

1 Introduction

- Dispersion of solute particles in an ice-covered turbulent flow.
- Depletion of solute through the bed layer.
- How is the solute concentration affected?
- Interaction of turbulence, ice cover, and bed processes governs transport.

2 Problem formulation

$$u(z) \frac{\partial C}{\partial x} = \frac{\partial}{\partial z} \left(K_z \frac{\partial C}{\partial z} \right),$$

$$K_z \frac{\partial C}{\partial z} \Big|_{z=z_b} = \beta C(x, z_b), \quad K_z \frac{\partial C}{\partial z} \Big|_{z=z_i} = 0,$$

$$u(z)C(0, z) = \frac{1}{2} (\delta(z - z_0) + \delta(z - z_1))$$

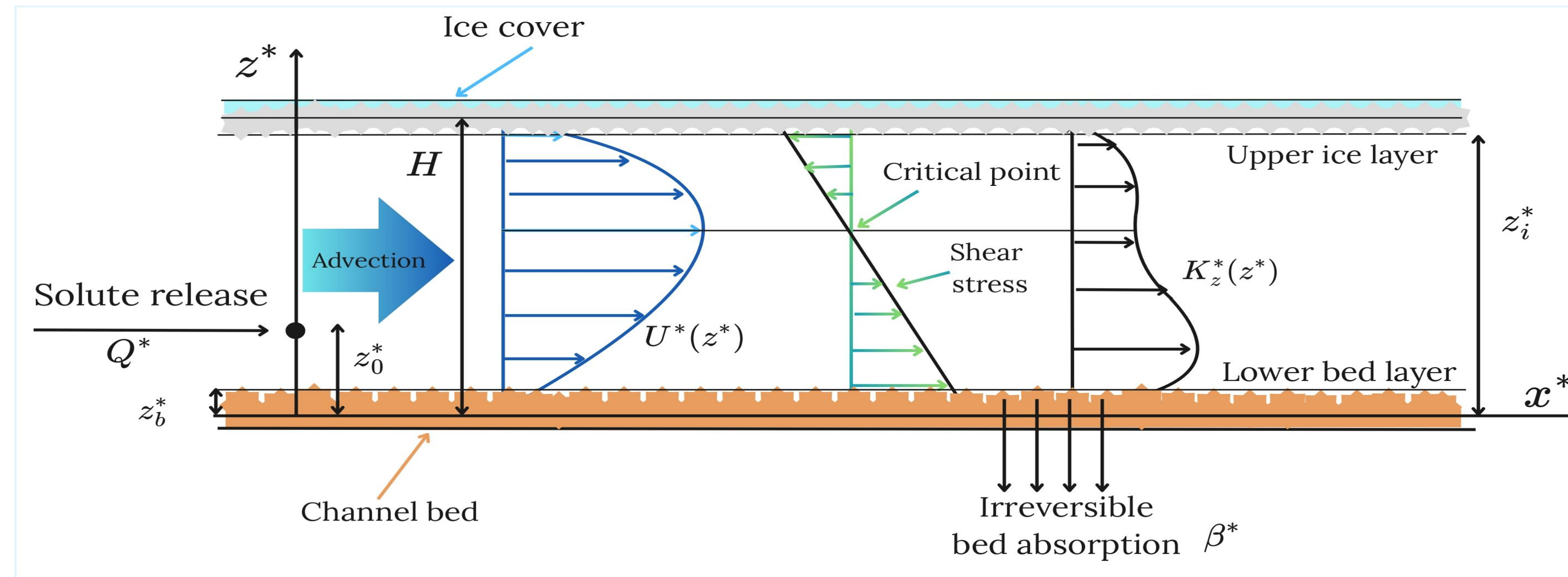
$C(x, z)$: Solute concentration,
 K_z : Turbulent diffusivity
 β : Bed absorption rate

Advection-diffusion with variable diffusivity
Bed acts as a reactive boundary (sink term via β)

3 Methodology

- Crank Nicolson implicit scheme-unconditionally stable.
- 20000 horizontal grid points and 500 vertical grid points.
- Discretized with second order accuracy.
- Resulting system solved using Thomas algorithm- efficient for solving large tridiagonal systems.

4 Schematic



5 Velocity and Diffusivity profile

The dimensional asymmetric velocity and diffusivity profiles are given by

$$U^*(z^*) = K_0 \left(\frac{z^*}{H} \right)^{\frac{1}{m_b}} \left(1 - \frac{z^*}{H} \right)^{\frac{1}{m_i}}$$

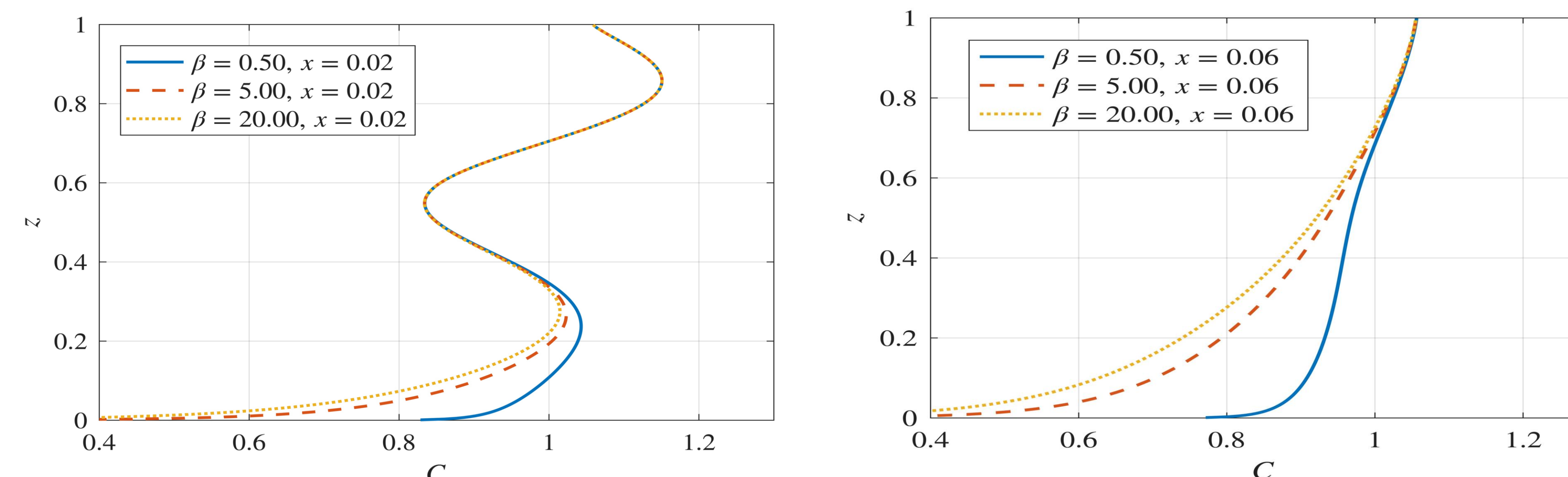
$$K_z^*(z^*) = 2\kappa u_b^* \beta z^* (1 - z^*/H) \left[1 + \alpha \left(\frac{z^*}{HZ_0^*} - 1 \right)^2 \right]$$

where $u(z) = \frac{U^*(z^*)}{U^*(z^*)}$, $K_z = \frac{K_z^*}{K_z^*}$, $z = \frac{z^*}{H}$

- m_b and m_i are friction factors associated with bed and ice cover respectively.
- α and β depend on m_b , m_i and κ is the von Kármán constant.

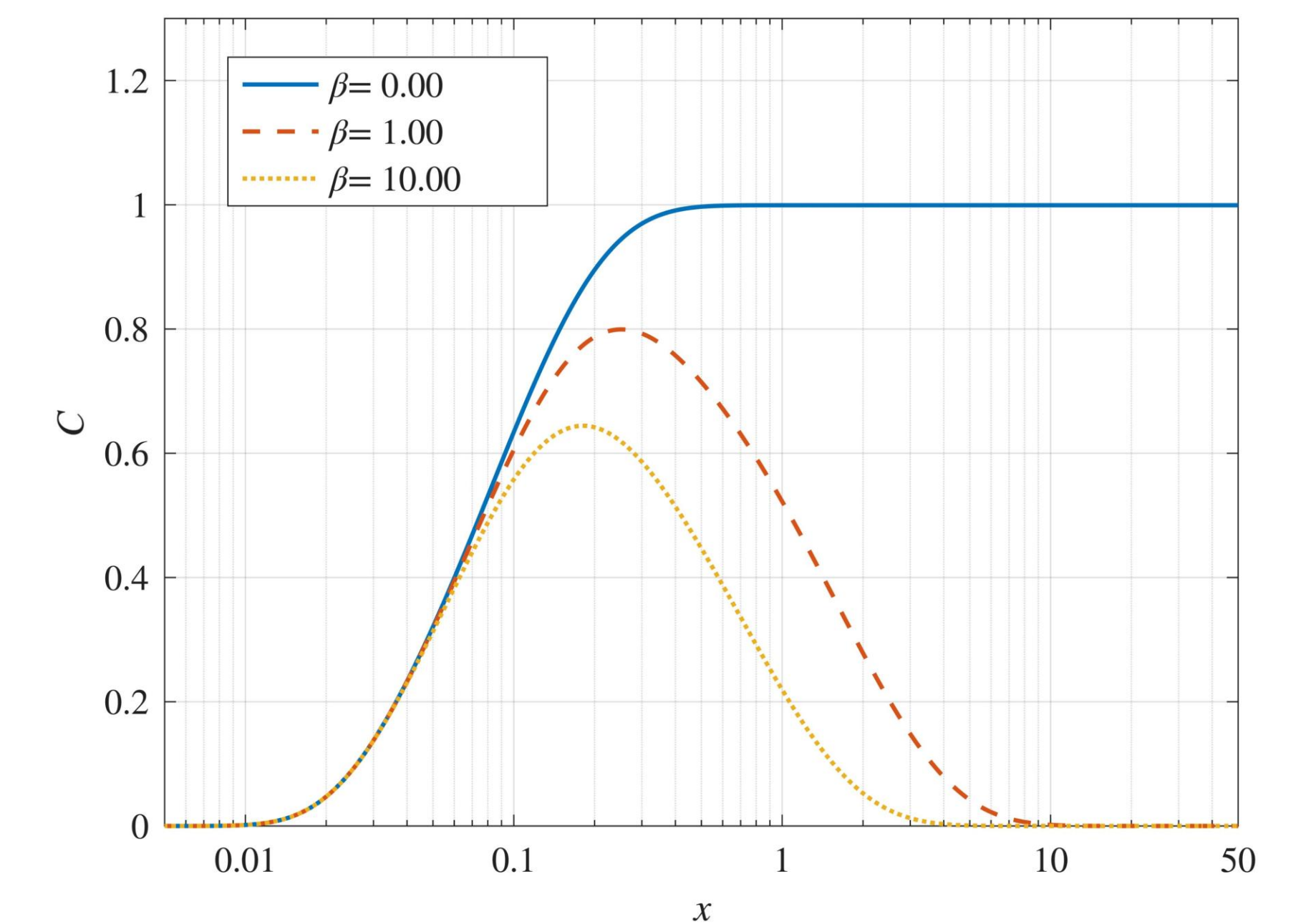
6 Results

The following figure shows the vertical concentration profiles for different bed absorption parameters for sources at $z_0 = 0.3$, $z_1 = 0.8$



The bed absorption parameter β significantly influences near-bed concentration, with its effects propagating vertically with increasing downstream distance.

Figure below shows longitudinal concentration profiles for various β values at $z = 0.8$ for source at $z_0 = 0.25$.



For $\beta > 0$, concentration decreases downstream while $\beta = 0$ results in asymptotic uniform concentration.

7 Concluding remarks

- Bed absorption greatly impacts near bed concentrations while concentration near ice-cover almost remains unchanged.
- Non-zero value of bed absorption leads to zero concentration in the far field.
- For zero bed absorption rate, the concentration reaches unity in far-field for $x > 0.6$ indicating complete mixing.
- Highlights the critical role of bed processes in transport modeling in ice-covered flows.

References

S. Paul, B. Kanungo, S. N. Sahu, K. Ghoshal, "Dispersion of solute from an elevated continuous point source in turbulent flow through an ice-covered channel," Applied Mathematical Modelling, vol. 151, 116512, 2025. doi: 10.1016/j.apm.2025.116512

