When extreme value theory fails: the case of precipitation

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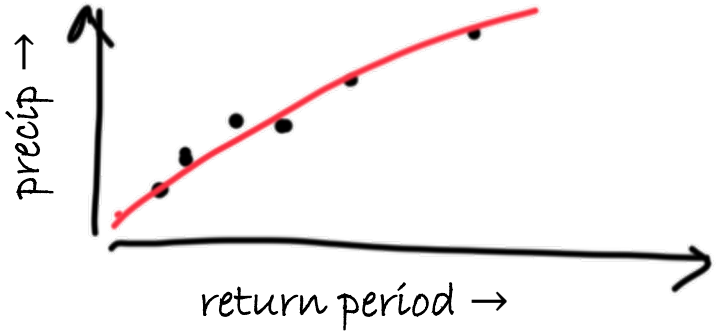
Background

We are interested in rare precipitation extremes with low annual exceedance probability

Difficult task, decades of research efforts



Extreme value theory (EVT) gives an elegant solution:

there is one possible limit distribution for the maxima of asymptotically large samples

↕ the GEV distribution, comprises three types of tails

↕ the number of events in each year should be "large enough"

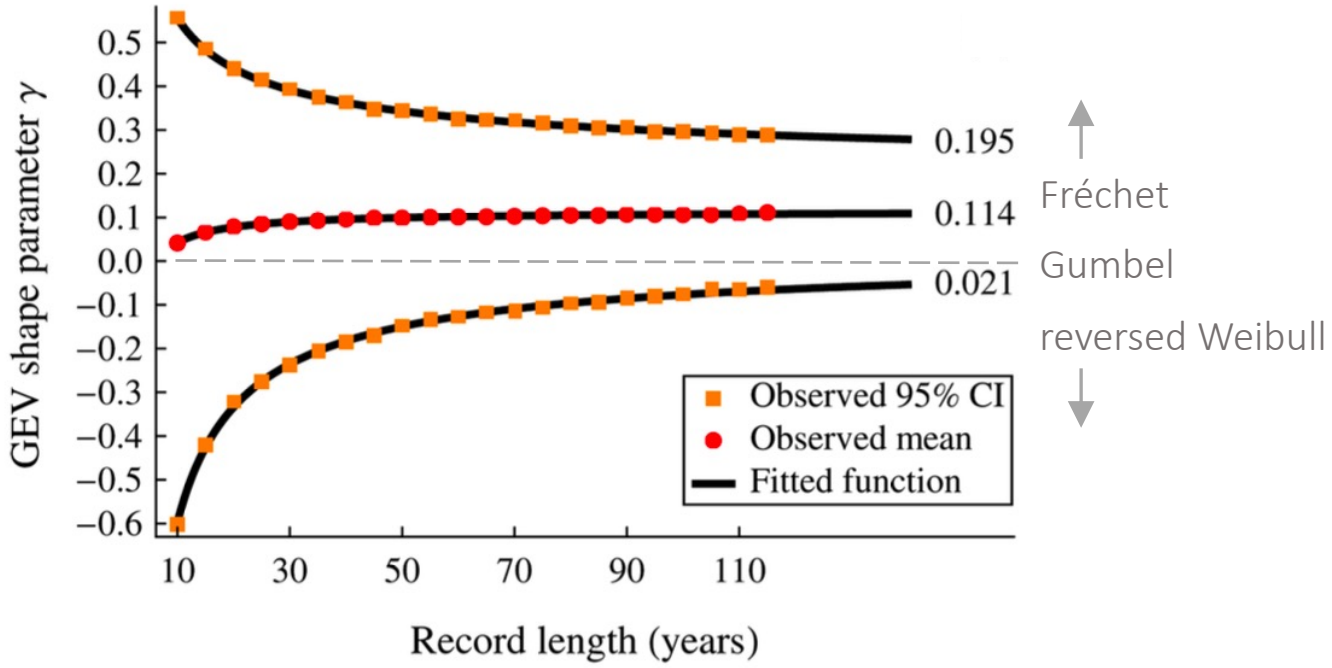
Fisher & Tippet, 1928; von Mises, 1936; Gnedenko, 1943; Jenkinson, 1955

A contradiction between physics and statistics?

EVT finds that extreme precipitation belongs to the Fréchet type (power-law tail)
Papalexiou & Koutsoyiannis, WRR, 2013

Physics arguments (and observations) suggest Weibull tails (exponential family)
Wilson & Toumi, GRL, 2005

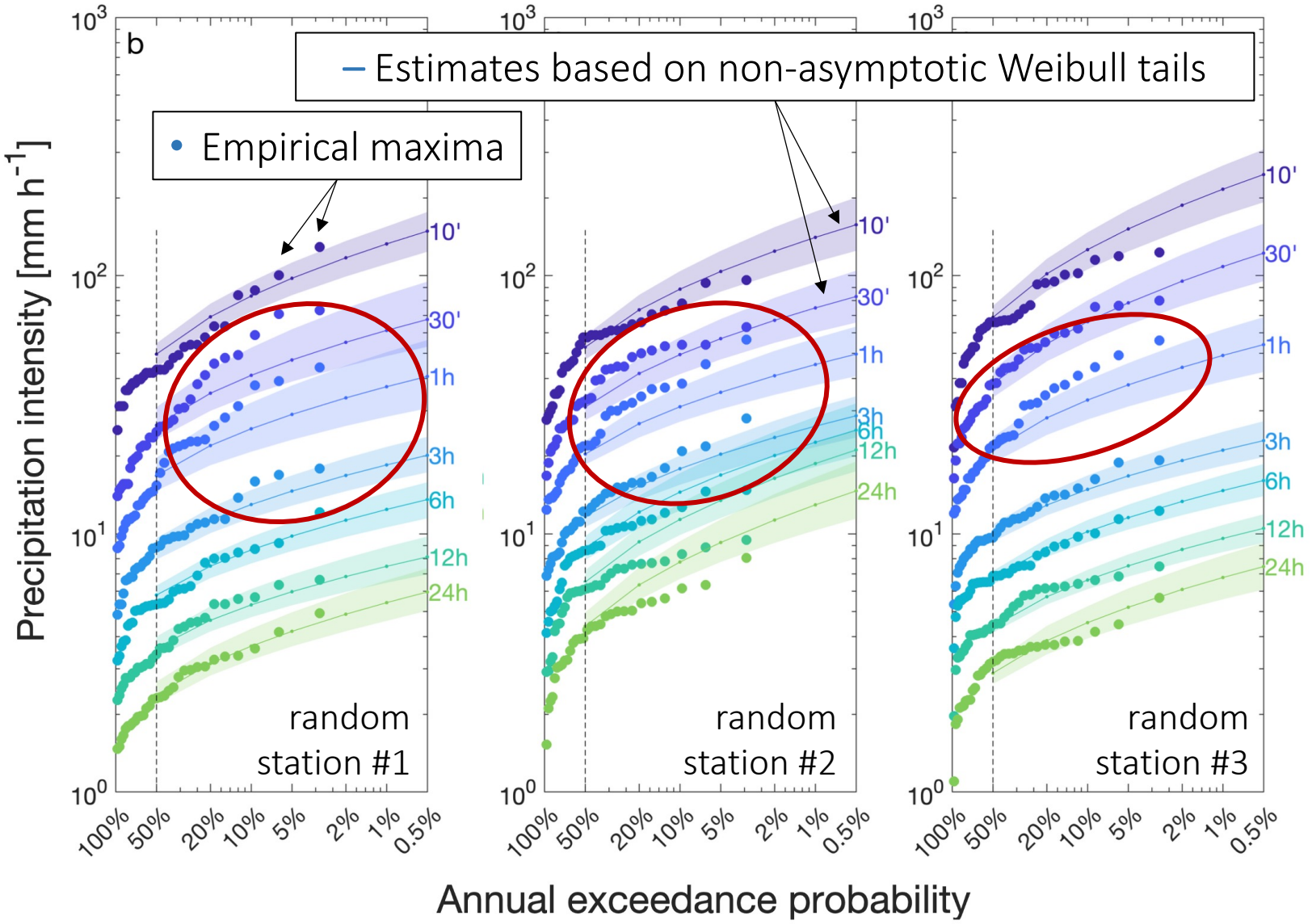
Asymptotically, Weibull tails converge to Gumbel type – convergence is very slow



sample size $>10^7$ needed
Papalexiou & Koutsoyiannis, WRR, 2013

Papalexiou & Koutsoyannis, WRR, 2013

Still, Weibull tails sometimes seem too light



Sometimes, Weibull tails seem too light for sub-daily durations

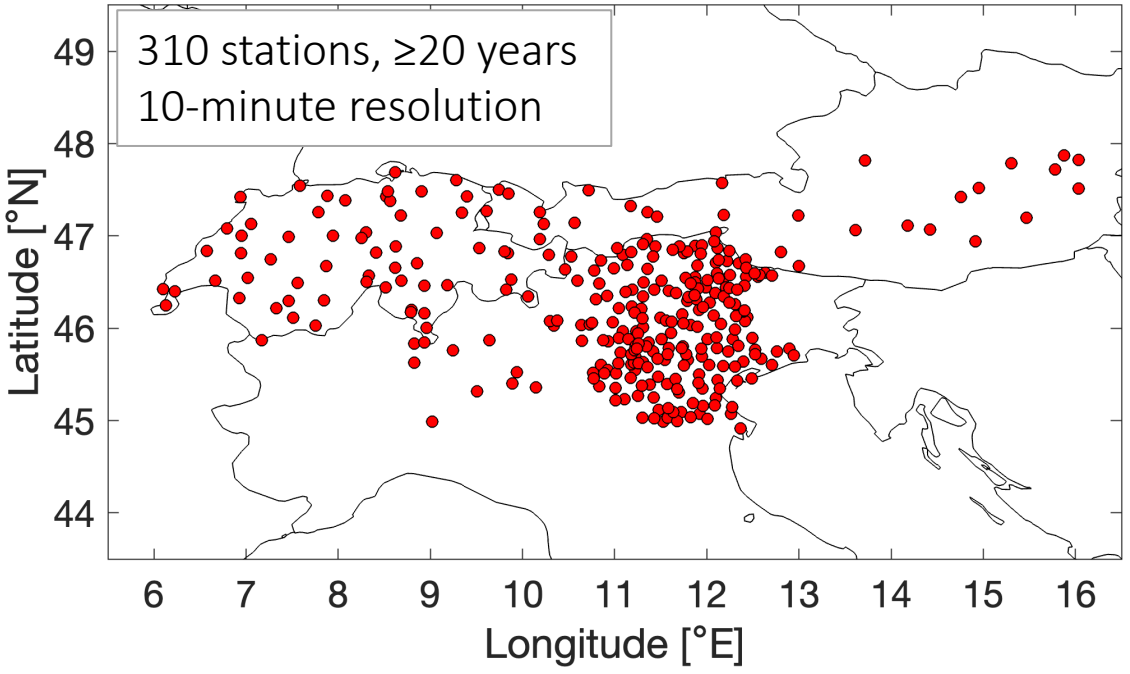
Is our (simplified) physics wrong?

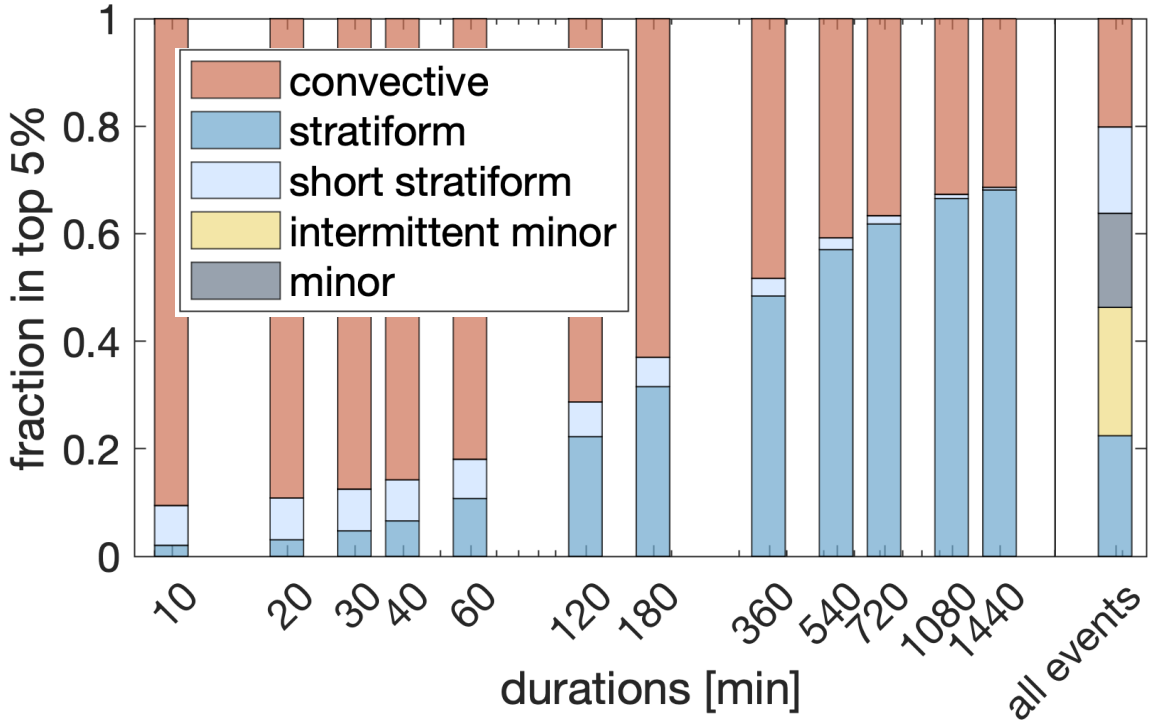
We hypothesize the discrepancy is due to process heterogeneity (e.g., convective VS stratiform)

Hundreds of stations with storm typology

We investigate hundreds of sub-daily precipitation records in the Alps

Storms are classified into “convective-like” and “stratiform-like” (and minor types)
(Papacharalampous & al, 2025, https://doi.org/10.31223/X56M9W)





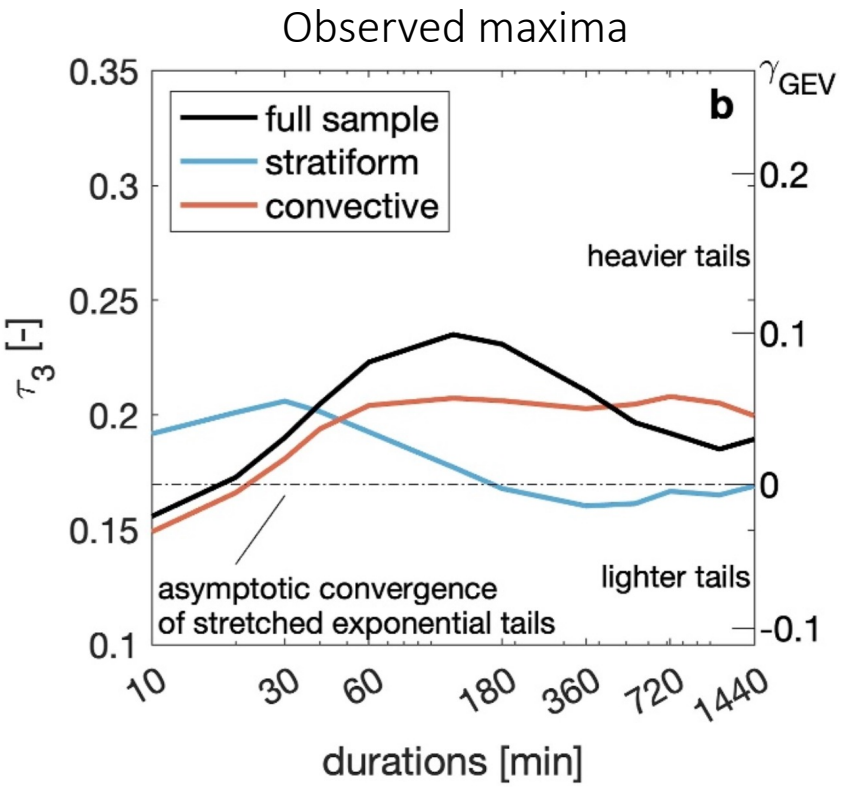
Full sample heavier than both parent processes

Tail heaviness measured using the third L-moment ratio (τ_3)
(Hosking, JRSS-B, 1990)

At some durations, the full sample shows heavier tails than both parent processes

Asymptotic convergence is not reached

(asymptotically, tail heaviness should converge to the tail of the heaviest component)

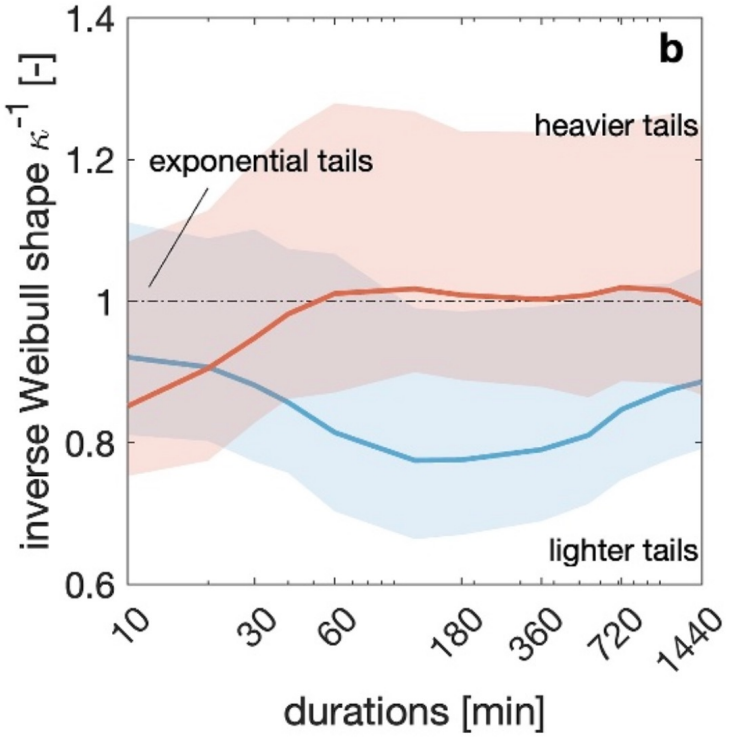


Weibull mixture explains observations

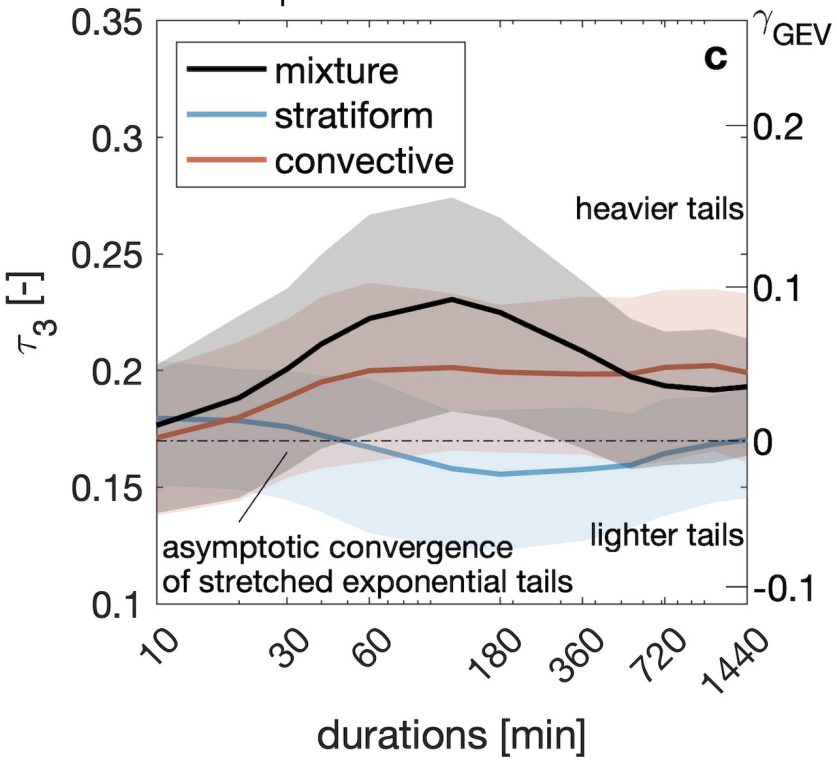
We describe both parent processes using the Weibull tails predicted by physics

Maxima from the mixture have the same tail heaviness of observed maxima

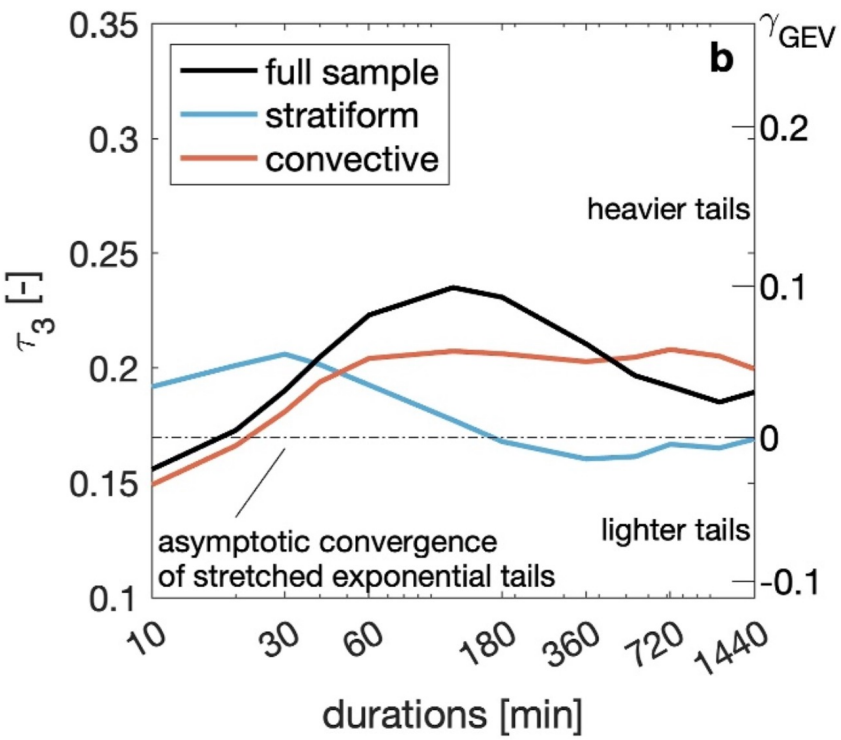
Weibull tails



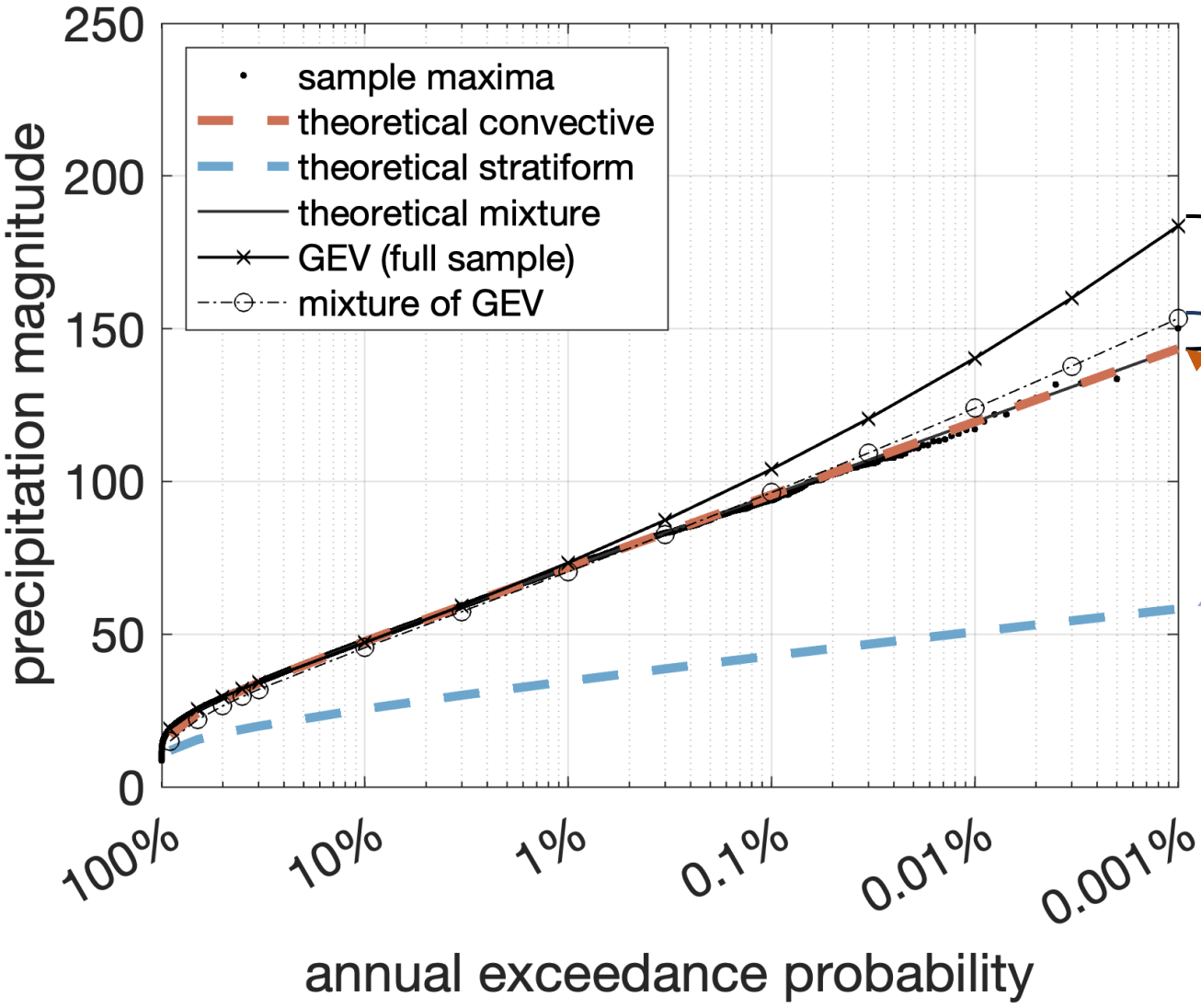
Samples from Weibull tails



Observed maxima



Biased EVT estimates due to heterogeneity



Non-convergence + mixture

Non-convergence of Weibull tails

True mixture

Parent processes

Synthetic data (10^5 years) based on the average characteristics observed at 2-hour duration

Take-home messages

Reality is not asymptotic (no real contradiction between EVT results and physics)

Sub-daily precipitation is well described by a mixture of Weibull-tailed processes

Assuming EVT convergence on such mixtures leads to biases

Full paper

Marra & al, GRL, 2026

Thank you!

Codes & data

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Supplementary slides

From physics to statistics (1/2)

Focusing on the atmospheric column and neglecting local evaporation, precipitation *P* over a time window of interest (duration) is given by:

$$P = -\int_0^{z'} \overline{\kappa \nabla \cdot (q\rho \mathbf{V})}\, dz = -\int_0^{z'} \overline{\kappa \frac{\partial q\rho w}{\partial z}}\, dz = \overline{\kappa (q\rho w)}_{z'}$$

$\mathbf{V}$: horizontal wind
ρ: density
q: specific humidity
w: vertical velocity
κ: efficiency term

Assuming mutual independence between q, ρw, and κ, one gets:
(Fankhauser, 1988; Palmen, 1958; Alpert, 1986; Magagi & Barros, 2004; Abbs, 1999)

$$P = \overline{\kappa} \cdot \overline{q} \cdot \overline{\rho w}$$

Wilson & Toumi, GRL, 2005

From physics to statistics (2/2)

If the averaging period (duration) is sufficiently long, for the central limit theorem one gets the product of three normal distributions:

$$P \sim \Phi_{\overline{\kappa}} \cdot \Phi_{\overline{q}} \cdot \Phi_{\overline{\rho w}}$$

$\mathbf{V}$: horizontal wind
ρ: density
q: specific humidity
w: vertical velocity
κ: efficiency term

The product of independent normal random variables will be Weibull right-tail equivalent:
(Frisch & Sornette, 1997; Laherrere & Sornette, 1998)

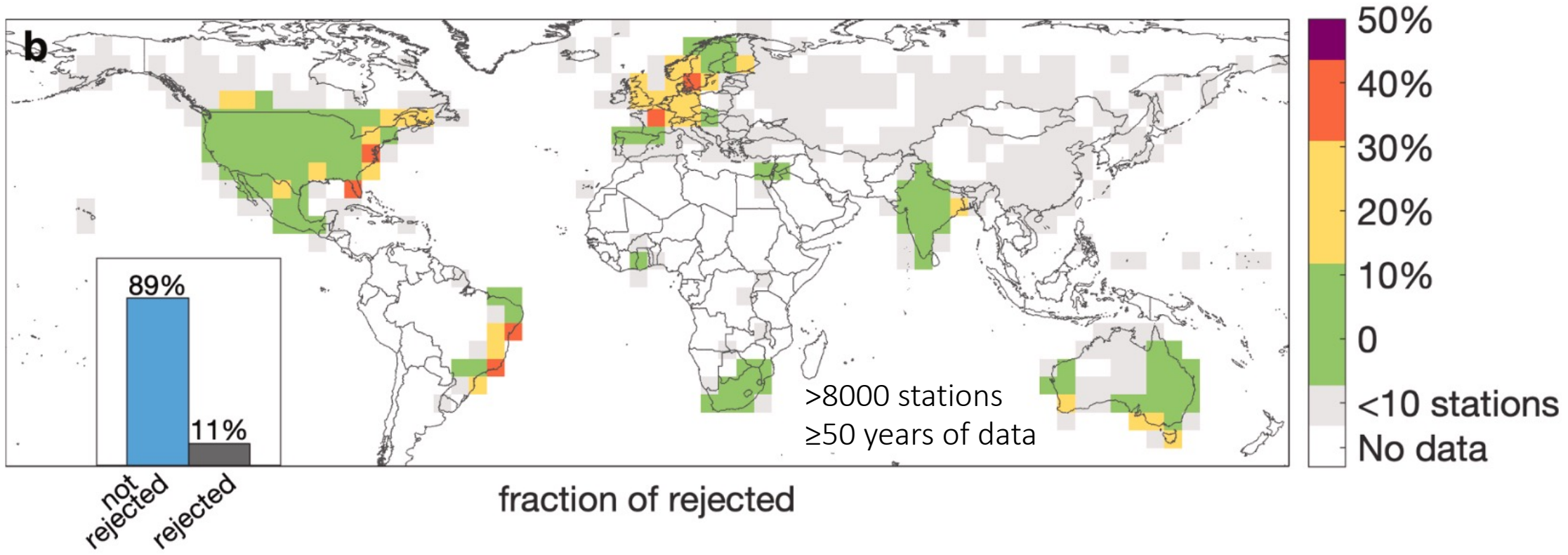
$$F(x) \sim 1 - \exp\left[-\left(\frac{x}{a}\right)^b\right]$$

Wilson & Toumi, GRL, 2005

Weibull tails in observations

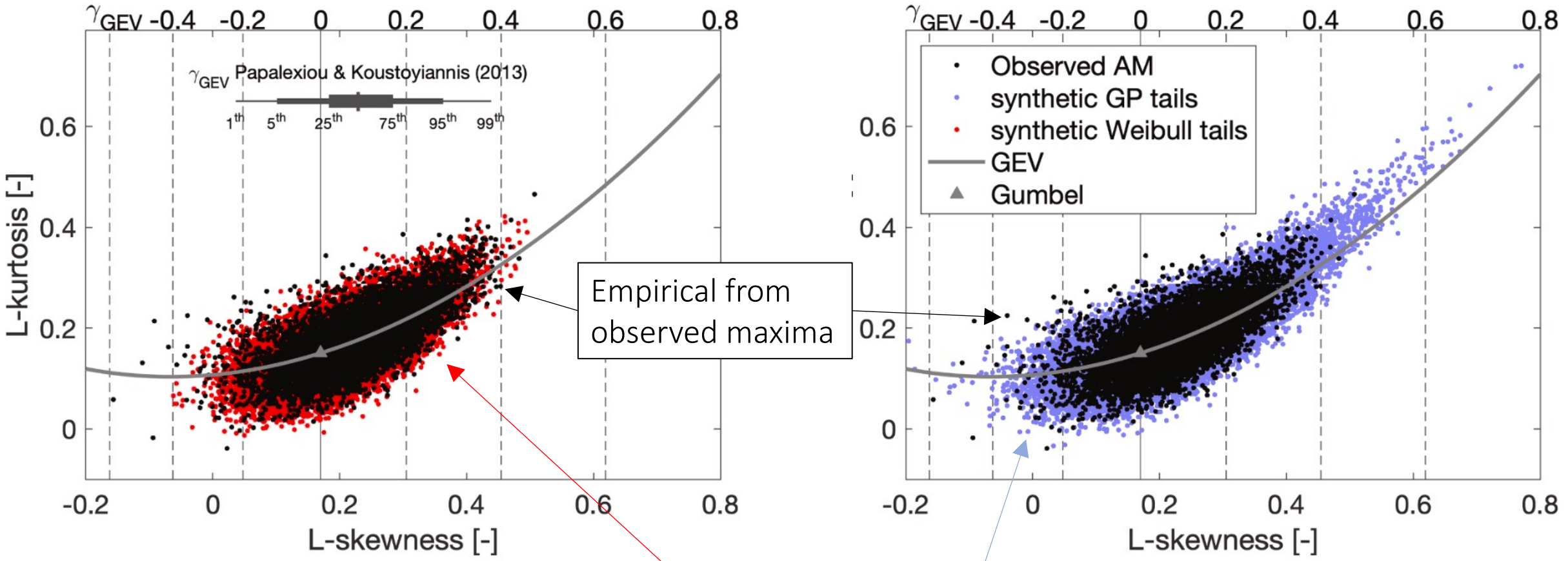
It is possible to check whether the observed extremes (e.g., annual maxima) are likely samples from a Weibull-equivalent parent

Weibull tails are not rejected over most of the globe



Marra & al, AdWR, 2023

Weibull tails & EVT tails



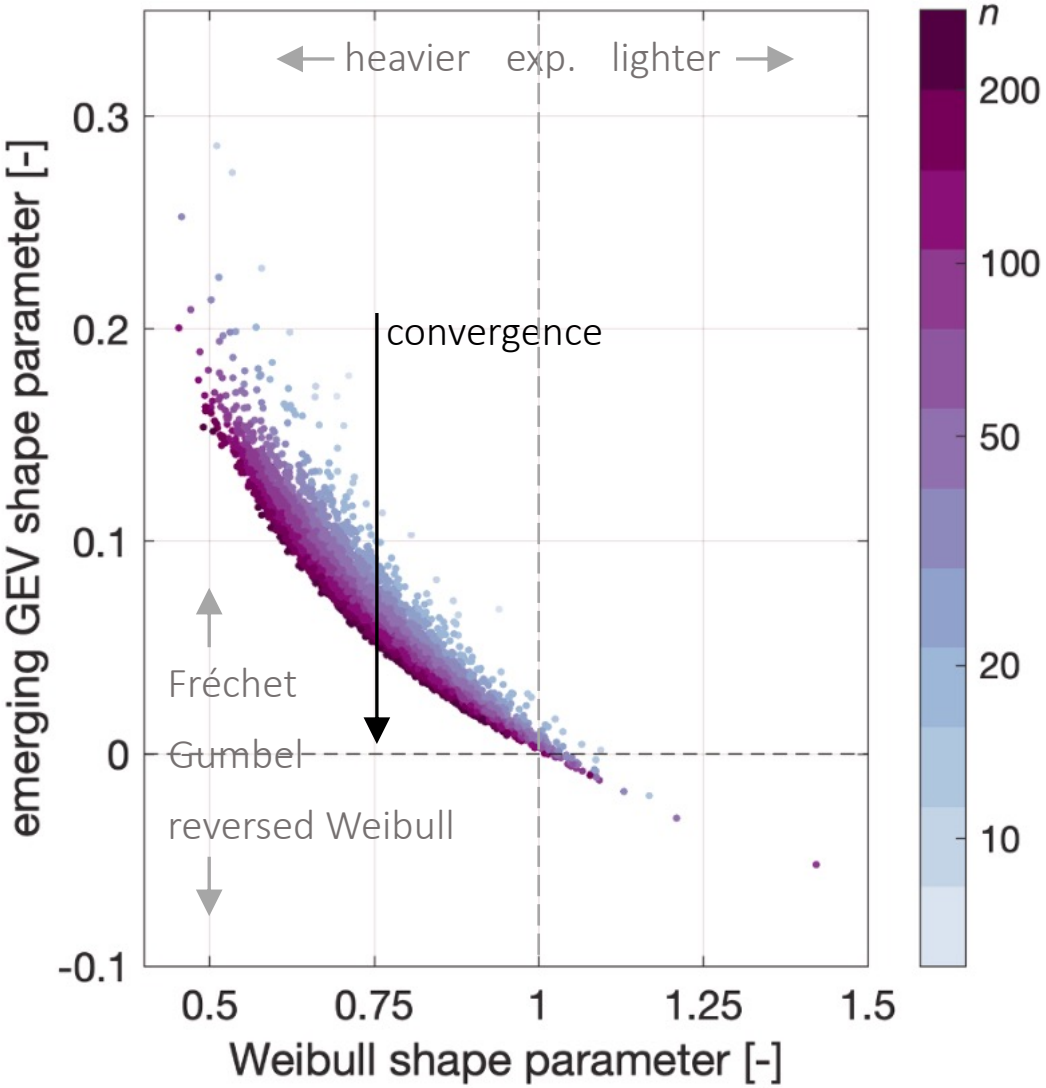
Empirical from observed maxima

Montecarlo from Weibull tails

Montecarlo from GP tails

Marra & al, AdWR, 2023

Asymptotic convergence of Weibull tails is slow



Weibull tails converge to the Gumbel type of EVT

Convergence of Weibull tails is very slow

Papalexiou & Koutsoyiannis, WRR, 2013

sample size >10^7 needed

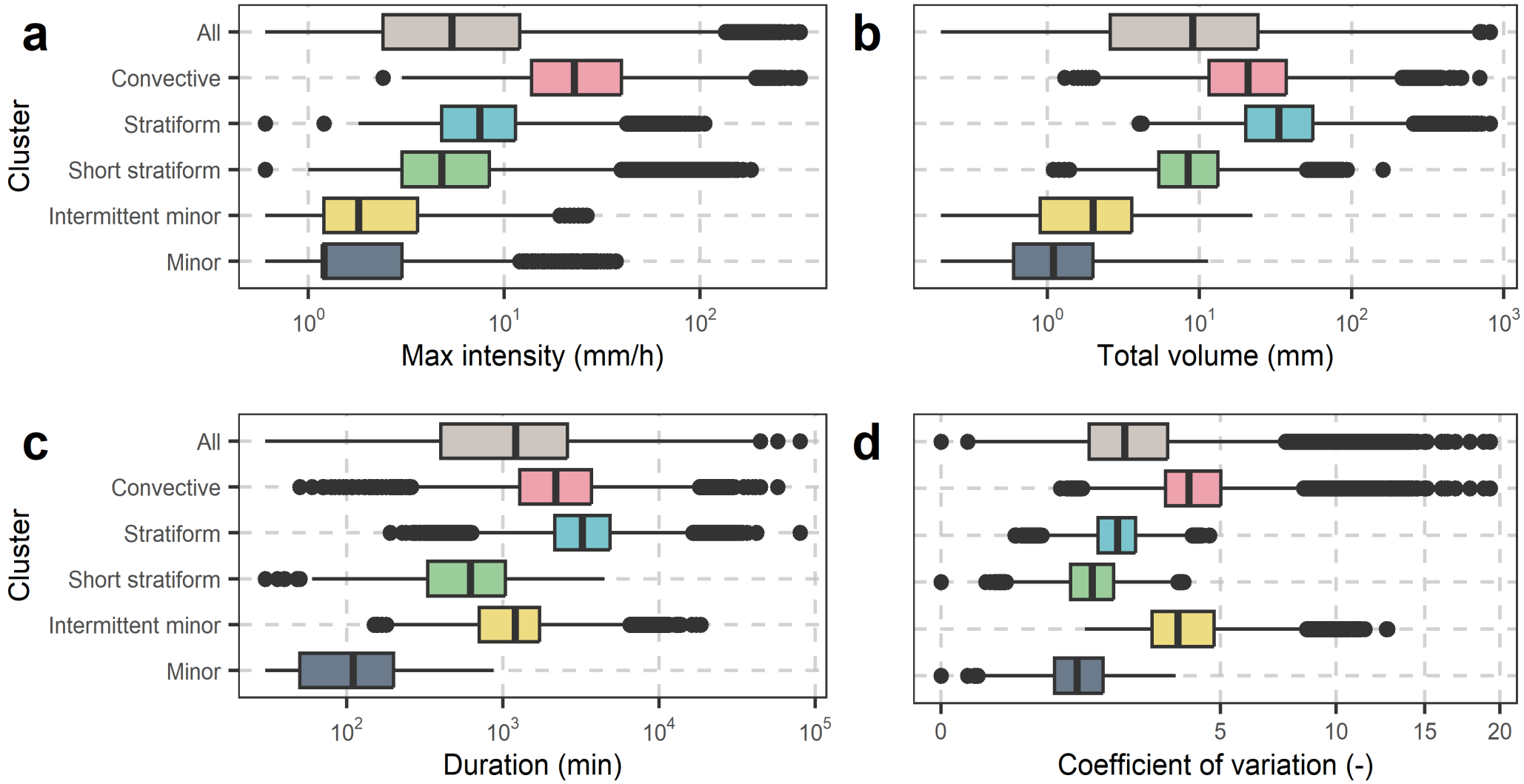
Non-convergent Weibull tails appear as Fréchet

Koutsoyiannis, HJS, 2004
Papalexiou & Koutsoyiannis, WRR, 2013
Marra & al, AdWR, 2023

All in order: no contradiction

Marra & al, AdWR, 2023

Storm typology based on four features



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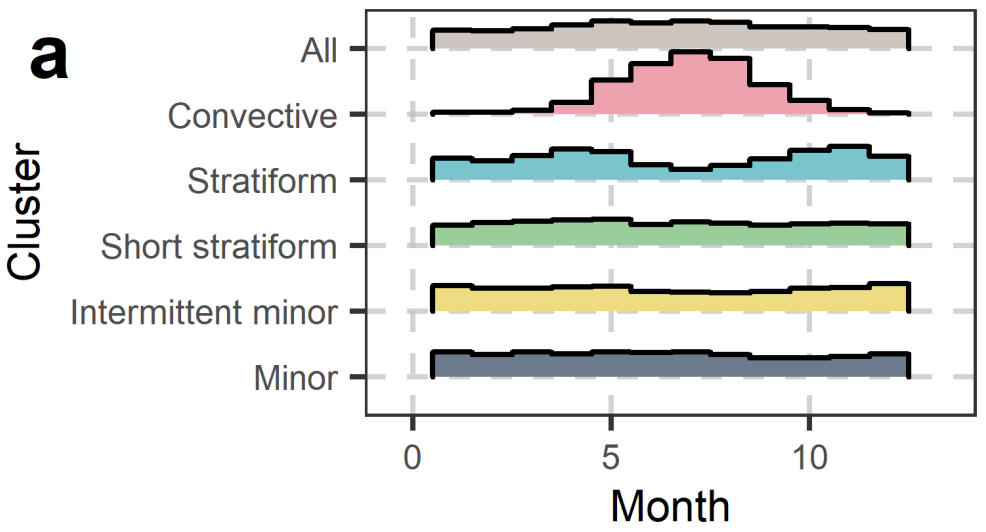
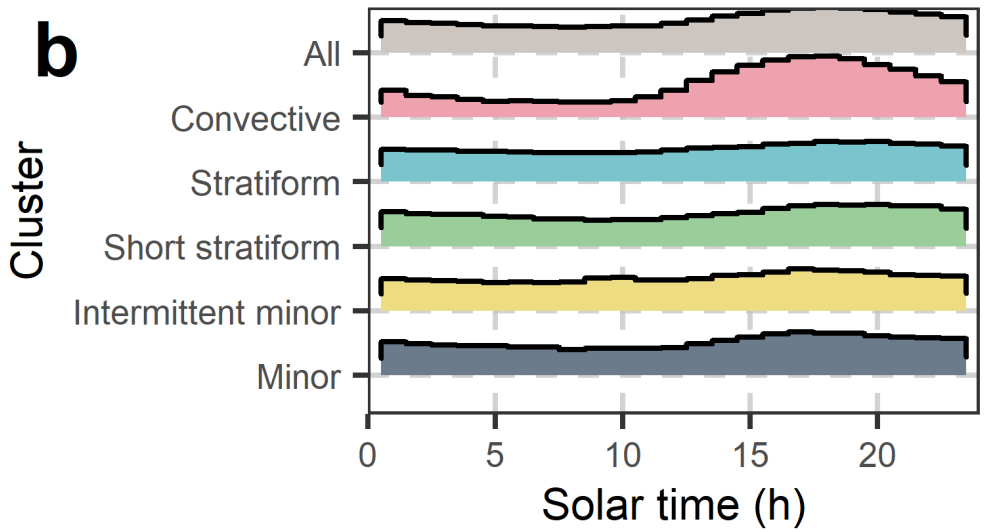
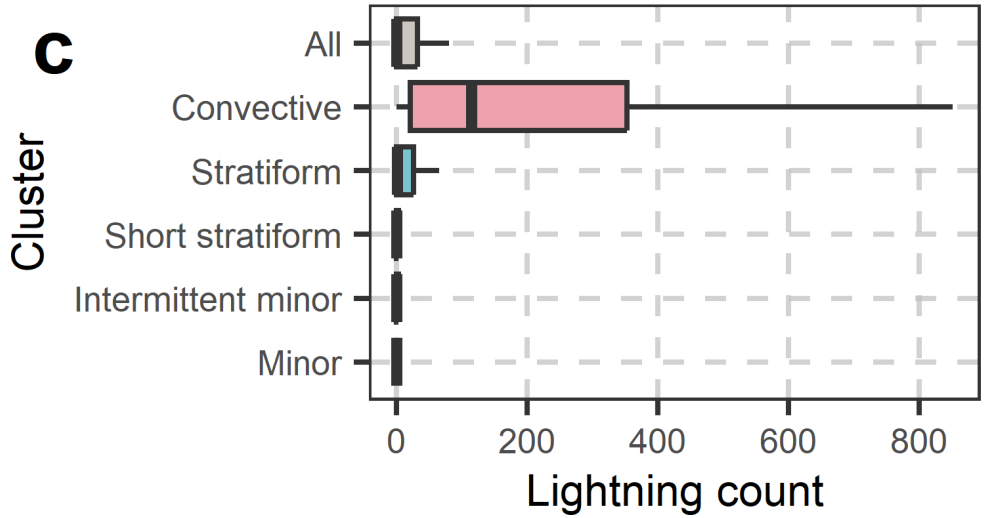
Validation using independent features







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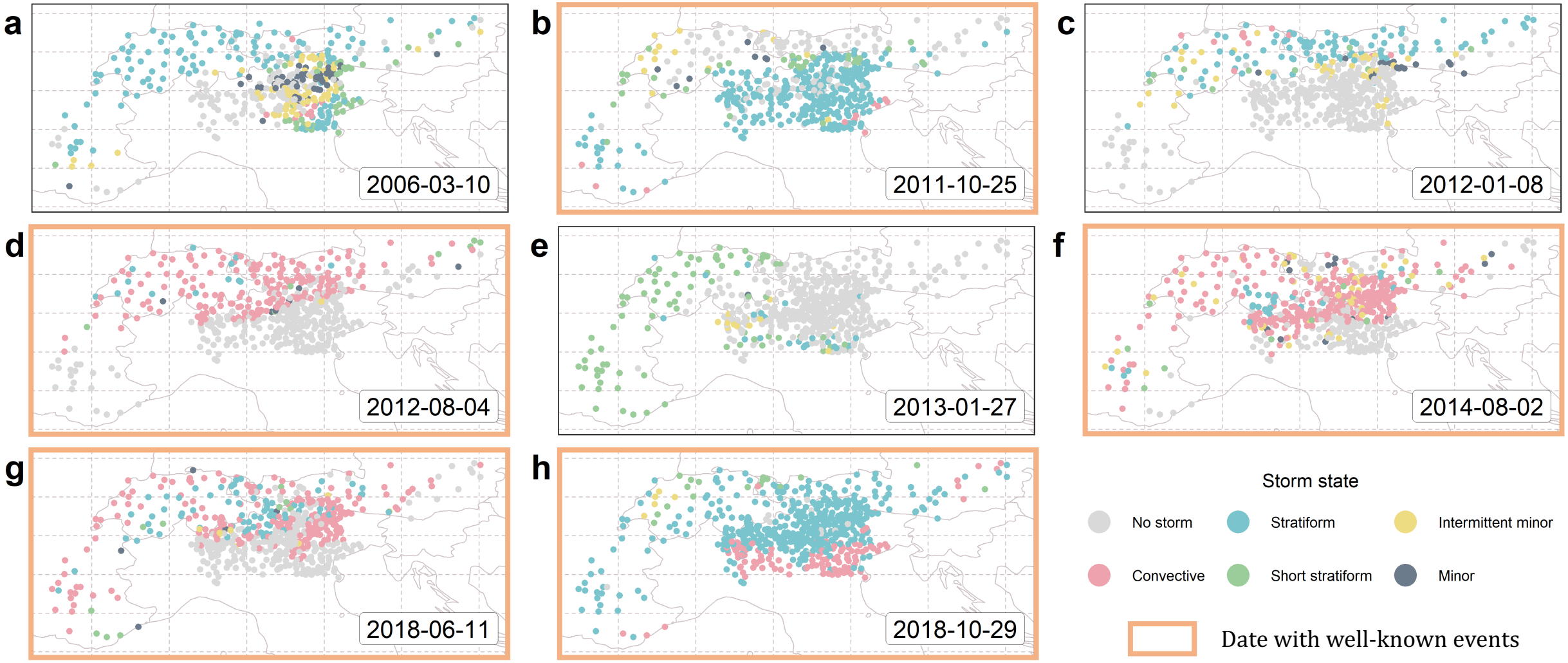
Well known and random events



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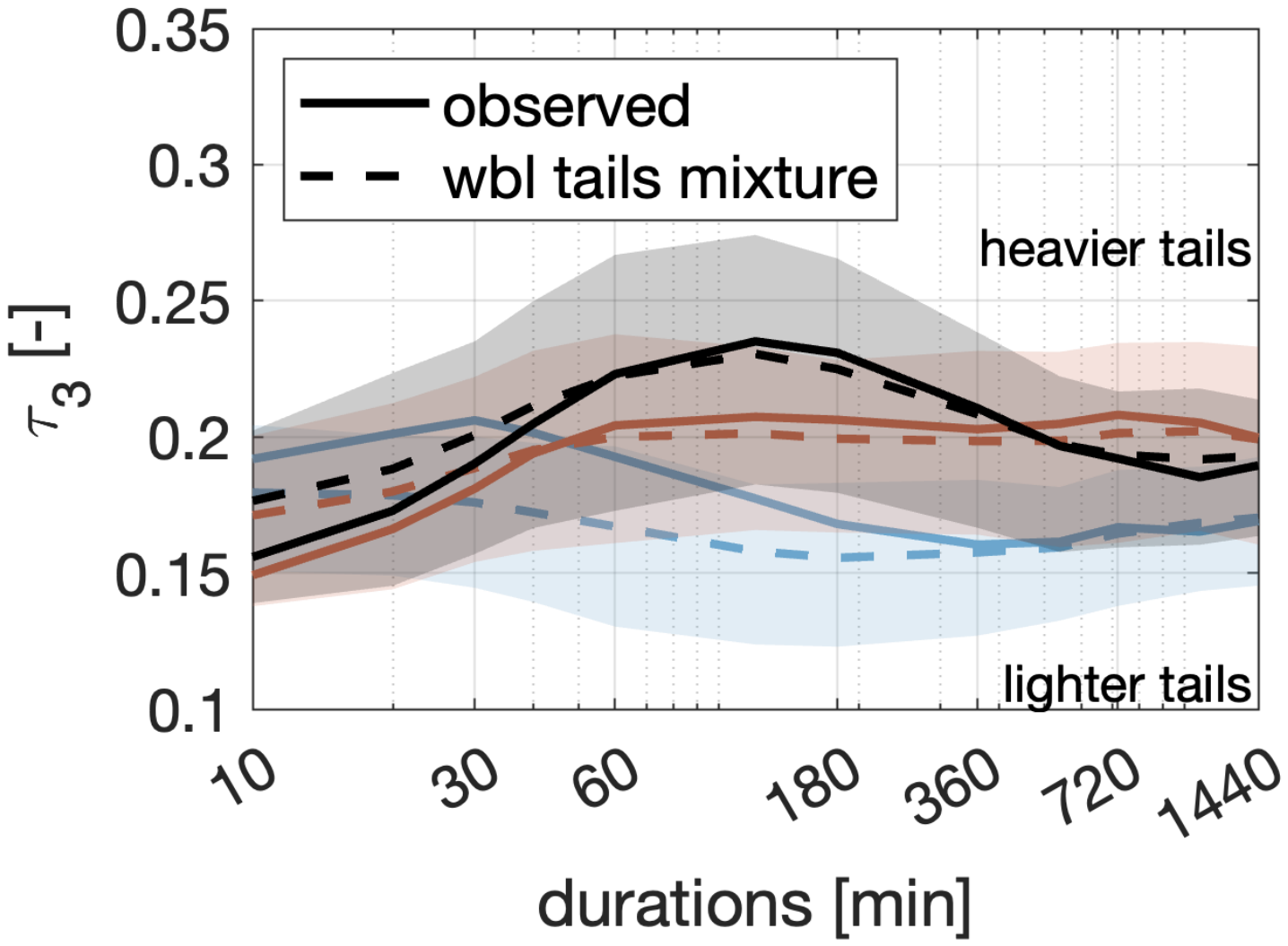
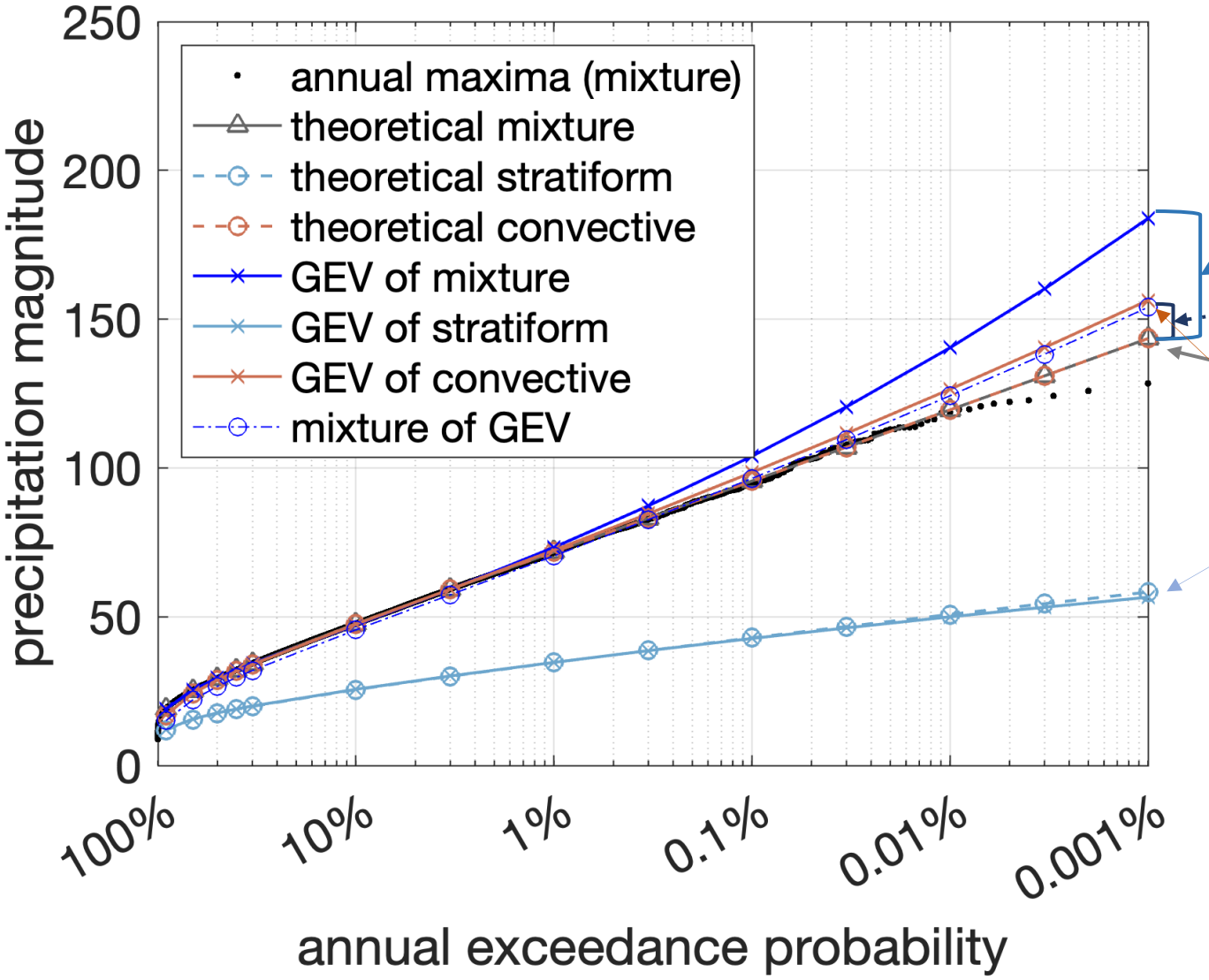
Different processes may lead to extremes



Average from ~300 stations in the Alps (France, Switzerland, Italy, Germany)

modified from: Marra & al, GRL, 2026

Biased EVT estimates due to heterogeneity



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