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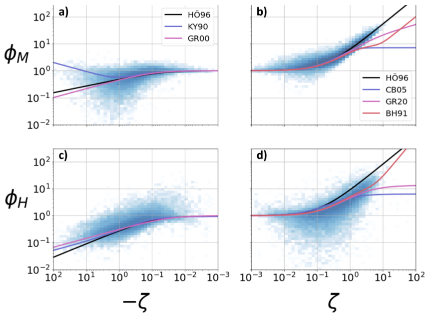
Anabatic Flows over Idealized Mountain Ridges and the Relation between Slope Angle and Turbulence Anisotropy

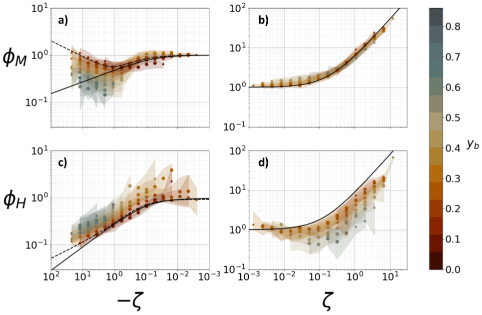
Andreas Rauchöcker, Ivana Stiperski, Alexander Gohm

Wien, 05.06.2026

Scaling including Degree of Anisotropy



Mosso et al. (2024)



Mosso et al. (2024)

From Reynolds stress tensor to anisotropic contribution

Reynolds stress tensor

$$\tau = \overline{u_i'u_j'} = \begin{pmatrix} \overline{u'u'} & \overline{u'v'} & \overline{u'w'} \\ \overline{v'u'} & \overline{v'v'} & \overline{v'w'} \\ \overline{w'u'} & \overline{w'v'} & \overline{w'w'} \end{pmatrix} \tag{1}$$

Isotropic part

$$i_{ij} = \frac{2}{3}(\overline{u'u'} + \overline{v'v'} + \overline{w'w'}) = \frac{2}{3}\overline{u_k'u_k'} \tag{2}$$

Anisotropic part

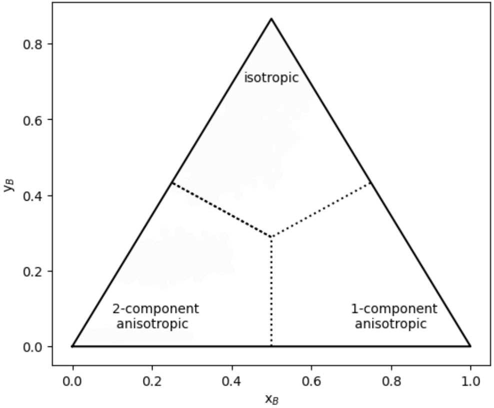
$$a_{ij} = \overline{u_i'u_j'} - \frac{2}{3}\overline{u_k'u_k'} \tag{3}$$

- Isotropic $\rightarrow$ invariant towards coordinate rotation
- Anisotropy $\rightarrow$ **Information about directionality**

Anisotropy - Baricentric map

- Isotropic $\rightarrow$ invariant towards coordinate rotation
- **Information about directionality**
- x_b, y_b functions of non-dimensionalized, anisotropic part of Reynolds stress tensor:

$$b_{ij} = \frac{\overline{u'_i u'_j}}{\overline{u'_k u'_k}} - \frac{1}{3}\delta_{ij} \qquad (4)$$



TKE Budget

The turbulent kinetic energy budget under Boussinesq approximation is given by Stull (1988)

$$\underbrace{\frac{\partial}{\partial t}\frac{\overline{u_i'^2}}{2}}_{\text{Tendency}} = \underbrace{-u_j\frac{\partial \overline{u_i' u_i'}}{\partial x_j}}_{\text{Advection}} \underbrace{-\overline{u_i' u_j'}\frac{\partial \overline{u_i}}{\partial x_j}}_{\text{Shear}} + \underbrace{\delta_{i3}\overline{u_i'\theta'}\frac{g}{\overline{\theta}}}_{\text{Buoyancy}} + \underbrace{f\epsilon_{ij3}\overline{u_i' u_j'}}_{\text{Coriolis}}$$

$$\underbrace{-\frac{1}{2}\frac{\partial \overline{u_j' u_i' u_i'}}{\partial x_j}}_{\text{Turbulent transport}} \underbrace{-\frac{1}{\rho}\frac{\partial \overline{u_j' p'}}{\partial x_j}}_{\text{Pressure correlation}} + \underbrace{\nu\overline{u_i'\frac{\partial^2 u_i'}{\partial x_j^2}}}_{\text{Dissipation}} \quad (5)$$

Free convection - Flat vs. Valley slope

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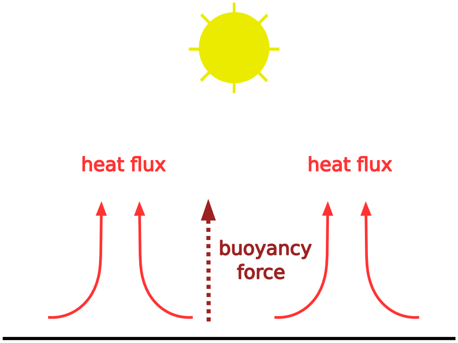
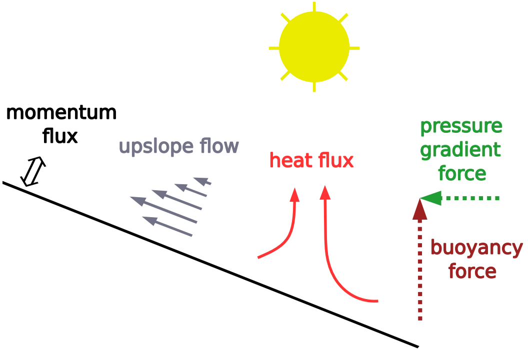
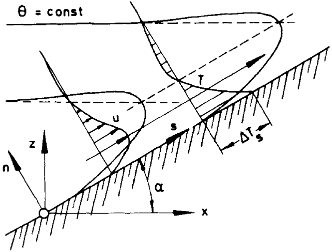
Flat



Slope



Analytical solution by Prandtl (1944)



Adapted from Schumann (1990)

Characteristic depth D

$$D = \left(\frac{4 K_M K_H}{N^2 \sin^2 \alpha} \right)^{1/4} \tag{6}$$

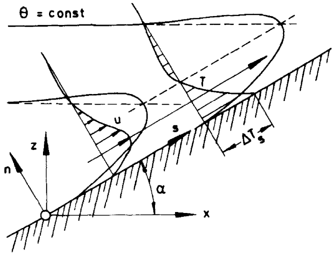
Temperature perturbation profile ΔT

$$\Delta T(n) = d_s \exp\left(-\frac{n}{D}\right) \cos\left(-\frac{n}{D}\right) \tag{7}$$

Wind speed profile u

$$u(n) = d_s \frac{g}{\theta_0 N} \left(\frac{K_H}{K_M} \right)^{1/2} \exp\left(-\frac{n}{D}\right) \sin\left(-\frac{n}{D}\right) \tag{8}$$

Analytical solution by Prandtl (1944)

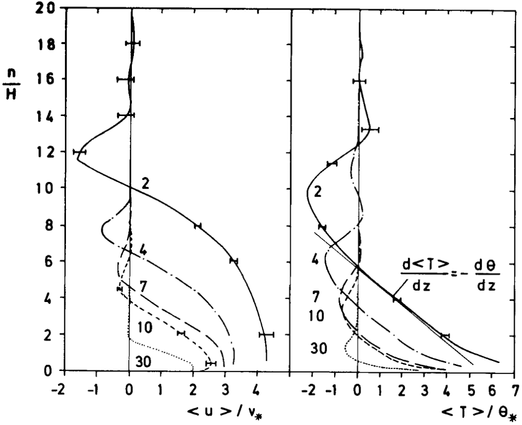


Adapted from Schumann (1990)

- driving parameters
  - surface heating
  - background stratification
  - slope angle
  - (surface roughness)

- Steeper slopes → shallower and weaker upslope flow
- Less steep slopes → deeper and stronger upslope flow

Numerical simulations from Schumann (1990)



Main findings

- Deeper and stronger for shallower slope angle
- Small slope angles: TKE production primarily by buoyancy
- Steeper slopes: shear production more important

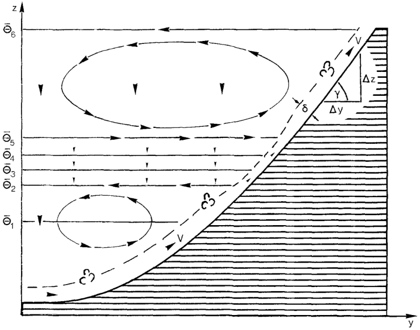
Limitations

- Coarse resolution (by today's standards)
- Tilted domain → no valley, homogeneous slope angle

Analytical Model from Vergeiner and Dreiseitl (1987)

Local balance

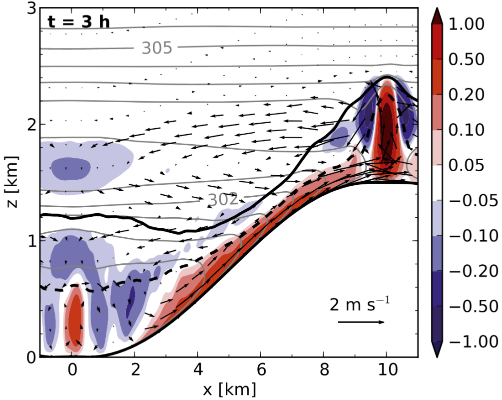
$$\underbrace{-\rho c_p v \frac{\partial \overline{\theta_0}}{\partial z}}_{\text{adiabatic cooling}} \underbrace{- \rho c_p \frac{\partial}{\partial z}\left(v\delta(\theta - \overline{\theta_0})\right)}_{\text{cooling by entrainment}} + \underbrace{\frac{\overline{w'\theta'_0}}{\tan\gamma}(1-q)}_{\text{warming by sensible heat}} = 0 \qquad (9)$$



parameter sensitivity

- Background stratification $\frac{\partial \overline{\theta}}{\partial z}$
- Surface forcing $\overline{w'\theta'_0}$
- Slope angle γ
- Slope shape?

Numerical simulations from Schmidli (2013)



Sinusoidal ridge + periodic → valley configuration

Main findings

- Two circulation cells for deep valley
- Subsidence in valley center causes warming there
- Entrainment enhanced in valley center compared to flat reference

Limitations

- (Coarse resolution)
- No terrain variation

Research questions

1. Does **turbulence anisotropy** in the slope flow depend on the **slope angle**? If there are differences, is this due to steeper slopes **shear** for steeper slopes?

2. Does this depend on the **shape** of a mountain ridge?

3. What is the impact of **different surface forcings and background stratification** on turbulence anisotropy?

Model Setup

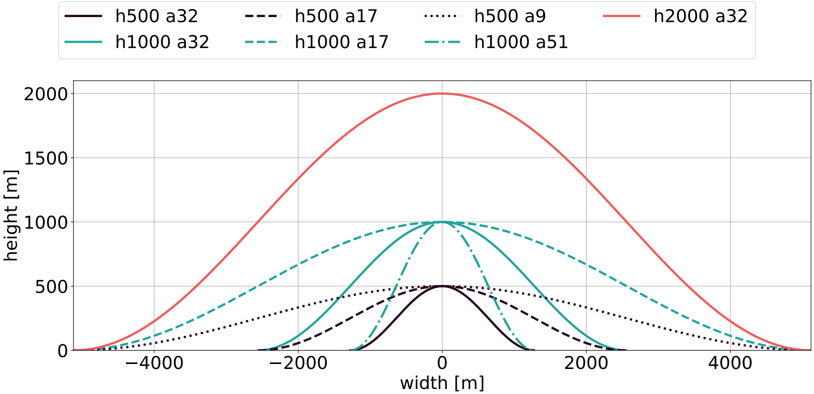
- Cloud Model 1 (CM1 Bryan and Fritsch, 2002)
- $\Delta x = \Delta y = 10$ m, $\Delta z \geqslant 5$ m, periodic boundaries
  - Grid points in x,y,z direction: [256, 512, 1024] x 512 x 285
- Purely thermal regime $\rightarrow u = v = w = 0$ m s^{-1} at initialization
- After 2.5 h spinup: 30-min and spanwise averages

- Idealized terrain - 2D sinusoidal ridge
  - sensitivity tests with triangular shapes ($\rightarrow$ pyramids)
- Convective $\rightarrow \overline{w'\theta'} = 0.12$ K m s^{-1}
  - sensitivity tests for $\overline{w'\theta'} = 0.06$ K m s^{-1}
- θ = 300 K at the surface, $d\theta/dz =$0.003 K m^{-1}
  - sensitivity tests for $d\theta/dz =$0.006 K m^{-1}

Research questions

1. Does the **structure of turbulence** in the slope flow depend on the **slope angle**? If there are differences, is this due to steeper slopes **shear** for steeper slopes?
2. Does this depend on the **shape** of a mountain ridge?
3. What is the impact of **different surface forcings and background stratification** on the structure of turbulence?

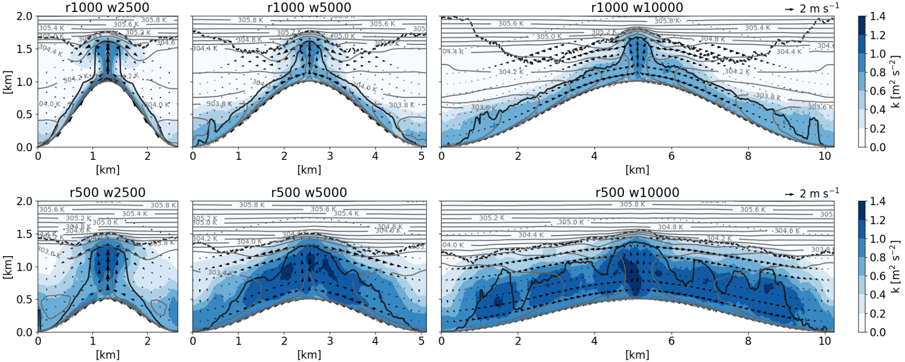
Terrain configuration: slope angle



max. slope angles between 9° and 51°

Slope Angle: Flow structures

Turbulent Kinetic Energy

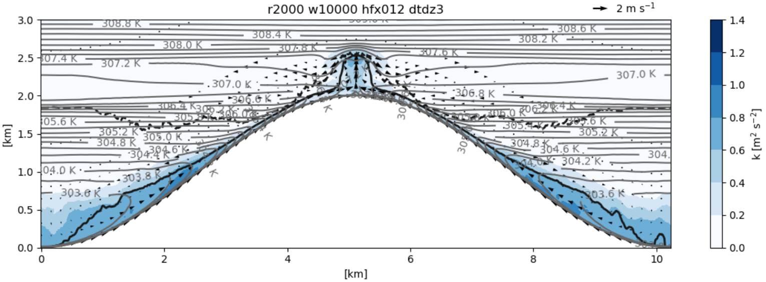


solid: slope flow layer depth δ, dashed: boundary layer height z_i

Steeper slopes → shallower slope flow layer, stronger flow

Slope Angle: Flow structures

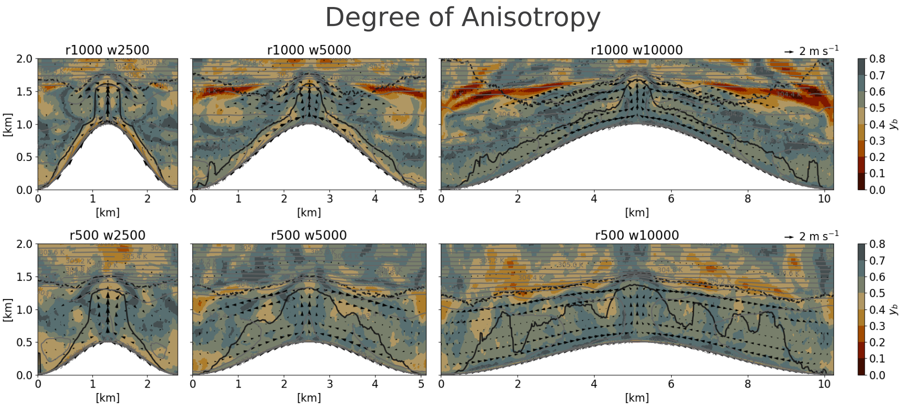
Turbulent Kinetic Energy



solid: slope flow layer depth δ, dashed: boundary layer height z_i

Higher ridge $\rightarrow$ 2 circulation cells

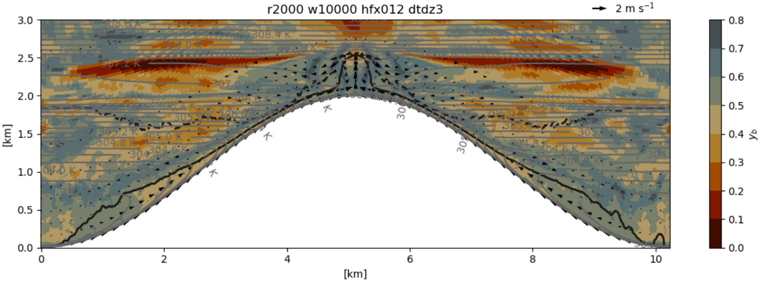
Slope Angle: Flow structures



solid: slope flow layer depth δ, dashed: boundary layer height z_i

Slope Angle: Flow structures

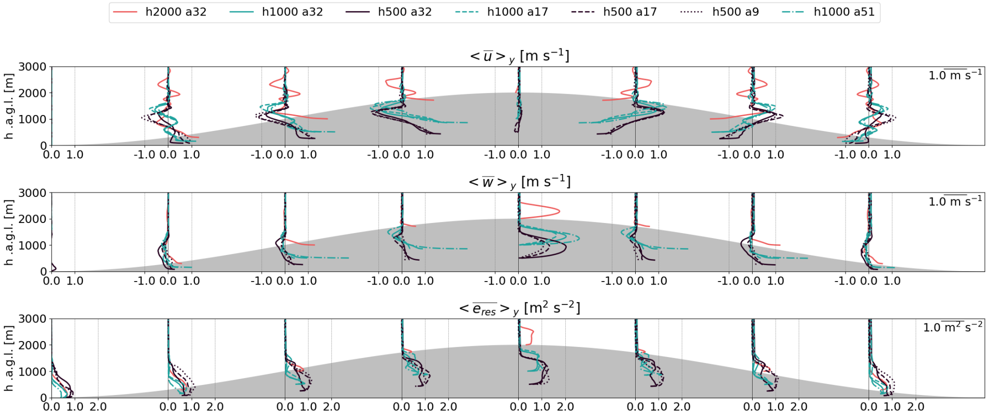
Degree of Anisotropy



solid: slope flow layer depth δ, dashed: boundary layer height z_i

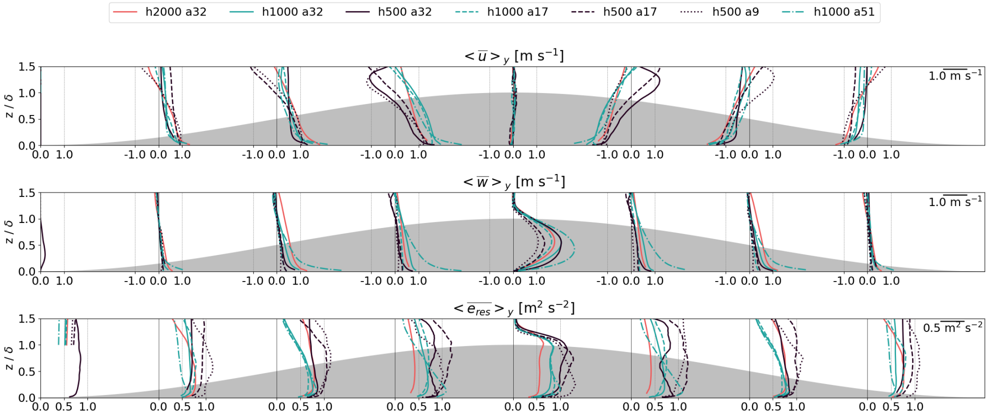
Slope Angle: Mean quantities: u, w

Full height

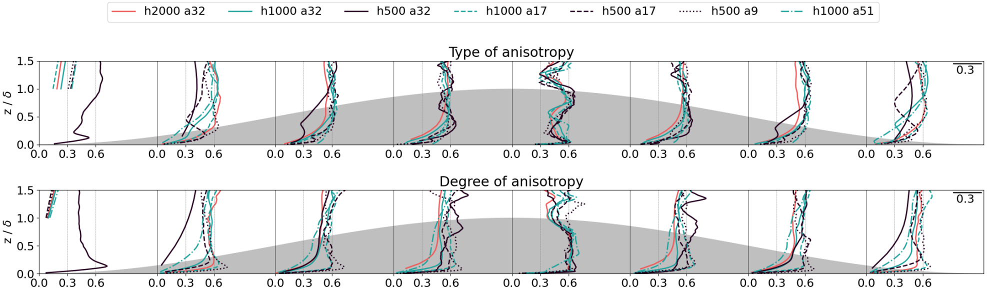


Slope Angle: Mean quantities: u, w

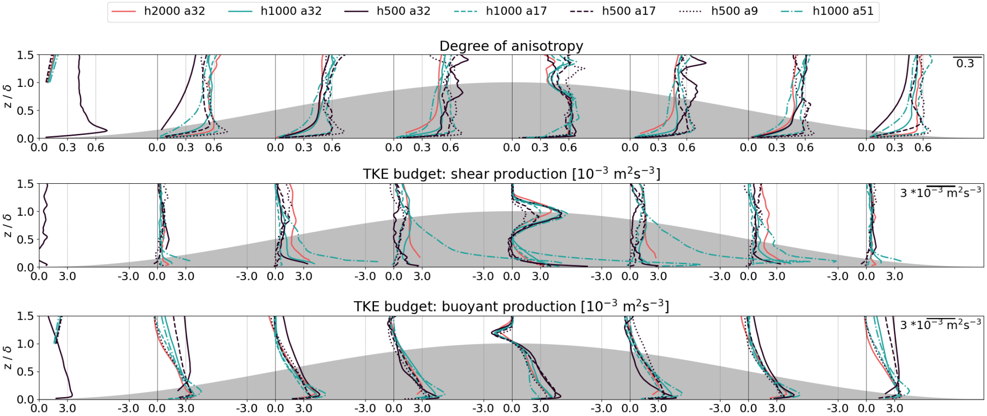
Slope flow layer



Slope Angle: Type and degree of anisotropy y_b

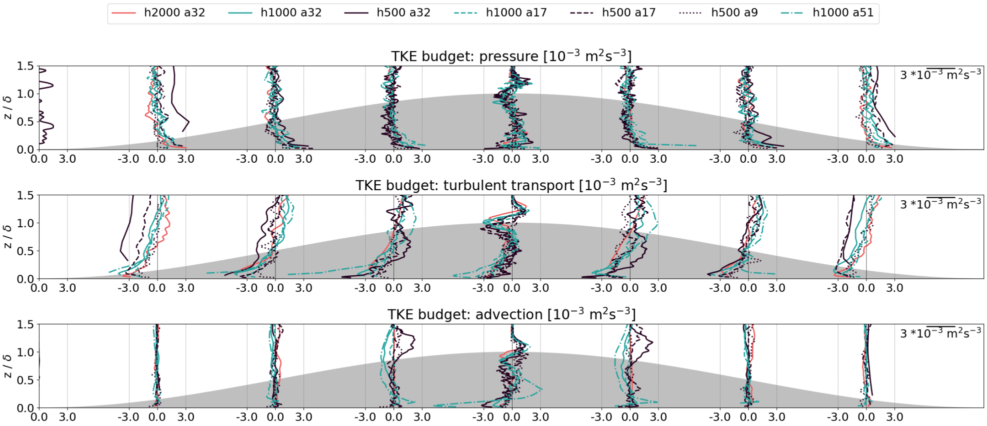


Slope angle: degree of anisotropy, buoyancy and shear



Steeper slopes → more anisotropic turbulence
due to enhanced shear production

Slope angle: pressure, transport and advection



Research questions

1. Does the **structure of turbulence** in the slope flow depend on the **slope angle**? If there are differences, is this due to steeper slopes **shear** for steeper slopes?

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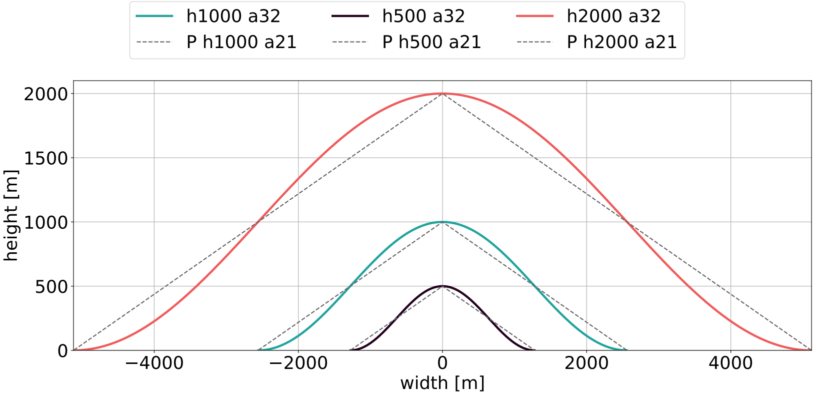
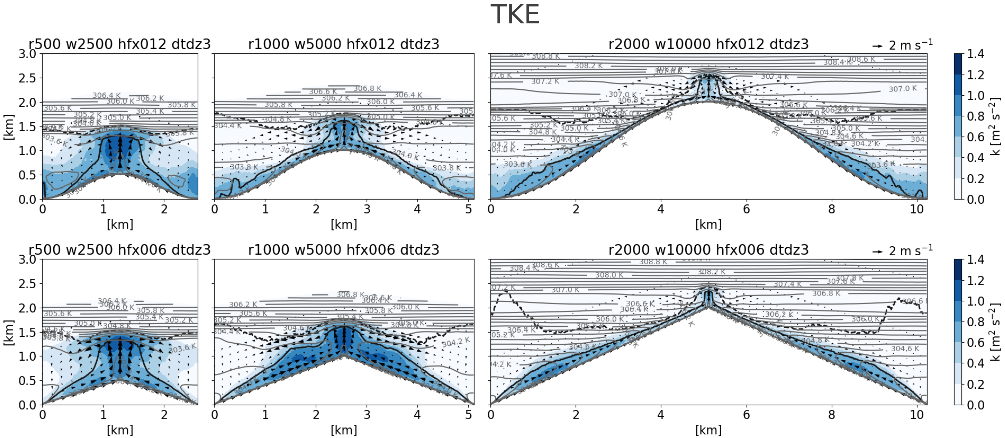
Terrain configuration sensitivity tests: ridge shape



Sinusoidal: max. slope angles of 32°

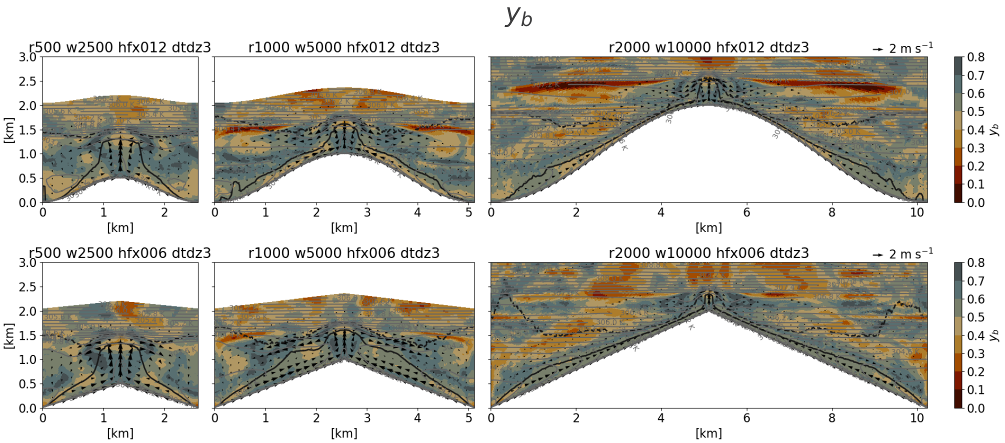
Pyramid: (constant) slope angle of 21°

Ridge shape: Flow structures



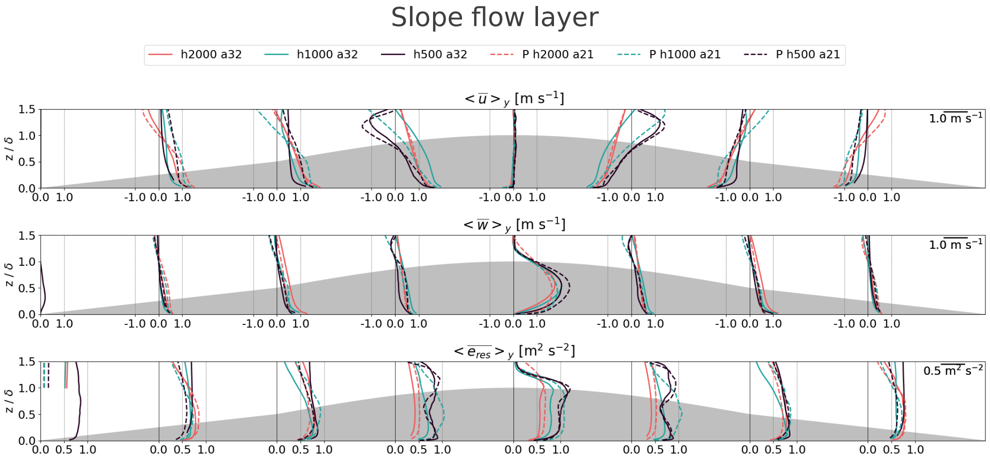
solid: slope flow layer depth δ, dashed: boundary layer height z_i

Ridge shape: Flow structures

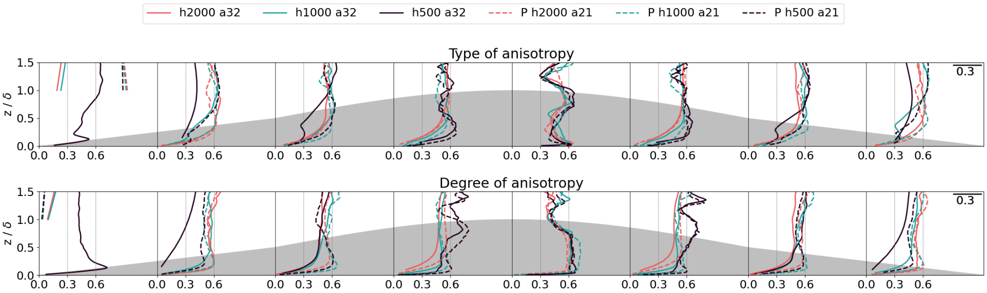


solid: slope flow layer depth δ, dashed: boundary layer height z_i

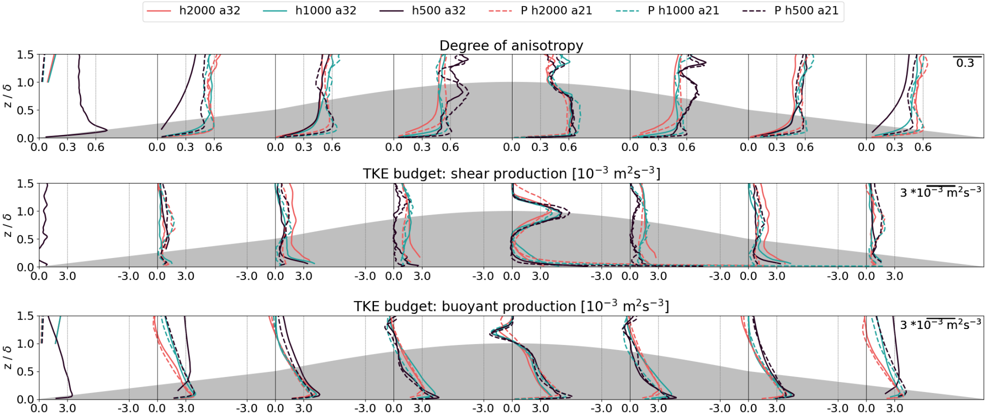
Ridge shape: Mean quantities: u, w



Ridge shape: Type and degree of anisotropy y_b

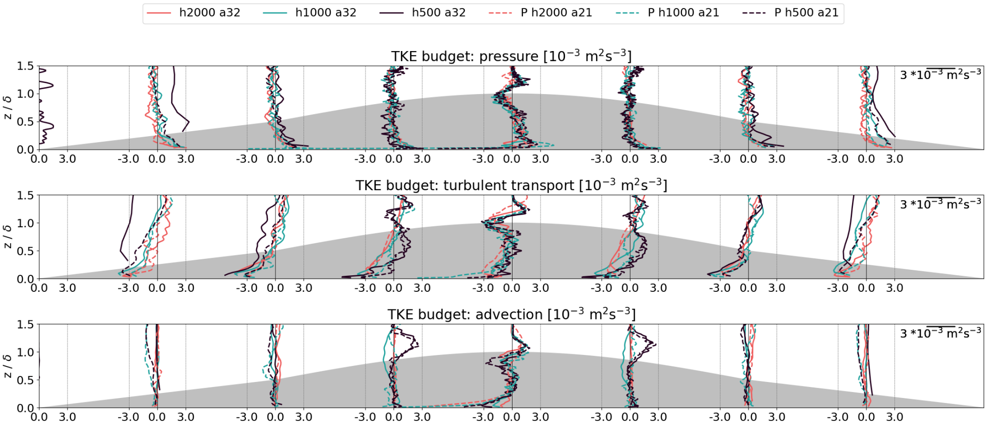


Ridge shape: degree of anisotropy, buoyancy and shear



Sinusoidal ridge → more anisotropic due to enhanced shear

Slope angle: pressure, transport and advection



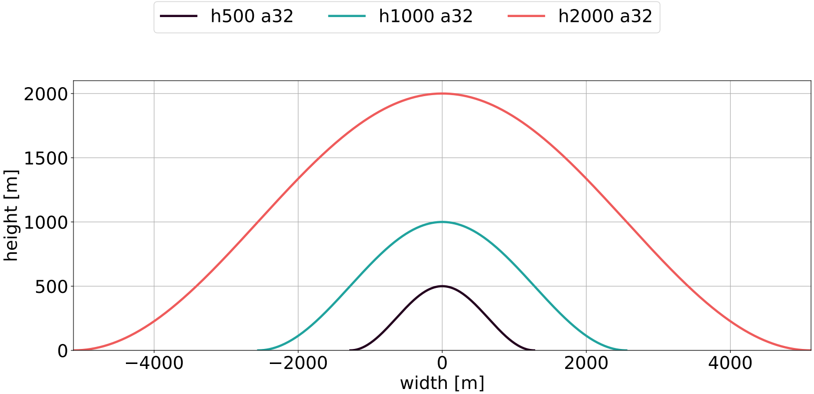
Research questions

1. Does the **structure of turbulence** in the slope flow depend on the **slope angle**? If there are differences, is this due to steeper slopes **shear** for steeper slopes?

2. Does this depend on the **shape** of a mountain ridge?

3. What is the impact of **different surface forcings and background stratification** on the structure of turbulence?

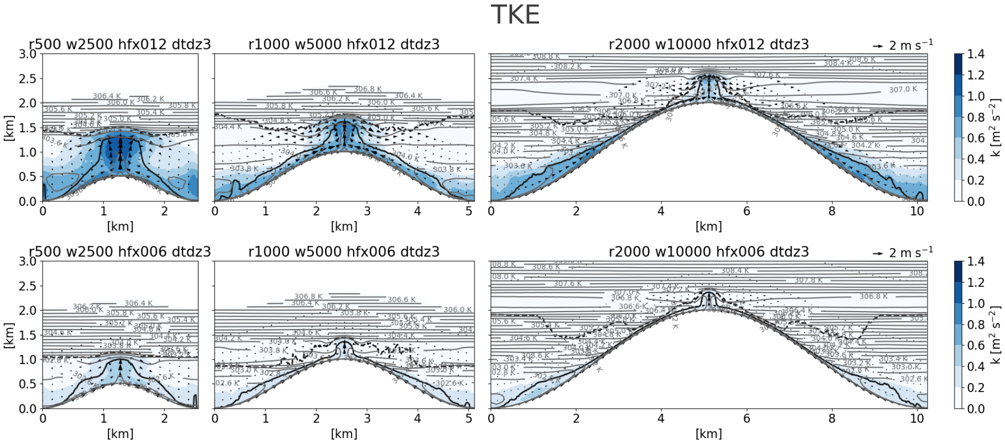
Terrain configuration sensitivity tests: heat flux



All cases: max. slope angles of 32°

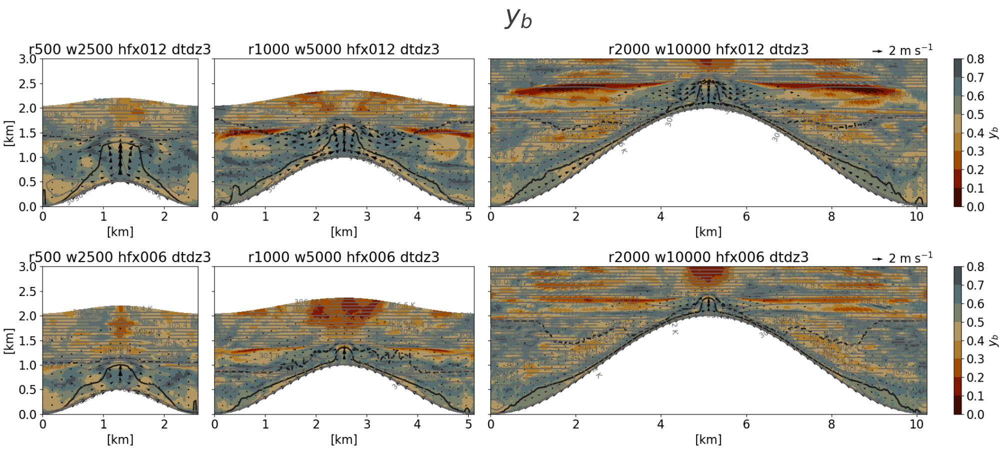
Surface forcing: $0.012\ \mathrm{K\ m\ s^{-1}}$ vs. $0.006\ \mathrm{K\ m\ s^{-1}}$

Surface Forcing: Flow structures



solid: slope flow layer depth δ, dashed: boundary layer height z_i

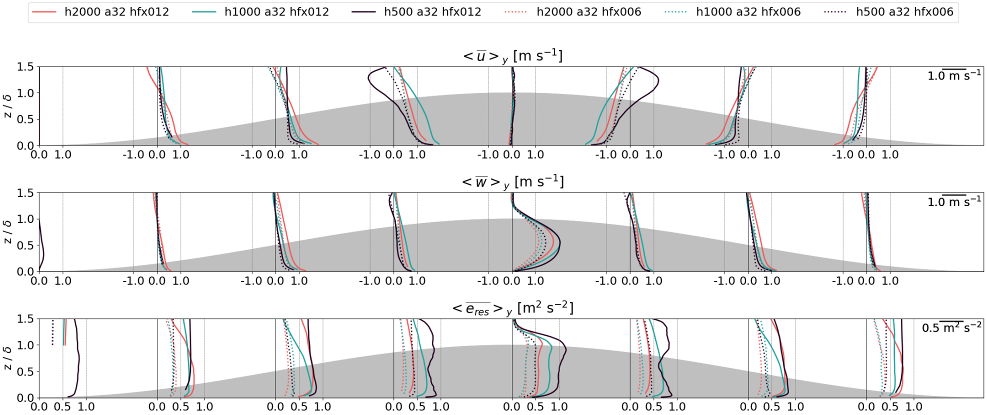
Surface Forcing: Flow structures



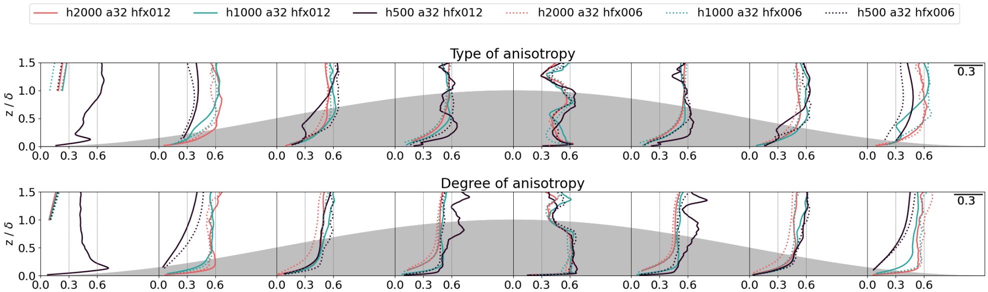
solid: slope flow layer depth δ, dashed: boundary layer height z_i

Forcing: Mean quantities: u, w

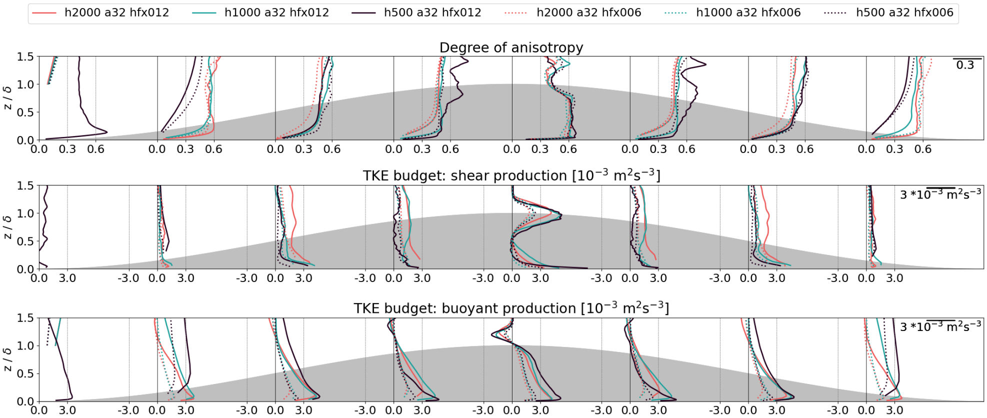
Slope flow layer



Forcing: Type and degree of anisotropy y_b

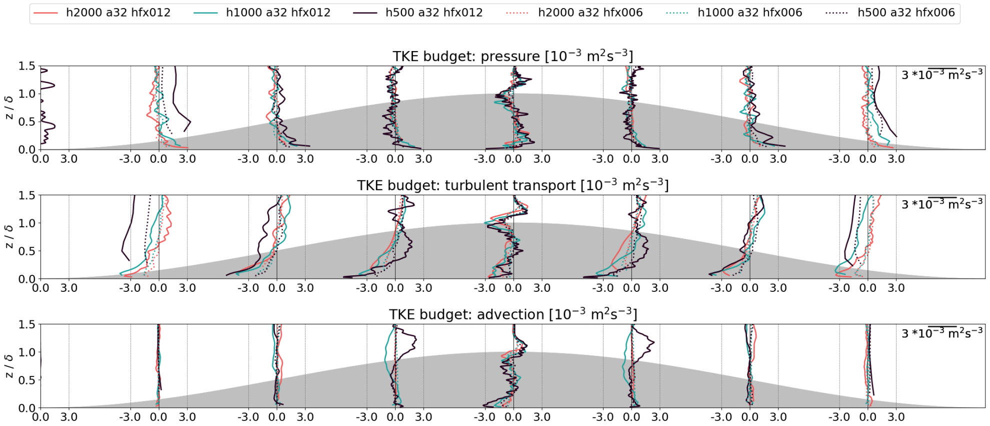


Forcing: degree of anisotropy, buoyancy and shear

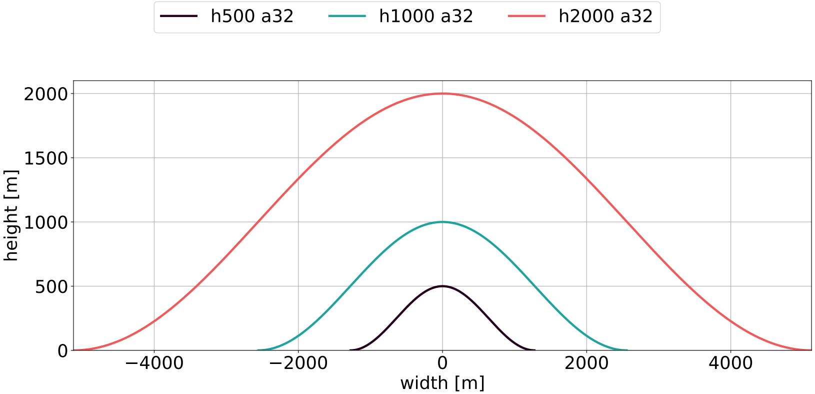


Lower heat flux → less buoyancy and shear production

Surface Forcing: pressure, transport and advection



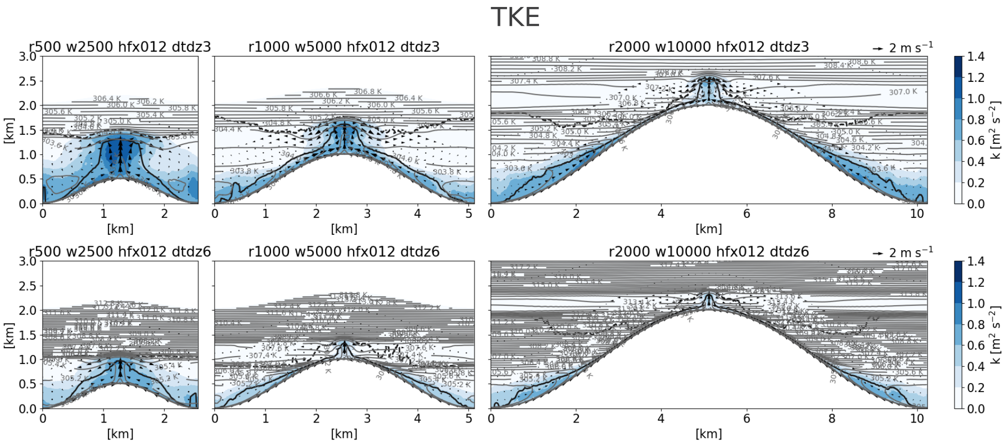
Terrain configuration sensitivity tests: stratification



All cases: max. slope angles of 32°

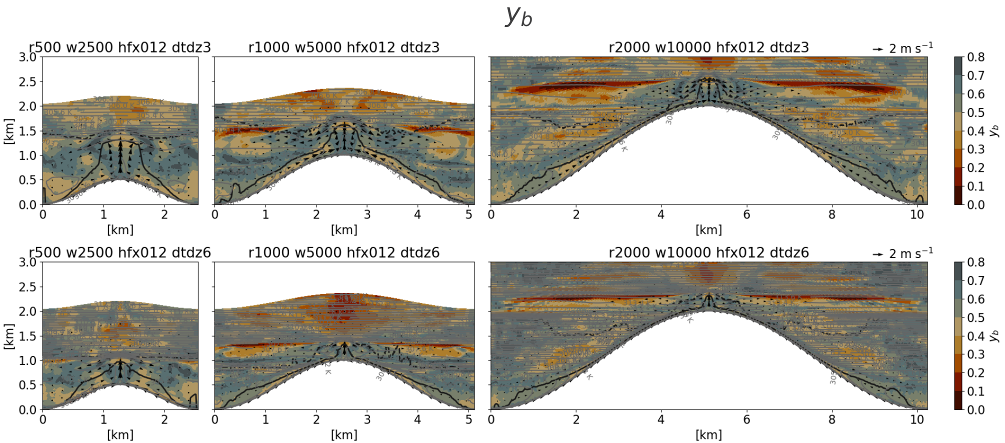
Background stratification: $0.003\ \mathrm{K\,m^{-1}}$ vs. $0.006\ \mathrm{K\,m^{-1}}$

Stratification: Flow structures



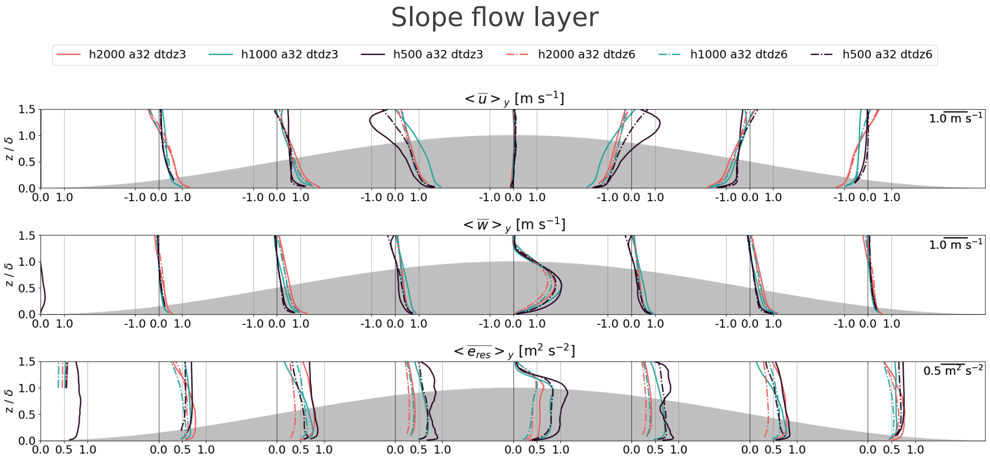
solid: slope flow layer depth δ, dashed: boundary layer height z_i

Stratification: Flow structures

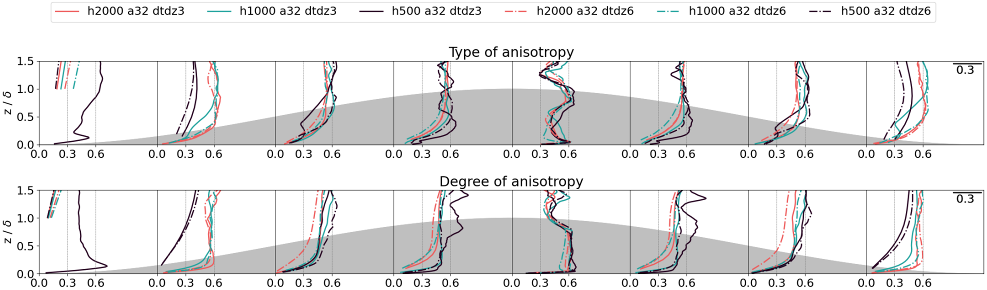


solid: slope flow layer depth δ, dashed: boundary layer height z_i

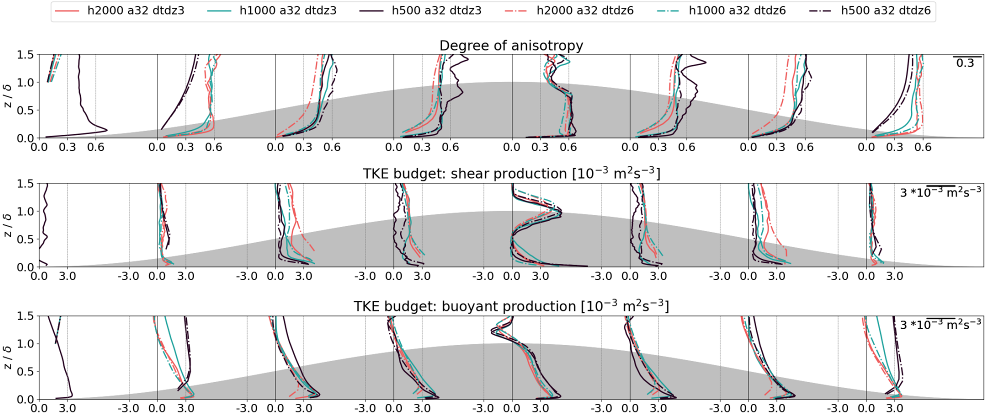
Stratification: Mean quantities: u, w



Stratification: Type and degree of anisotropy y_b

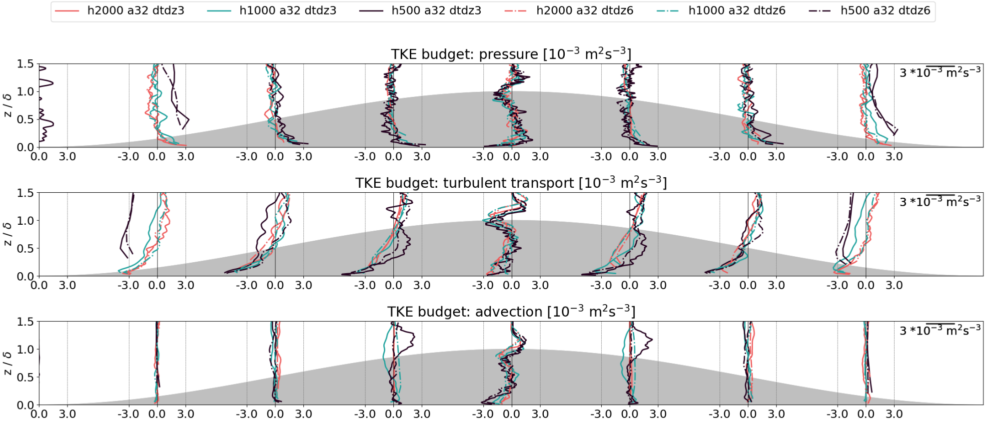


Stratification: degree of anisotropy, buoyancy and shear



Stronger stratification only changes regime , but not turbulence structure

Stratification: pressure, transport and advection



Conclusion

1. In anabatic flows over, turbulence is more anisotropic over steeper slopes
   - due to more shear production of TKE over steep slopes

2. Turbulence is more anisotropic when the slope angle is changing.
   - due to increased shear production over sinusoidal ridges compared to triangular ridges

3. Changes in the surface forcing and background stratification have less influence on turbulence anisotropy
   - BUT: Less surface heating and stronger stratification impact the flow regime
     - One or two circulation cells
     - If two cells, location if stable layer in between
   - BUT: Less surface heating and stronger stratification suppress turbulence
     - lower TKE
     - shallower slope flow layer

Acknowledgments

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Thank you for your interest

and feedback!

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Wien, 05.06.2026