中国海洋大学 OCEAN UNIVERSITY OF CHINA

未来海洋学院 Academy of Future Ocean

EGU26 GENERAL ASSEMBLY

The unified extreme value theory for characterizing changes in return periods and levels of *N*-year temperature anomalies

Presenter: Wei Ban

Supervisor: Prof. Jianping Li

Ocean University of China, Qingdao, China

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Return period (Once in *N* years)

An estimate of the average time interval between occurrences of an event (e.g., flood or extreme rainfall) of (or below/above) a defined size or intensity.

Generalized extreme value distribution (GEVD)

- **For maxima (GEVD)**

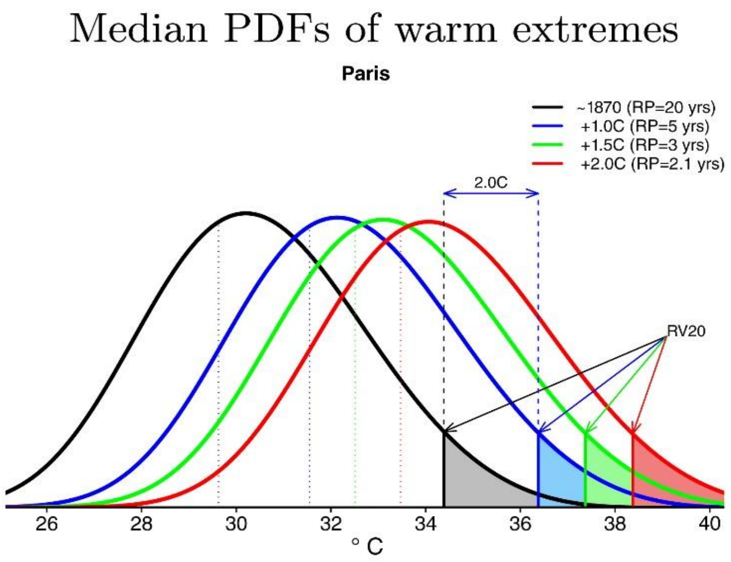
$$G_M(x) = \begin{cases} \exp\left\{-\exp\left[-\left(\frac{x-\mu}{\sigma}\right)\right]\right\}, \xi = 0 \\ \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}, \xi \neq 0, 1+\xi\left(\frac{x-\mu}{\sigma}\right) > 0 \end{cases}$$

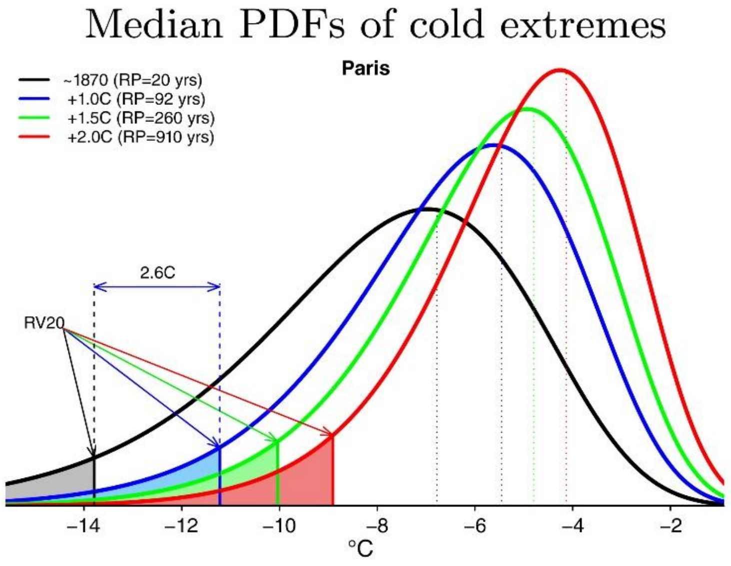
where μ, σ, and ξ are the location, scale, and shape parameters, respectively.

- **For minima (negative GEVD)**

$$G_m(x) = \begin{cases} 1-\exp\left[-\exp\left(\frac{x-\tilde{\mu}}{\sigma}\right)\right], \xi = 0 \\ 1-\exp\left\{-\left[1-\xi\left(\frac{x-\tilde{\mu}}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}, \xi \neq 0, 1-\xi\left(\frac{x-\tilde{\mu}}{\sigma}\right) > 0 \end{cases}$$

where $\tilde{\mu} = -\mu$, σ, and ξ are the location, scale, and shape parameters, respectively.



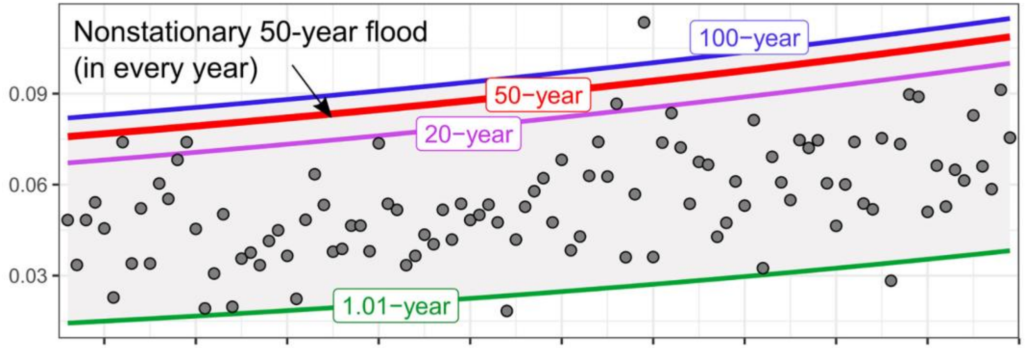


(Kharin et al., 2018)

1. Introduction: Previous studies

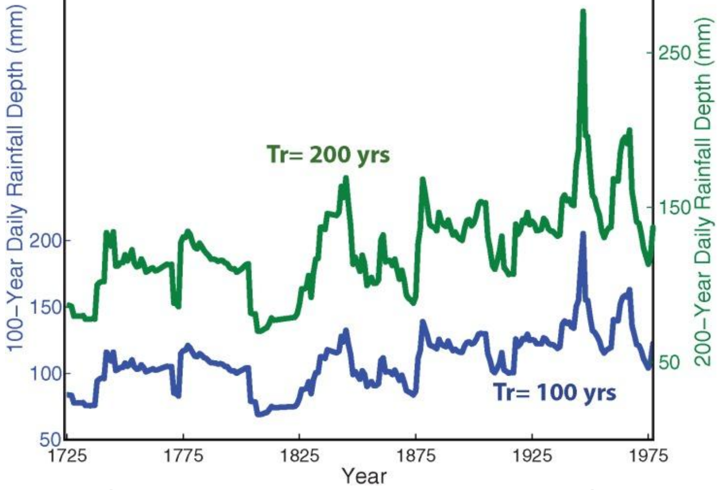
Non-stationary climate

- Incorporate time-dependent parameters or climate covariates into distributions



(Slater et al., 2021)

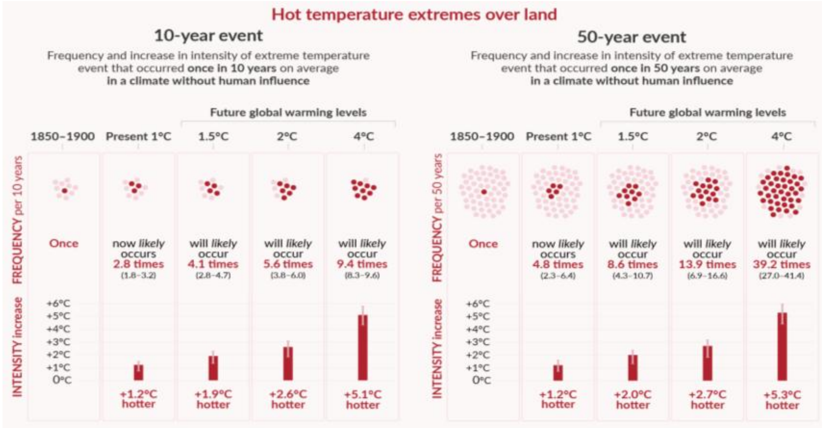
- Use a moving window to study changes in extremes



(Marani and Zanetti, 2015)

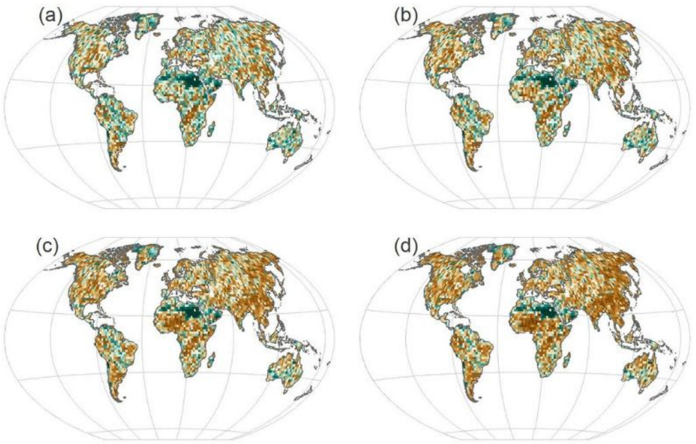
Climate model simulations

- Projected changes at different global warming levels



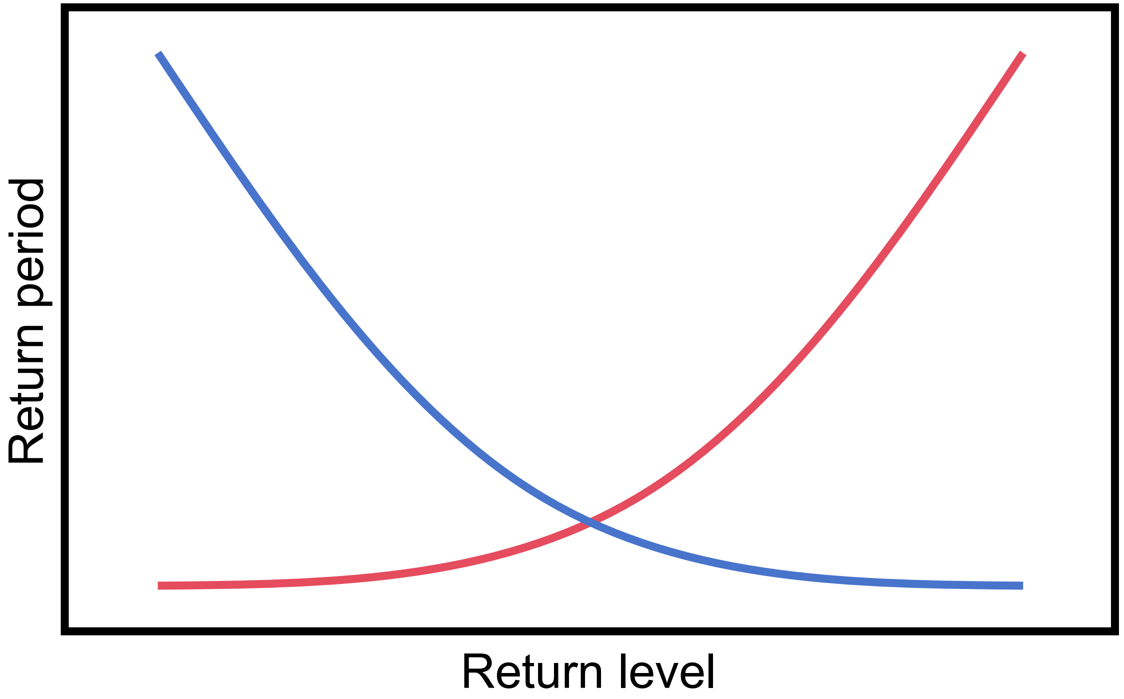
(IPCC AR6, 2021)

- Projected changes across two time periods



(Abdelmoaty and Papalexiou, 2023)

1. Introduction: Scientific questions



- Both extreme positive and negative events should have long return periods.
- Few studies proposed distributions for **both the upper and lower tails** (Naveau et al., 2016; Stein, 2021; de Carvalho et al., 2022).

- ☐ **Is there a method to simultaneously study the relationship between the return periods and return levels of positive and negative events in an anomalous series? What are the advantages compared to traditional approaches?**
- ☐ **Can this method be applied to study observed changes in the intensity and frequency of *N*-year temperature anomalies and enable prediction?**

2. Data and method

Data

Variables	Dataset	Land Component	SST Component	Resolution	Period	Reference Period
Surface Temperature	HadCRUT5	CRUTEM5	HadSST4	5°×5° Monthly	1850–2024	1961–1990
	Berkeley Earth	Berkeley	HadSST4	1°×1° Monthly	1850–2024	1951–1980
	NOAAGlobalTemp	GHCNv4	ERSSTv5	5°×5° Monthly	1850–2024	1971–2000
	GISTEMP	GHCNv4	ERSSTv5	2°×2° Monthly	1880–2024	1951–1980

Method

Extreme value theory (EVT): GEVD

Parameter estimation: maximum likelihood estimation (MLE)

Goodness-of-fit: MaxAE, MAE, RMSE

Return periods and levels

An anomalous series $\boldsymbol{X} = \{x_1, x_2, \cdots, x_n\} \rightarrow$ **positive and negative events**

The **probabilities** of positive and negative events:

Negative events: $P(\boldsymbol{X} \leq x) = F(x),\ x \leq 0,$

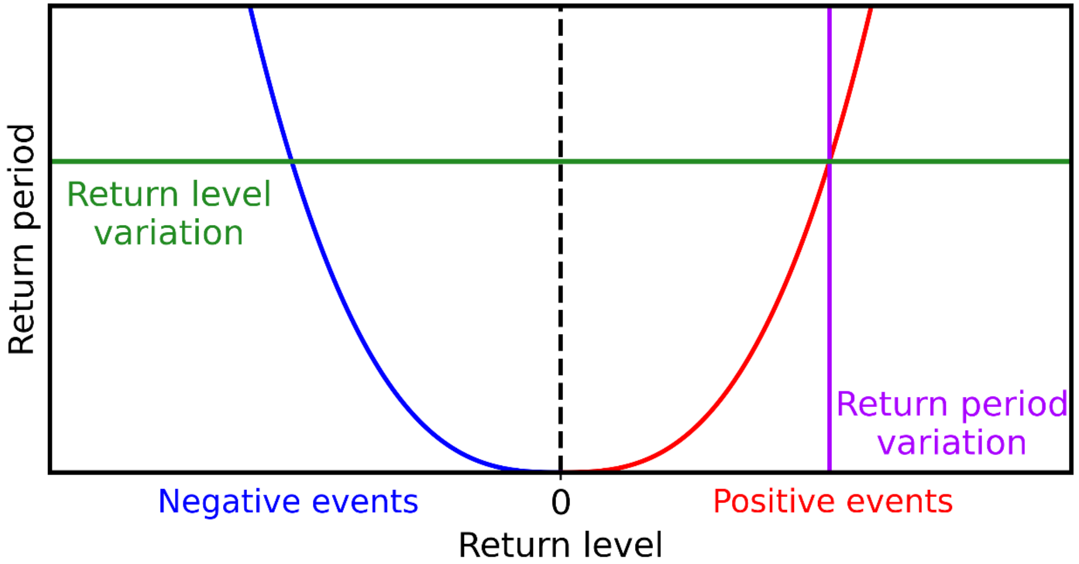
Positive events: $P(\boldsymbol{X} \geq x) = 1 - F(x),\ x > 0,$

The return period can be expressed as the reciprocal of a probability.

The **return periods** of positive and negative events:

$$RP(x) = \begin{cases} \dfrac{1}{P(\boldsymbol{X} \geq x)}, x > 0 \\ \dfrac{1}{P(\boldsymbol{X} \leq x)}, x \leq 0 \end{cases}$$

where $RP(x)$ and x respectively represent the return period and return level.



Return periods as a function of return levels of anomalous events.

Return level variation at a fixed return period.

Return period variation at a fixed return level.

Intensity and frequency

Frequency: once in *N* years

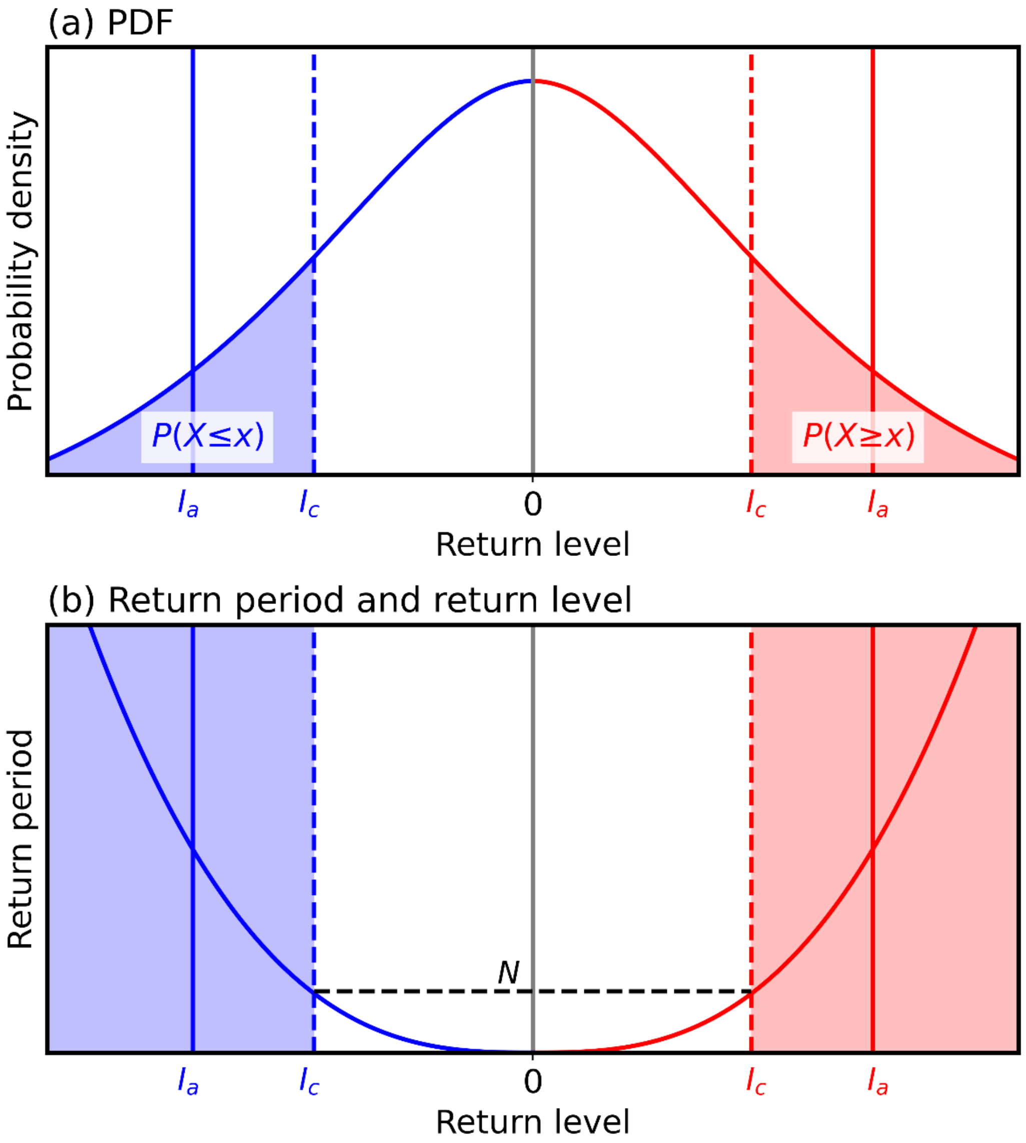
Intensity: $\boldsymbol{X} \geq x$ for positive events; $\boldsymbol{X} \leq x$ for negative events

Critical intensity (I_c) and average intensity (I_a):

$$I_c = x$$

$$I_a = \begin{cases} \frac{1}{n_+} \sum_{i:\, x_i \geq x} x_i \,, x > 0 \\ \frac{1}{n_-} \sum_{i:\, x_i \leq x} x_i \,, x \leq 0 \end{cases}$$

where x denotes the return level for *N*-year anomalous events, and n_+ and n_- represent the counts of positive events ($x_i \geq x$) and negative events ($x_i \leq x$), respectively.

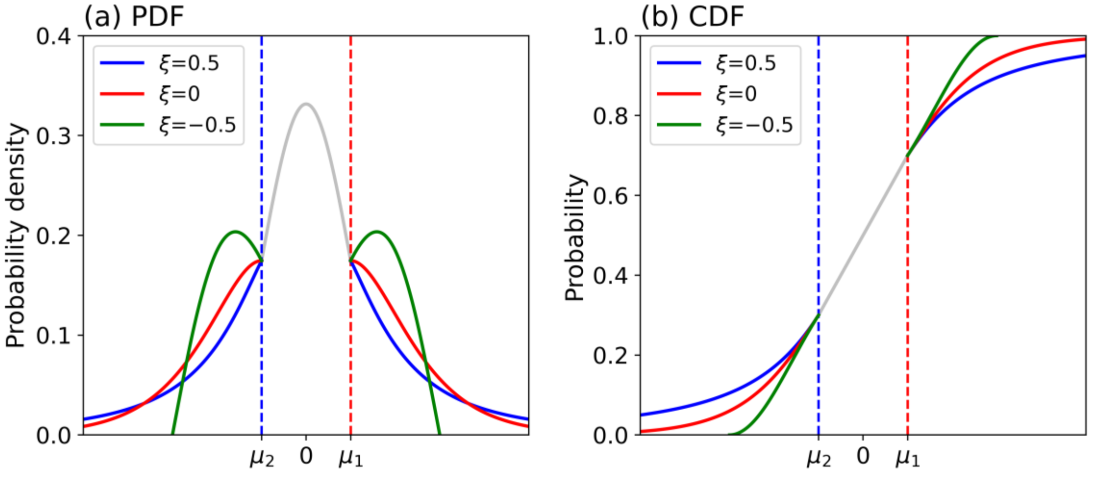


- **Interval extreme value distribution (IEVD)**

$$G(x)=\begin{cases}\begin{cases}\left\{\exp\left[-\exp\left(-\frac{x-\mu_1}{\sigma}\right)+1\right]-1\right\}\frac{1-k_1}{e-1}+k_1, \xi=0\\ \left\{\exp\left[-\left(1+\xi\frac{x-\mu_1}{\sigma}\right)^{-1/\xi}+1\right]-1\right\}\frac{1-k_1}{e-1}+k_1, \xi\neq 0, 1+\xi\left(\frac{x-\mu_1}{\sigma}\right)>0\end{cases}, x\geq\mu_1>0\\ \begin{cases}\left\{1-\exp\left[-\exp\left(\frac{x-\mu_2}{\sigma}\right)\right]\right\}\frac{ek_2}{e-1}, \xi=0\\ \left\{1-\exp\left[-\left(1-\xi\frac{x-\mu_2}{\sigma}\right)^{-1/\xi}\right]\right\}\frac{ek_2}{e-1}, \xi\neq 0, 1-\xi\left(\frac{x-\mu_2}{\sigma}\right)>0\end{cases}, x\leq\mu_2\leq 0\end{cases}$$

μ_1, μ_2: local parameters; σ: scale parameter; ξ: shape parameter; k_1, k_2: probability parameters

$$k_1=P(\boldsymbol{X}<\mu_1), k_2=P(\boldsymbol{X}\leq\mu_2)$$



Based on the traditional GEVD and negative GEVD, we construct a statistical model consistent with the UEVT.

3.2 Statistical model

- **Return periods and levels**

Positive events:

$$x_{N+} = \begin{cases} \mu_1 - \sigma \ln\left\{1 - \ln\left[\left(1 - \frac{1}{N} - k_1\right)\frac{\mathrm{e}-1}{1-k_1} + 1\right]\right\}, \xi = 0 \\ \mu_1 + \frac{\sigma}{\xi}\left\{\left[1 - \ln\left(\left(1 - \frac{1}{N} - k_1\right)\frac{\mathrm{e}-1}{1-k_1} + 1\right)\right]^{-\xi} - 1\right\}, \xi \neq 0 \end{cases}$$

where x_{N+} and N respectively represent the return level and return period of positive events.

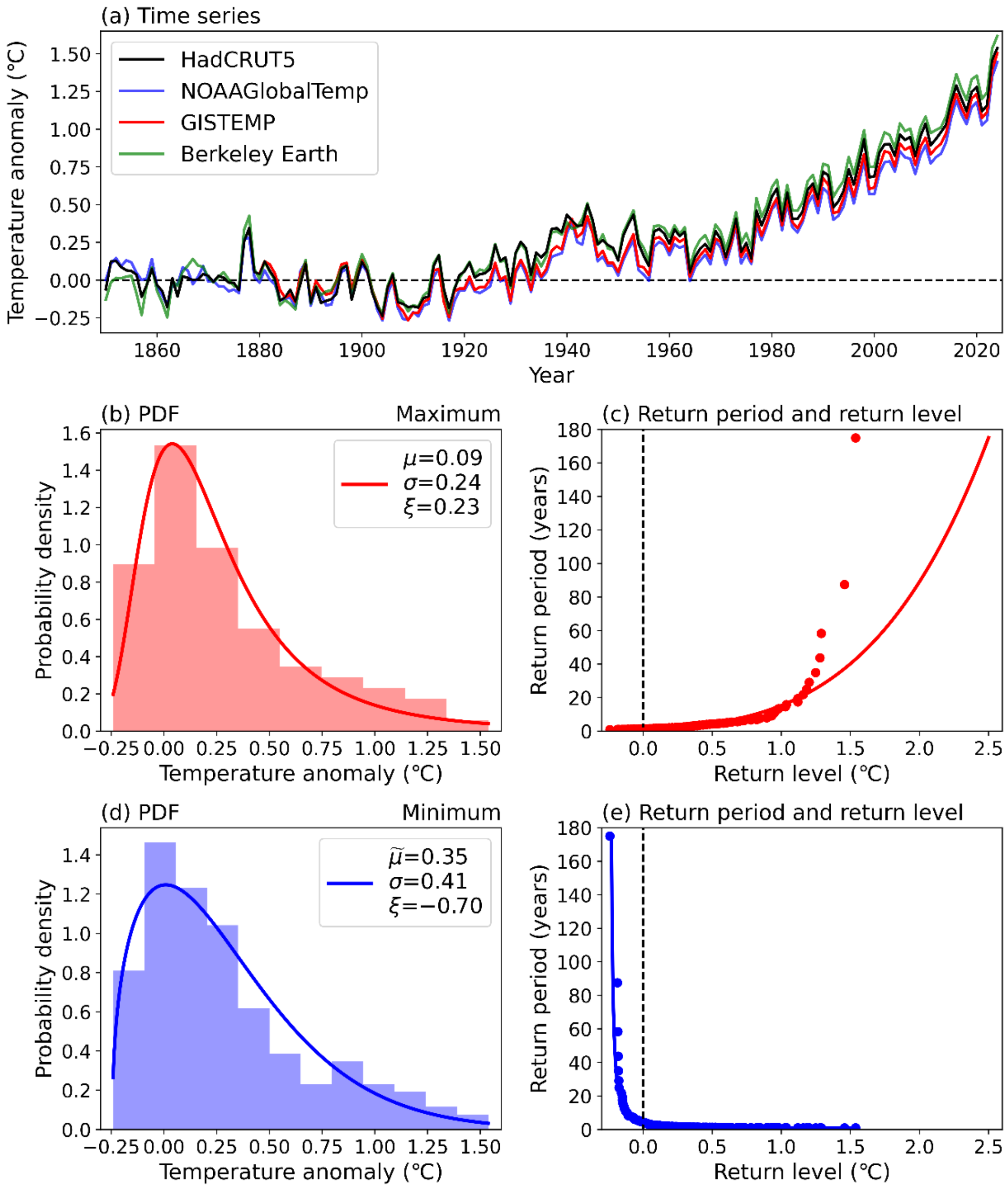
Negative events:

$$x_{N-} = \begin{cases} \mu_2 + \sigma \ln\left[-\ln\left(1 - \frac{e-1}{ek_2N}\right)\right], \xi = 0 \\ \mu_2 + \frac{\sigma}{\xi}\left\{1 - \left[-\ln\left(1 - \frac{e-1}{ek_2N}\right)\right]^{-\xi}\right\}, \xi \neq 0 \end{cases}$$

where x_{N-} and N respectively represent the return level and return period of negative events.

Based on the IEVD, the return level for a given return period N is obtained by inverting the CDF.

3.3 Comparative analysis of the traditional EVT and UEVT

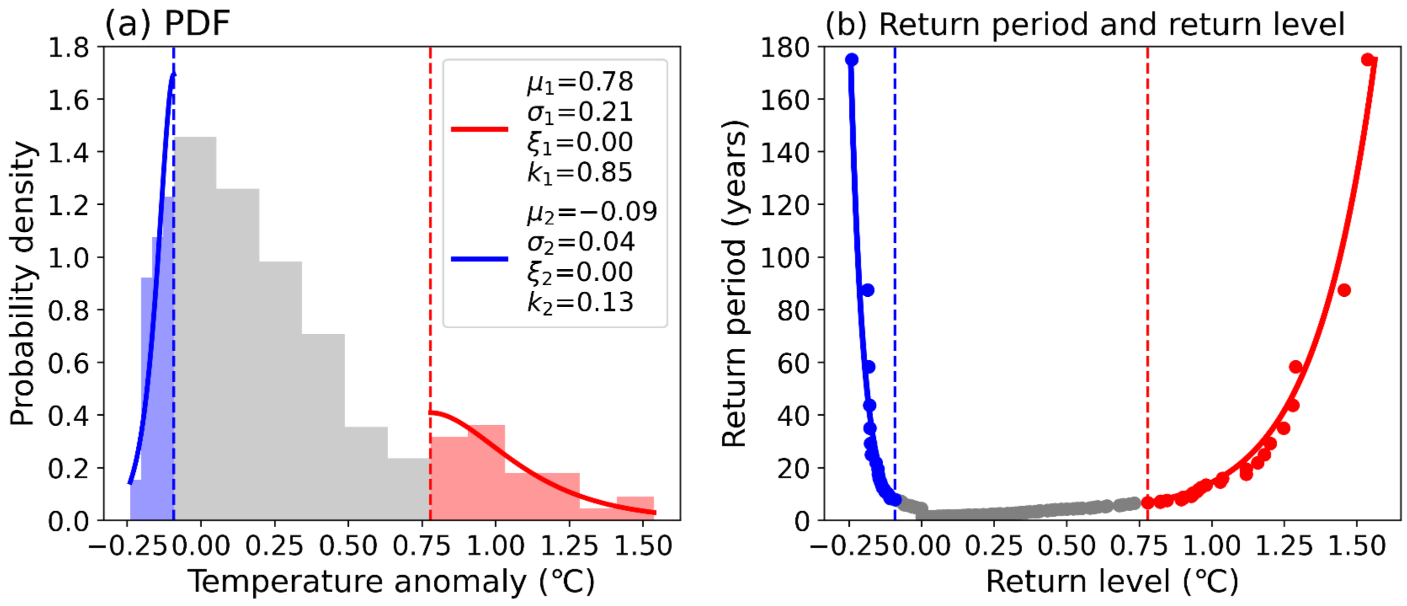


The traditional EVT

- Analyze events satisfying $\boldsymbol{X} \geq x$ (apply the **GEVD** to the temperature anomaly series):
  Warm extremes → long return periods (rare)
  Cold extremes → ~1-year return period (near-annual)
- Analyze events satisfying $\boldsymbol{X} \leq x$ (apply the **negative GEVD** to the temperature anomaly series):
  Cold extremes → long return periods
  Warm extremes → ~1-year return period

Both warm and cold extremes should correspond to **long return periods.**

3.3 Comparative analysis of the traditional EVT and UEVT



The UEVT

Both warm and cold extremes →

low probabilities & long return periods

- **Unified framework for both tails**
- **Flexible asymmetric distribution**
- **Accurate return period & return level estimates**

Warm anomalies

	MaxAE	MAE	RMSE
IEVD	19.6	3.7	6.3
GEVD	132.1	11.4	28.8

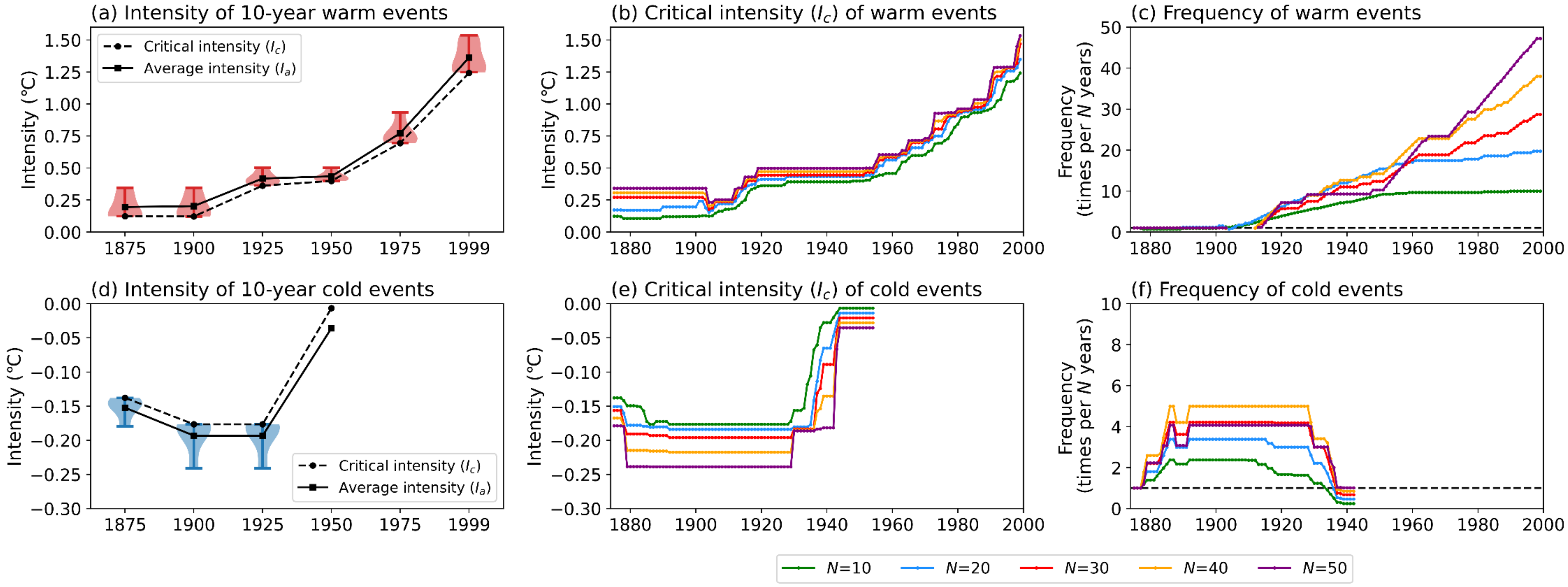
Cold anomalies

	MaxAE	MAE	RMSE
IEVD	40.6	6.4	10.6
GEVD	1444.1	72.8	308.3

IEVD provides significantly enhanced fitting capability for anomalous events

4.1 Applications of the UEVT: Observed changes

Temporal evolution of *N*-year temperature anomalies



- ***N*-year warm events: more frequent and intense**

Reference period: once per *N* years → Recent decades: ***N* times per *N* years**

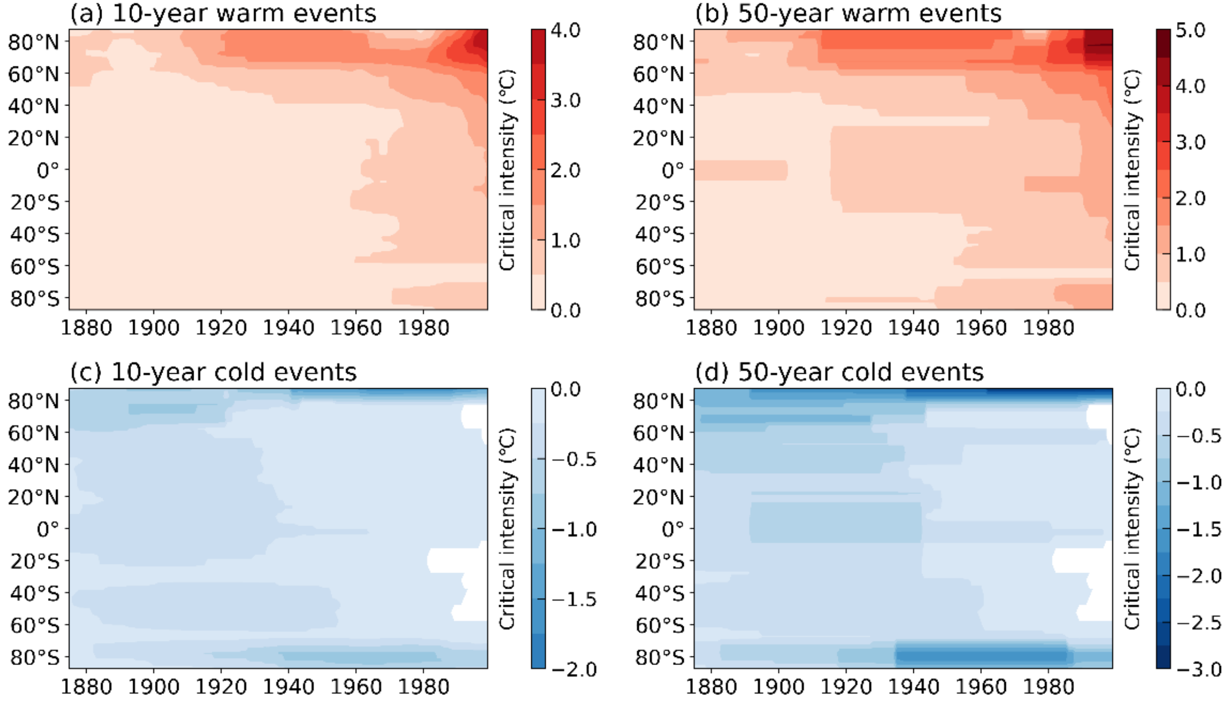
- ***N*-year cold events: less frequent and weaker**

initial increase → plateau → decline → disappeared

Influences of global warming and internal variability

4.1 Applications of the UEVT: Observed changes

Change in the I_c of zonally averaged temperature anomalies



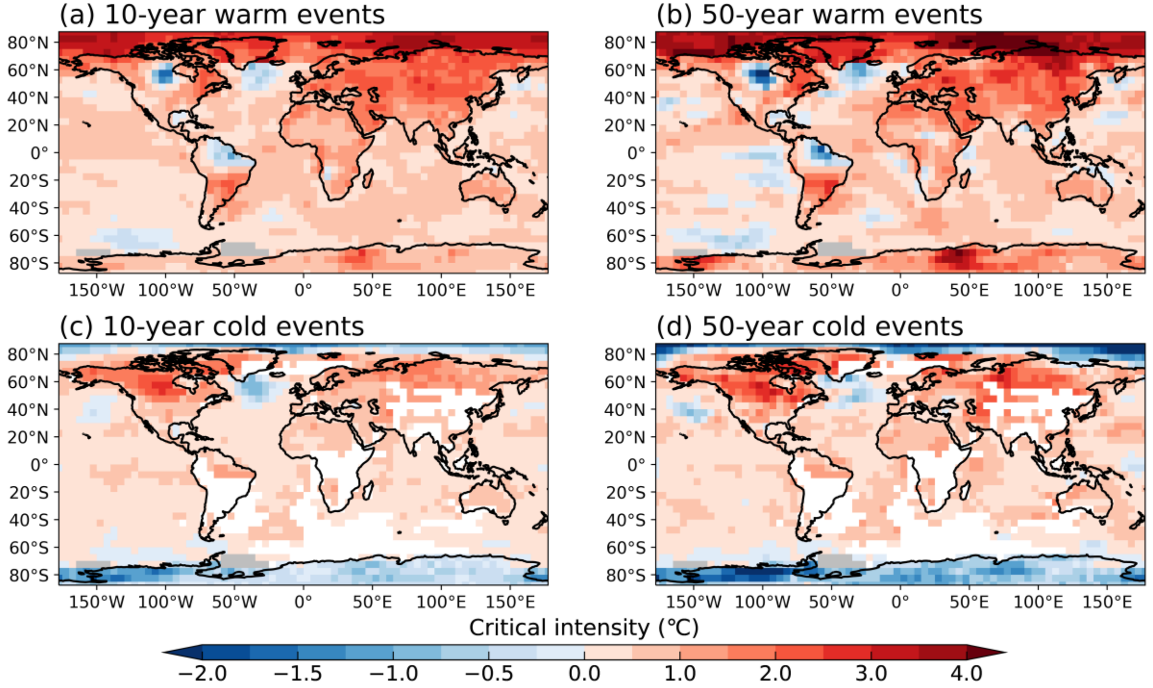
- **Intensifying warm events** (Arctic Amplification)

Expansion: Northern high latitudes → tropics and Southern Hemisphere

Accelerated tropical warming

- **Weakening cold events** (except polar regions)

Spatial distribution of I_c changes in temperature anomalies between 1974–2024 and 1850–1900



- Overall:

Intensifying warm events**, Weakening** cold events

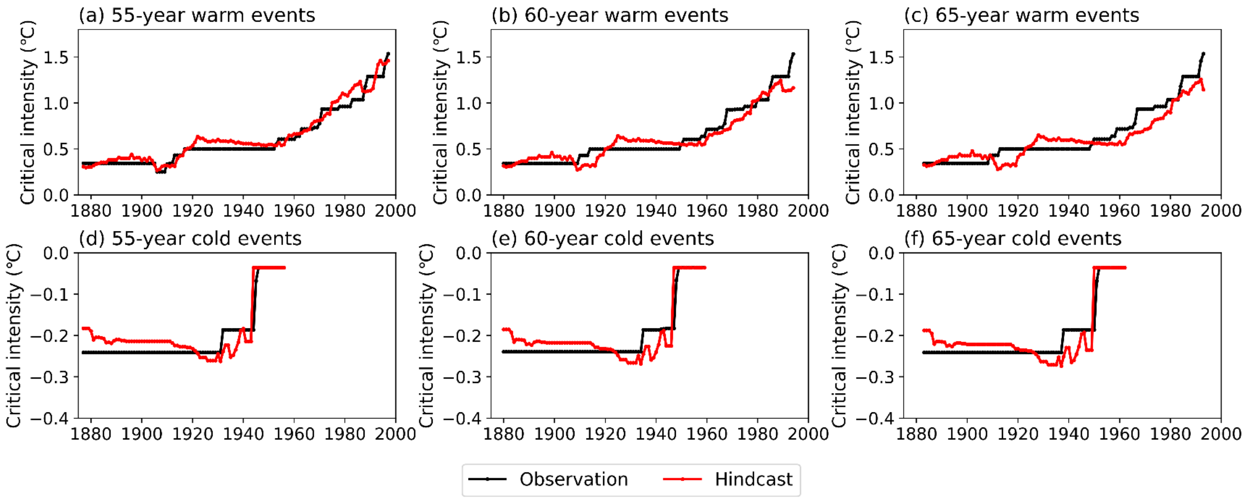
Hemispheric asymmetry and **land-ocean differences**

- Regional anomalies:

Spatial heterogeneity under global climate change

4.2 Applications of the UEVT: Hindcast results

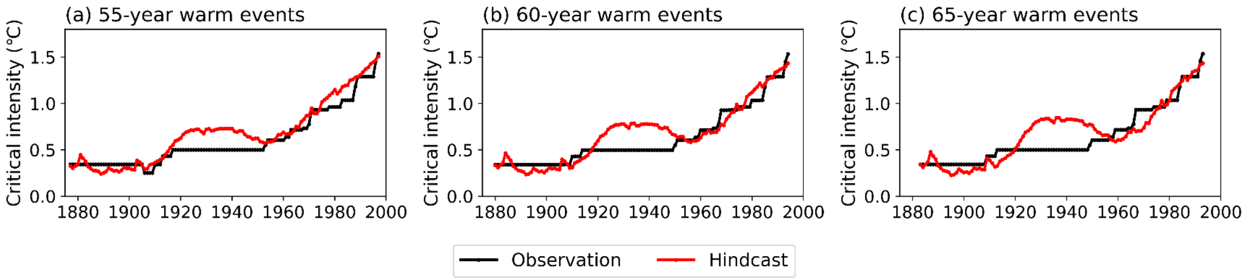
● Direct Hindcast



Hindcast successfully captures observed intensity trends for 55-, 60-, and 65-year temperature anomalies

- ➢ **Warm extremes:** delayed intensification for longer return periods
- ➢ **Cold extremes:** early intensification trend within the 51-year window

● Trend + Variability Hindcast



- ☐ **Superior performance** for strong positive trend events
- ☐ Captures **accelerated warming** effects
- ☐ Provides a **novel perspective** for extreme event prediction

$$y(t_i) = y_{trend}(t_i) + y^*(t_i), i = 1, 2, \cdots, n$$
$$y_{trend}(t_i) = at_i + b$$

$$RL(\tau_i) = RL_{trend}(\tau_i) + RL^*(\tau_i), i = 1, 2, \cdots, n - m$$
$$RL_{trend}(\tau_i) = a(t_i + N) + b$$

5. Conclusion

- Novel framework

UEVT and IEVD offer a superior, flexible, and accurate framework for analyzing *N*-year temperature anomalies.

  - UEVT: Simultaneously analyzes **return periods and return levels** of positive and negative events from the same anomaly series; **long return periods** for both types of extremes.
  - IEVD: Predicts temporal evolution of intensity and frequency for the events with longer return periods; **better goodness-of-fit** than traditional GEVD.

- Applications

Changes observed across **global, zonal-mean, and most regions** (the dominant role of **global warming**):

  - *N*-year warm events: more frequent and intense; *N*-year cold events: less frequent and weaker.
  - Some regions show weaker warm events or intensified cold events due to internal climate variability.

Hindcast: general consistency with observed changes.

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Thanks for your attention！

Email: bw@stu.ouc.edu.cn