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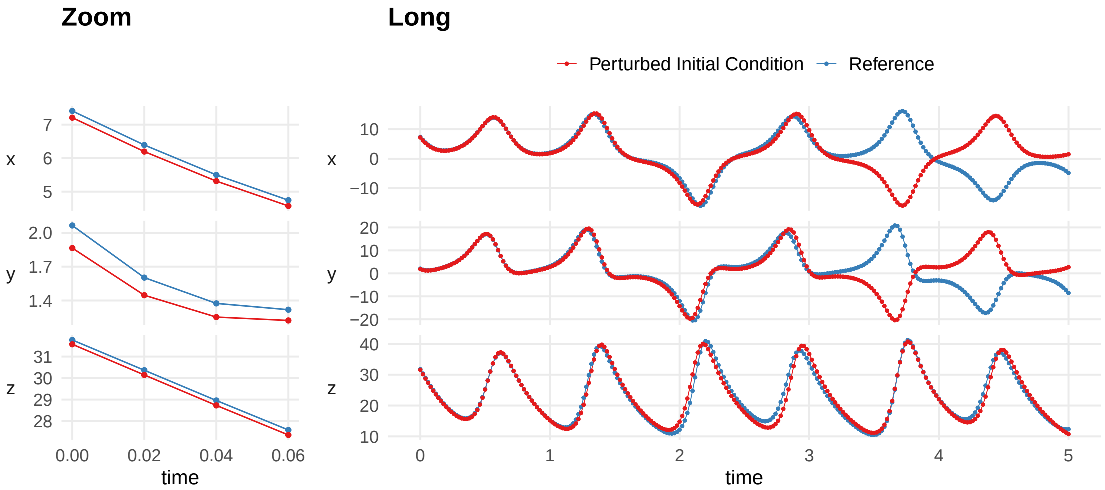
Sharing is encouraged


Zoom
Long
Perturbed Initial Condition
Reference
x
y
z
7
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5
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1.7
1.4
31
30
29
28
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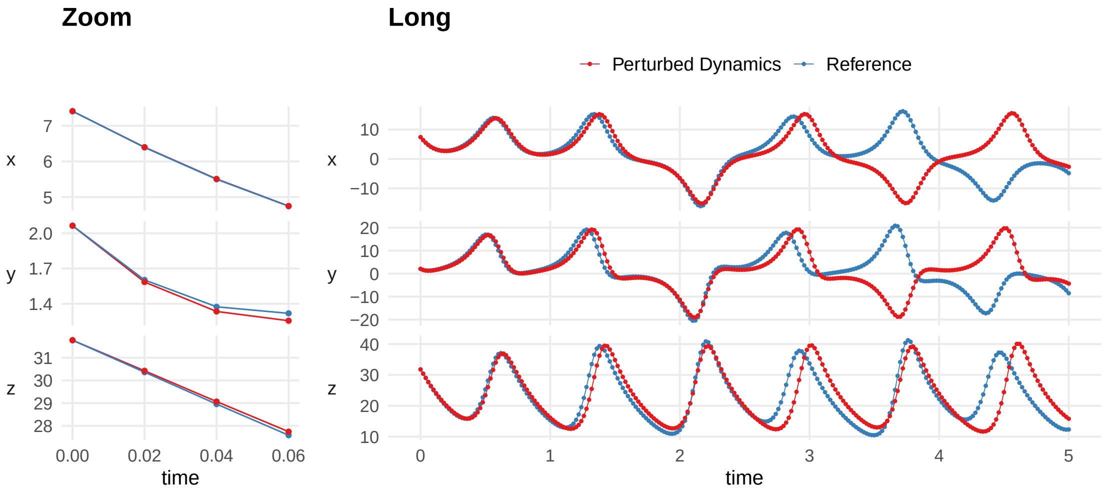
time


Zoom
Long
Perturbed Dynamics
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x
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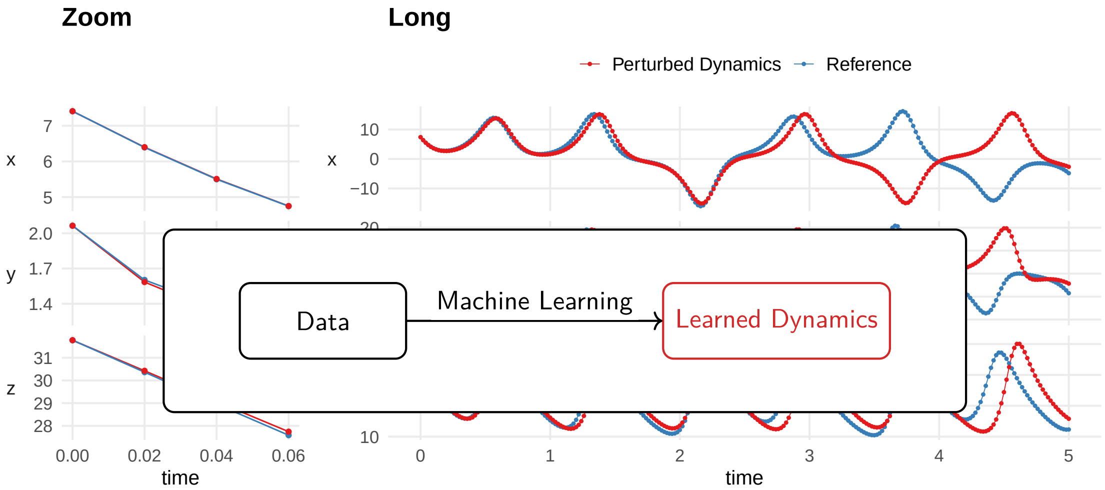
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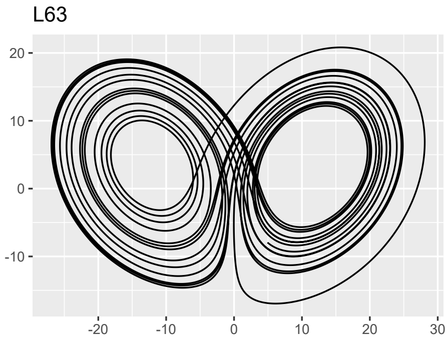
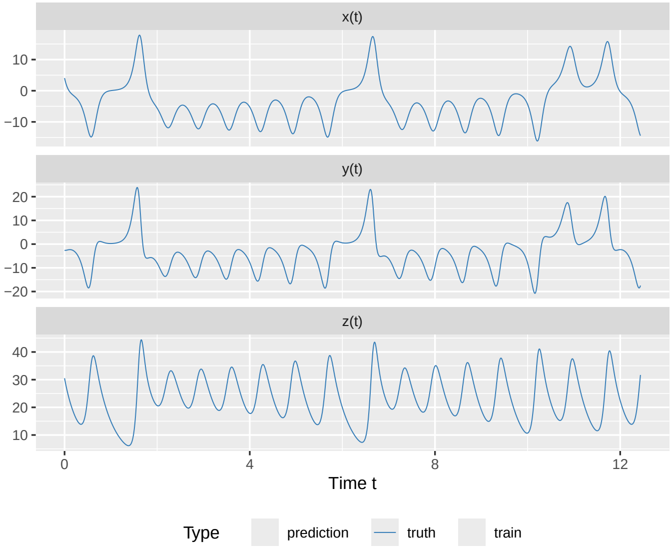
Data
Machine Learning
Learned Dynamics

$$\dot{x}(t) = 10(y(t) - x(t))$$
$$\dot{y}(t) = x(t)(28 - z(t)) - y(t)$$
$$\dot{z}(t) = x(t)y(t) - \frac{8}{3}z(t)$$





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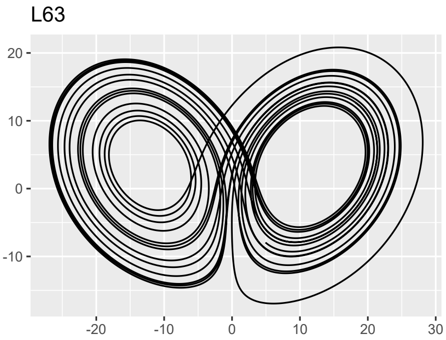

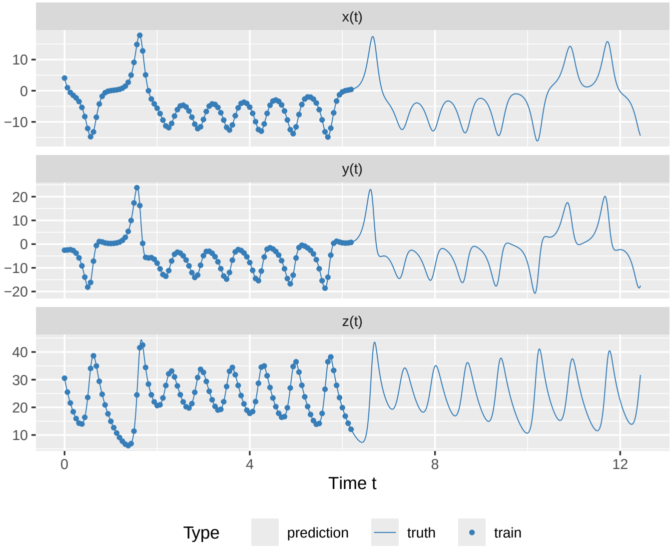

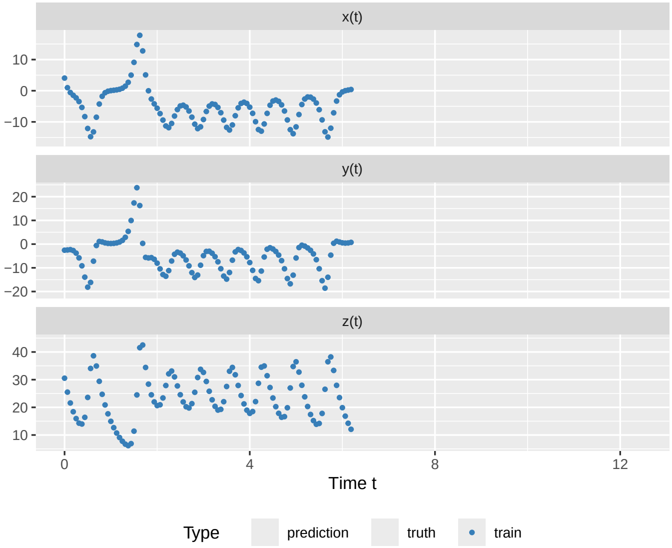
x(t)
10
0
−10
y(t)
20
10
0
−10
−20
z(t)
40
30
20
10
0
4
8
12
Time t
Type
prediction
truth
train

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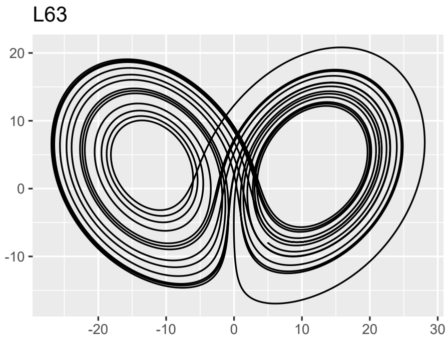

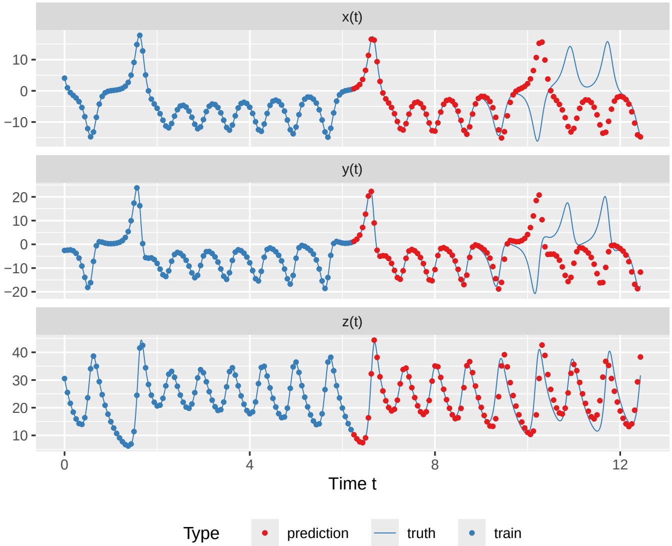

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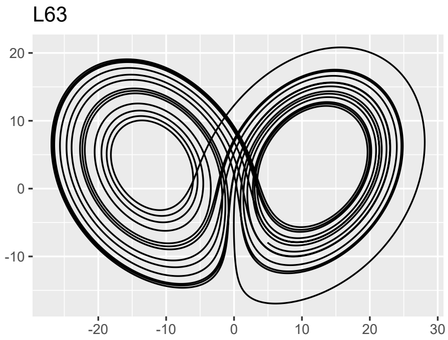
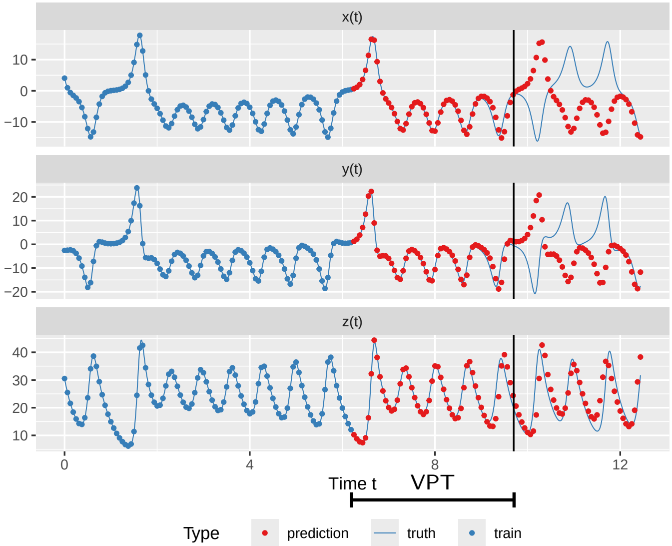
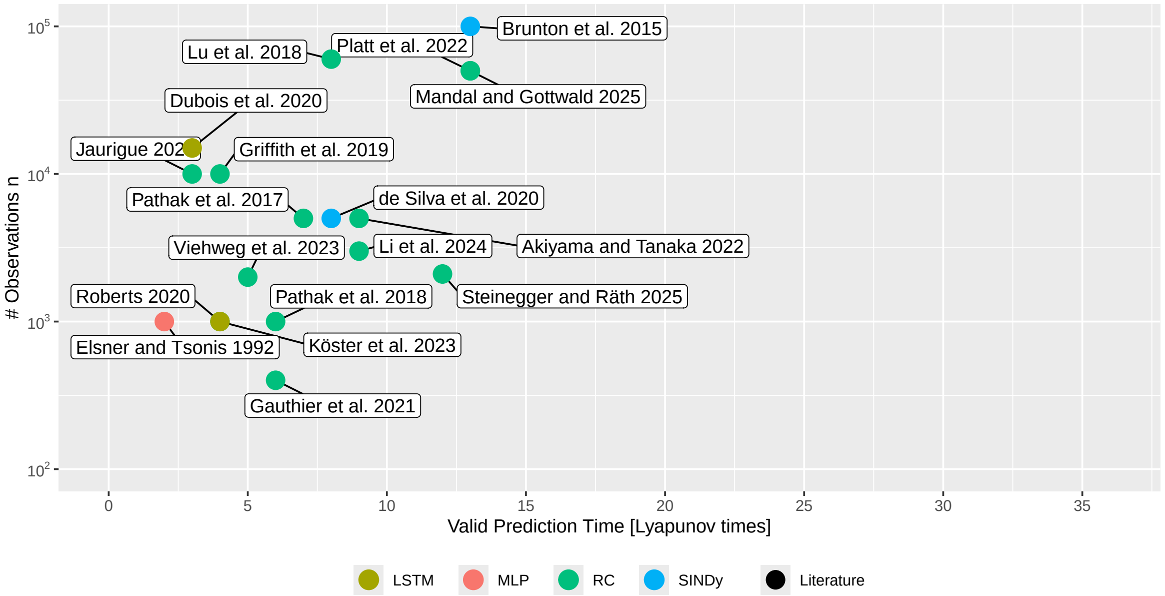




Literature



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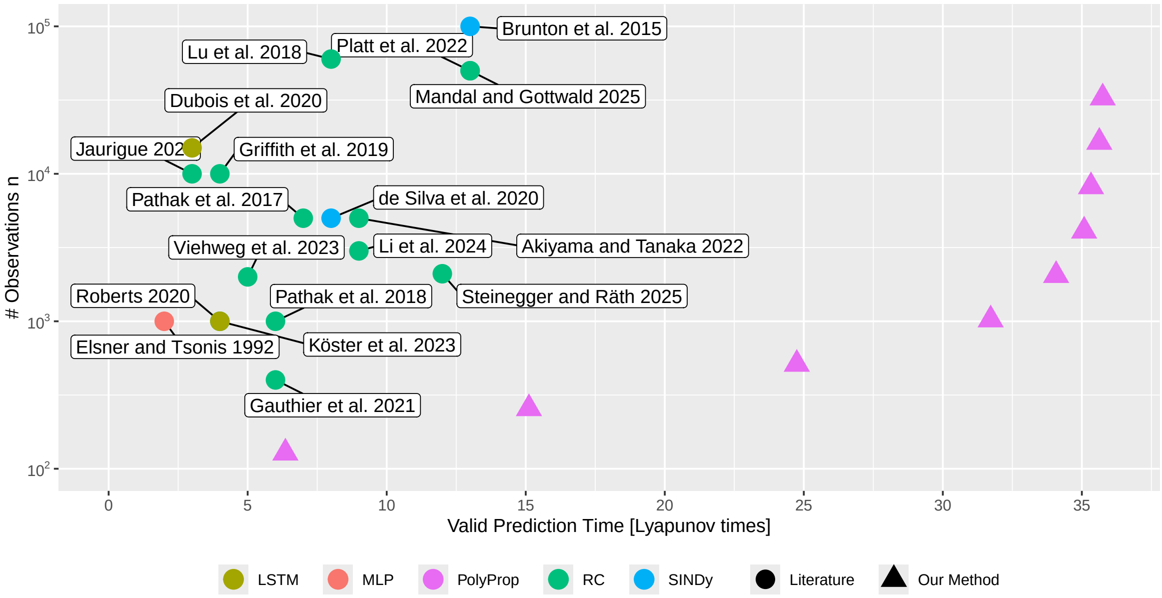
10^5
10^4
10^3
10^2
Observations n
0
5
10
15
20
25
30
35
Valid Prediction Time [Lyapunov times]
LSTM
MLP
RC
SINDy
Literature

Literature



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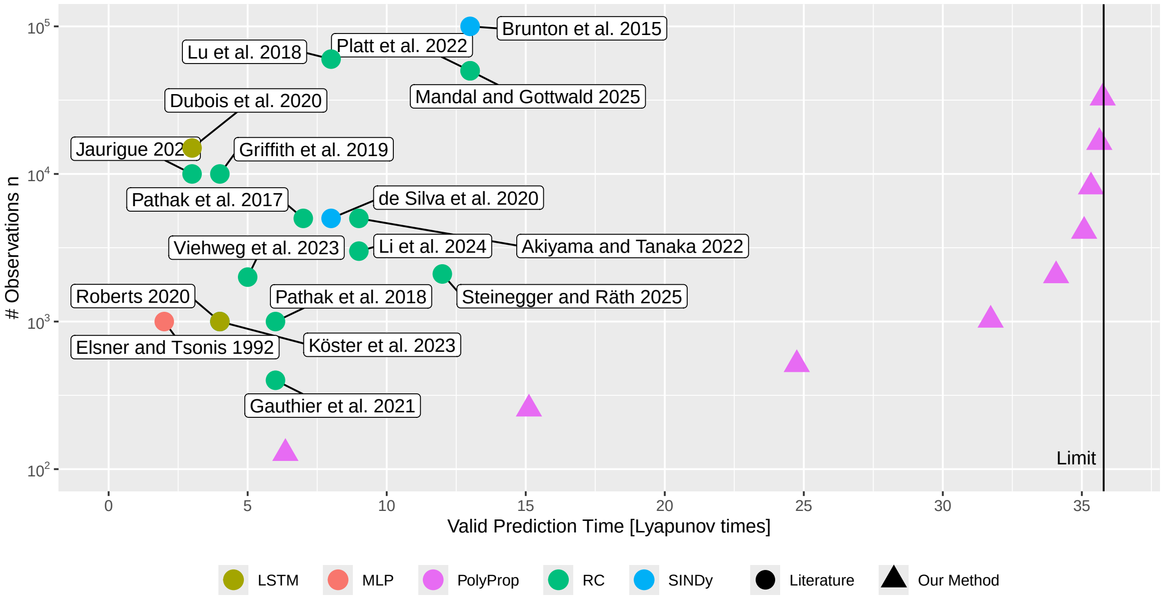

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Limit
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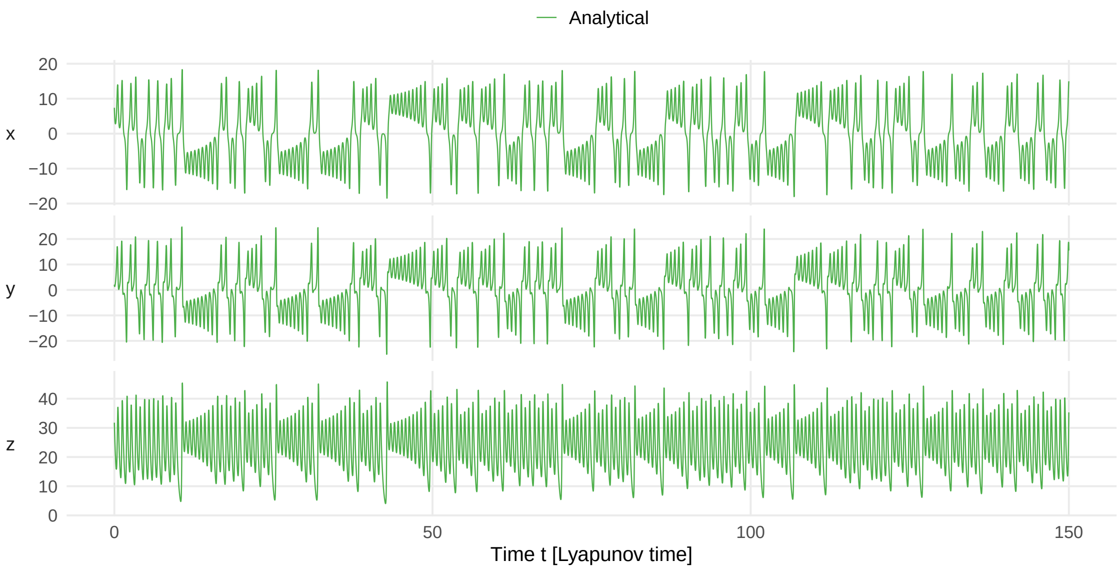
Lorenz 63 – Ground Truth and Machine Precision Limit



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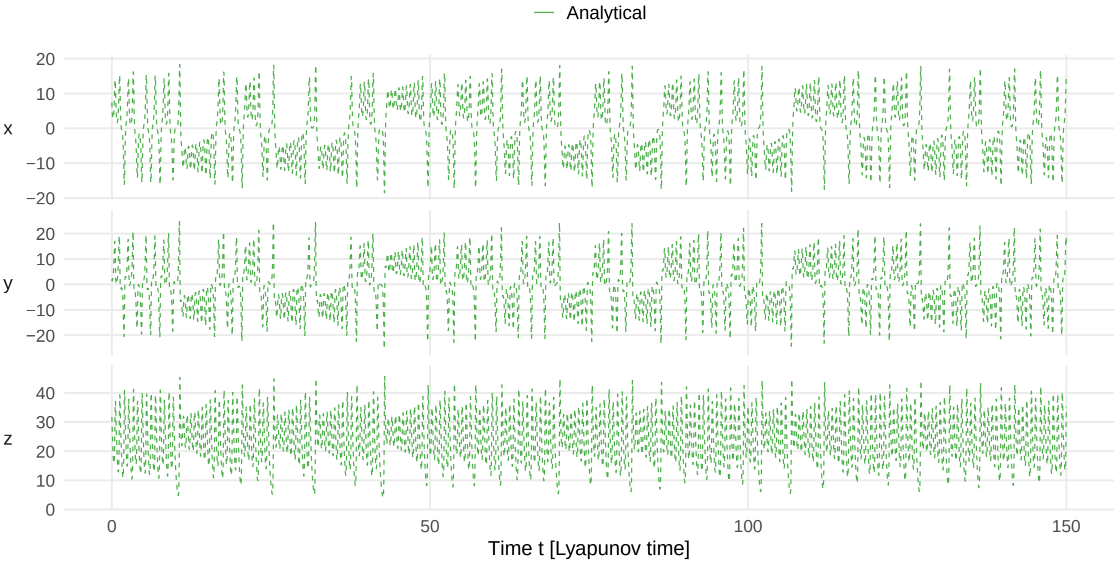

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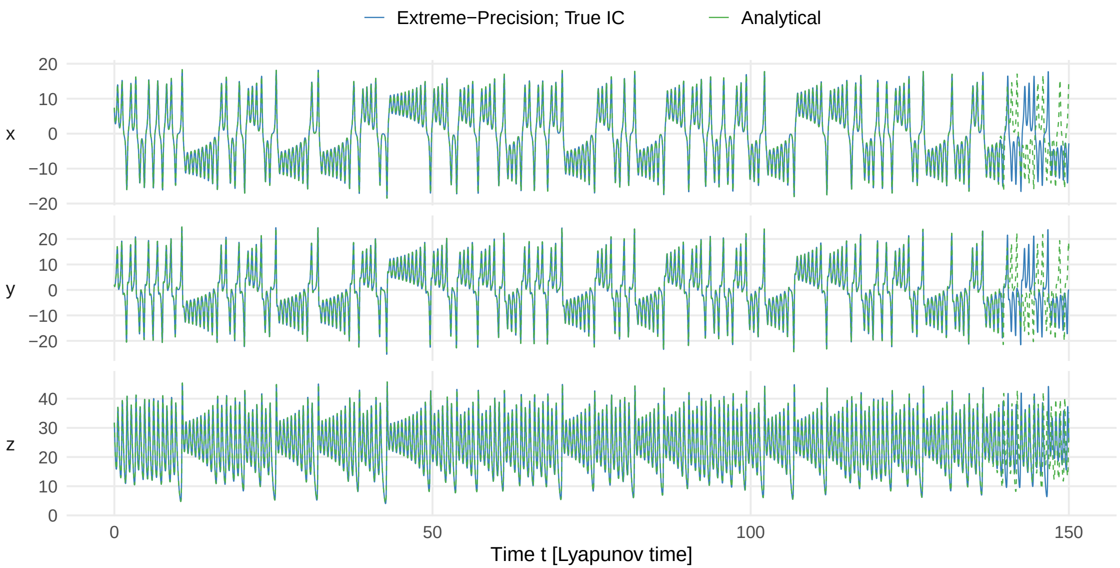

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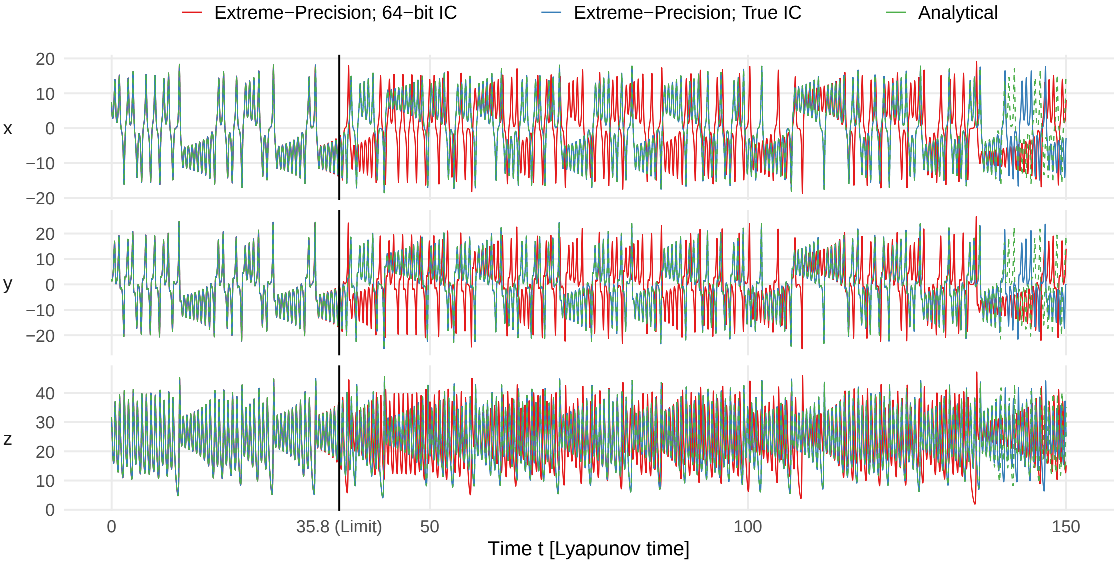

With Initial Condition Error equal to 64-bit Rounding Error:

	VPT [Lyapunov time]
Limit	35.8* $\approx -\ln(\mathtt{EPS}_{\mathtt{64bit}})$
PolyProp	35.6*
max precision RK45	31.6
standard RK4	21.6
previous best learned	13

* statistically indistinguishable

PolyProp does not have access to the ODE but to $n = 2^{14} \approx 16{,}000$ 64-bit observations at time step $\Delta t = 2^{-5} \approx 0.03$

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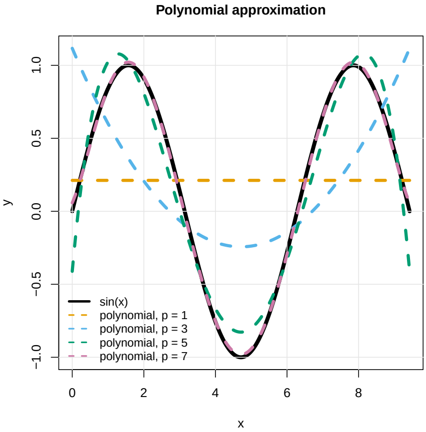

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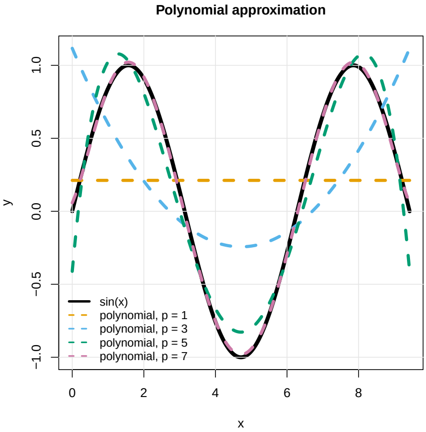

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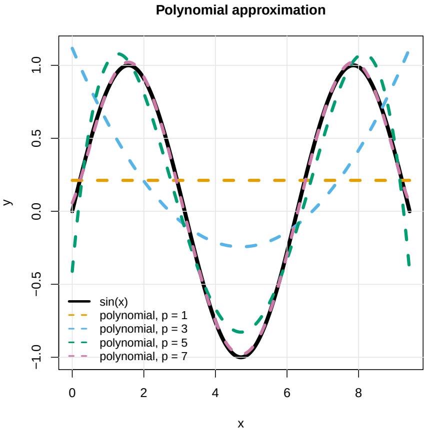
- OLS linear regression: $u(t + \Delta t) = \beta \cdot \mathrm{poly}_p(u(t))$



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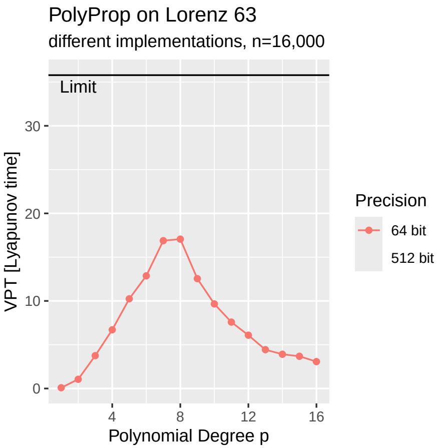

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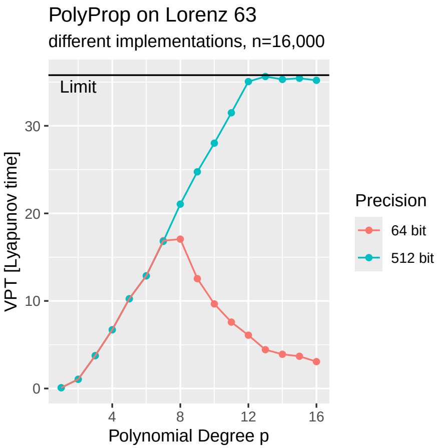
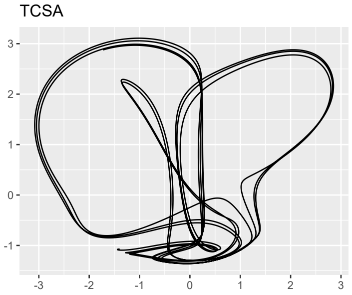
- Solution: Use **512-bit** arithmetic internally (observations stay **64-bit**)



Thomas Cyclically Symmetric Attractor

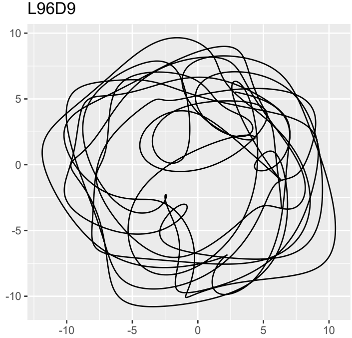
$$\dot{x} = \sin(y) - bx$$
$$\dot{y} = \sin(z) - by \qquad \text{with } b = 0.208$$
$$\dot{z} = \sin(x) - bz$$



Lorenz-96

$$\dot{x}_k = (x_{k+1} - x_{k-2})x_{k-1} - x_k + 8$$

with $x_{-1} = x_{d-1}$, $x_0 = x_d$, $x_{d+1} = x_1$.
Tested in dimensions $d \in \{5, \dots, 9\}$.



Limitations and Conclusion

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- State dimension: $d < 10$ ok
  $d > 100$ infeasible ($\to$ localization)
- Full state observation required
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Thank You!