

Supplementary material to oral talk

Searching for the links between minor sudden stratospheric warmings in the Southern Hemisphere and polar vortex parameters

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The stability of the polar vortex depends on both u and v components

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right) = \rho \vec{f} + \nabla_j (p_{ij} \vec{e}_i)$$

$$p_{ij} \approx -p \delta_{ij}$$

$$\vec{f} = \vec{g} + 2[\vec{v} \times \vec{\omega}] = \vec{g} + 2 \det \begin{pmatrix} \vec{e}_r & \vec{e}_\theta & \vec{e}_\varphi \\ 0 & -v & u \\ \omega \cos(\theta) & \omega \sin(\theta) & 0 \end{pmatrix}$$

$$\vec{f} = -\vec{e}_r 2u\omega \sin(\theta) + \vec{e}_\theta 2u\omega \cos(\theta) + \vec{e}_\varphi 2v\omega \cos(\theta) + g\vec{e}_r$$

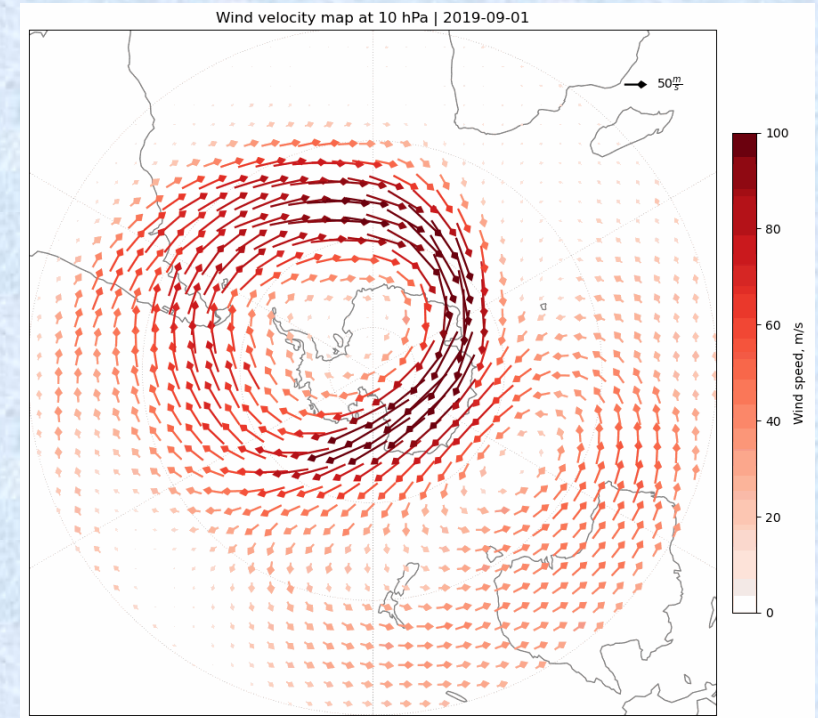
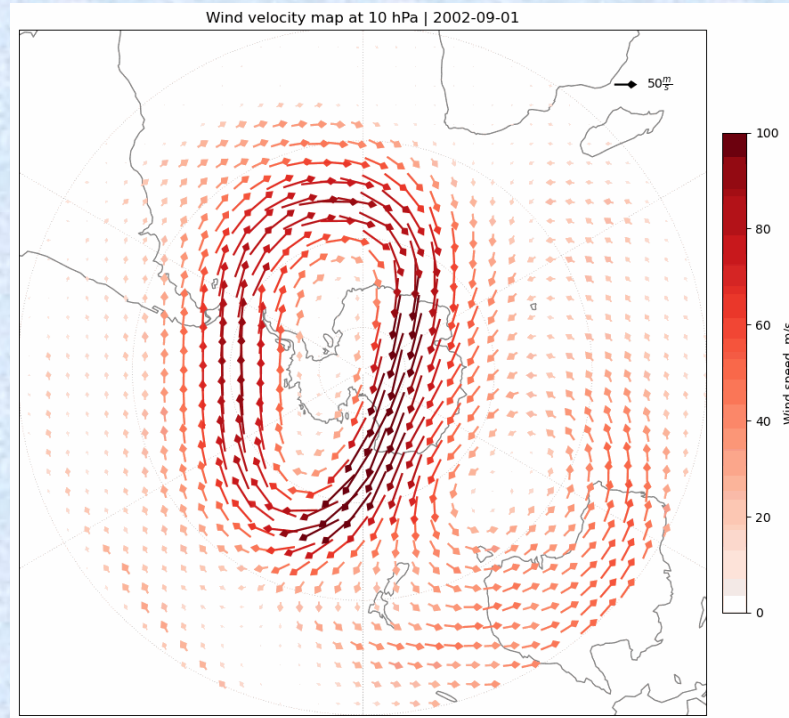
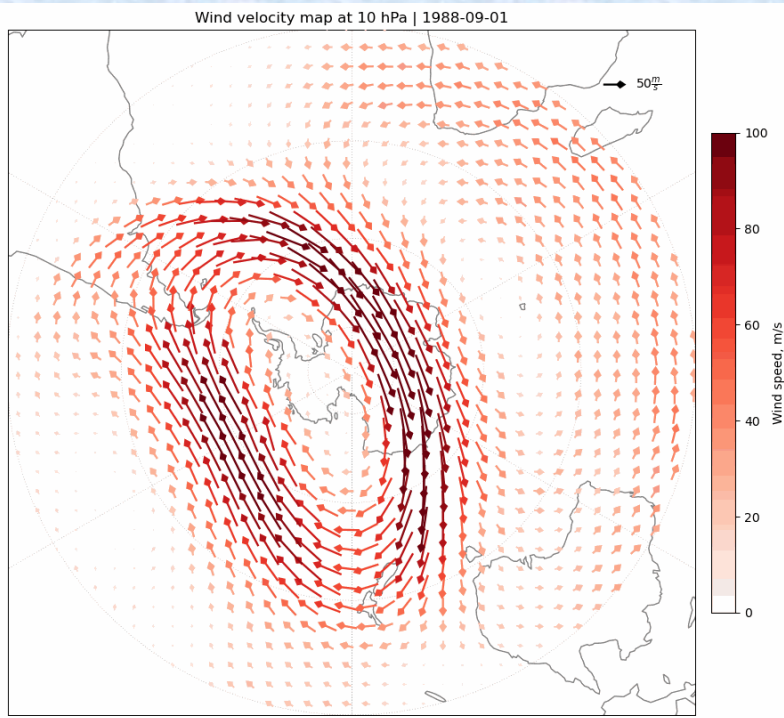
$$\theta = 90^\circ - \varphi$$

$$\frac{Du}{Dt} = 2v\omega \cos(\theta) - \frac{1}{\rho} \vec{\nabla} p \cdot \vec{e}_\varphi$$

$$\frac{Dv}{Dt} = 2u\omega \cos(\theta) - \frac{1}{\rho} \vec{\nabla} p \cdot \vec{e}_\theta$$

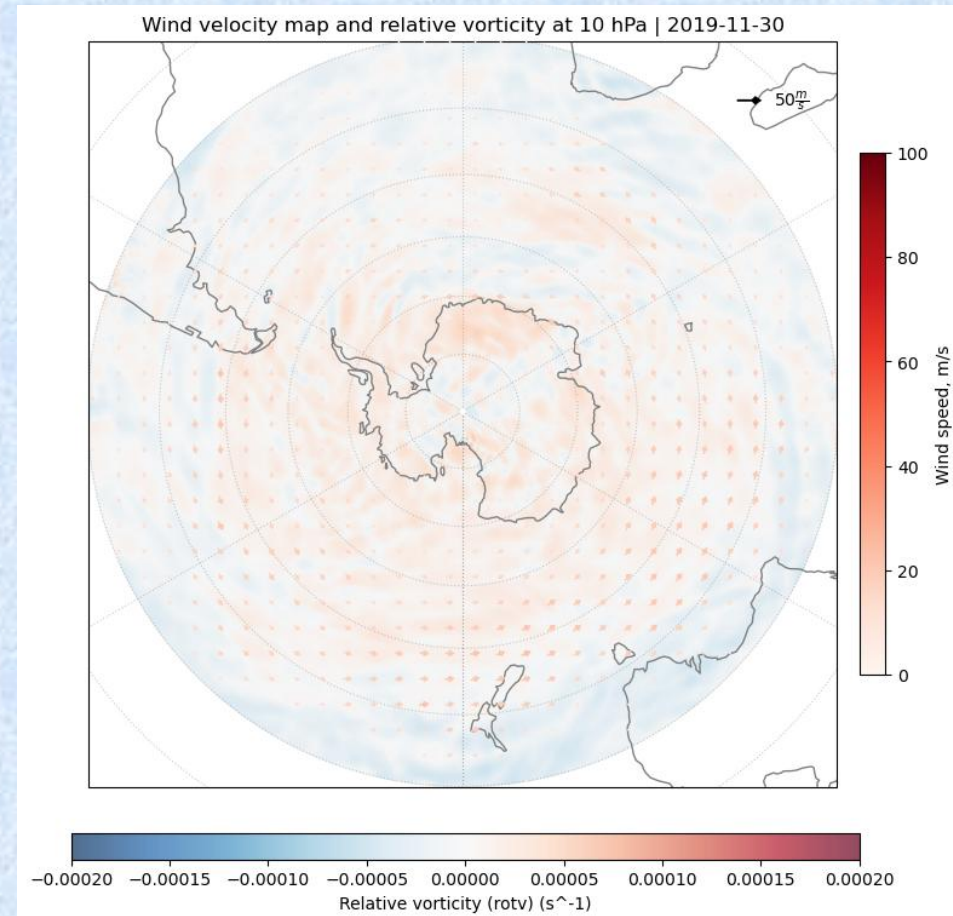
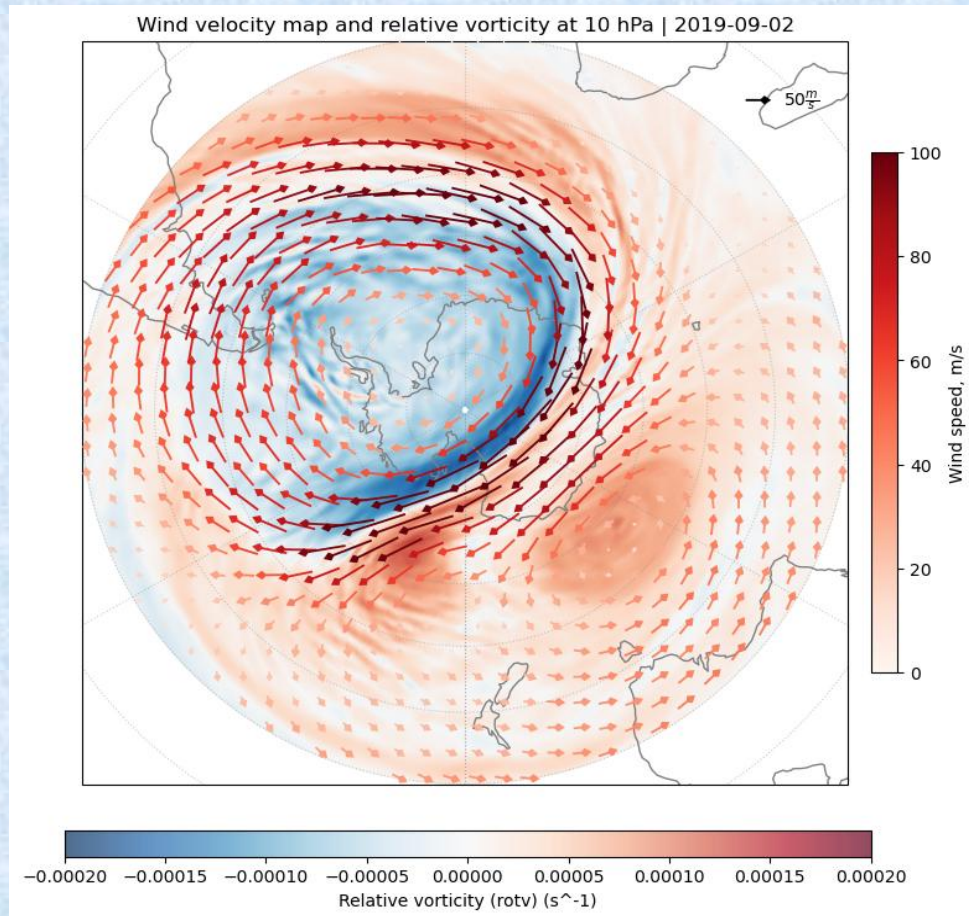
\vec{v} - wind velocity,
 u - longitudinal component,
 v - latitudinal component
 ω - angular velocity of Earth rotation

\vec{v} -field animations show deformation of the vortex in the presence of anticyclones



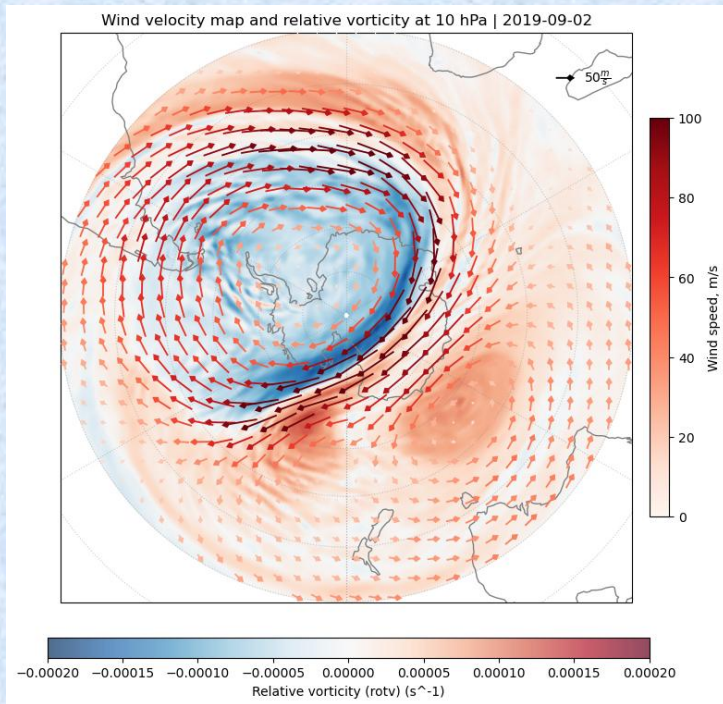
SSW in 1988, 2002, 2019 over Antarctica

The vortex strength should be considered in vortex area calculations



Area where $rot(\vec{v}) < 0$ could be the area of polar vortex, but the value of strength is not taken into the account

Circulation parameter may be used for vortex strength characteristic



$$C = - \sum_{\Delta S_i (\vec{e}_r \cdot \text{rot} \vec{v} < 0, \varphi < -\varphi_{max}^{\circ})} \vec{e}_r \cdot \text{rot} \vec{v}_i \Delta S_i$$

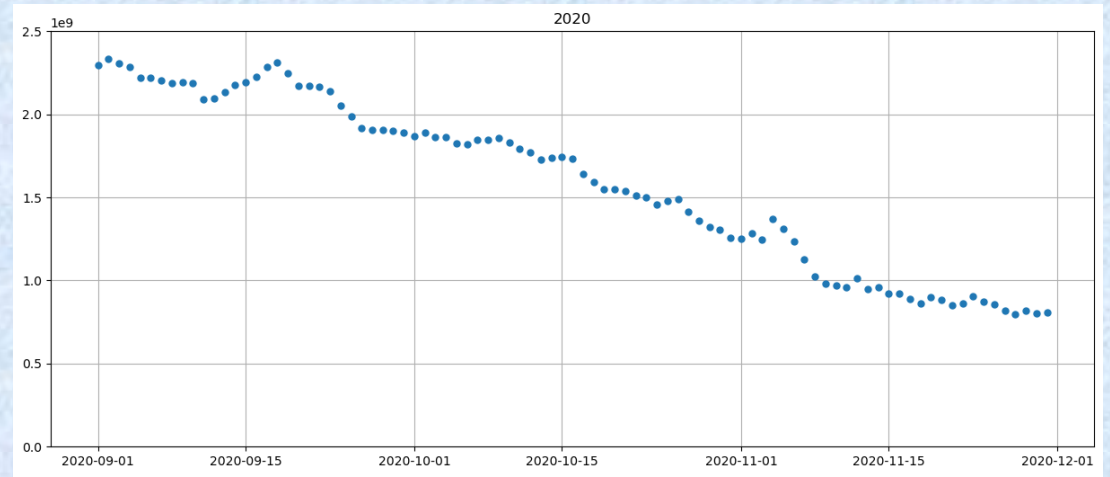
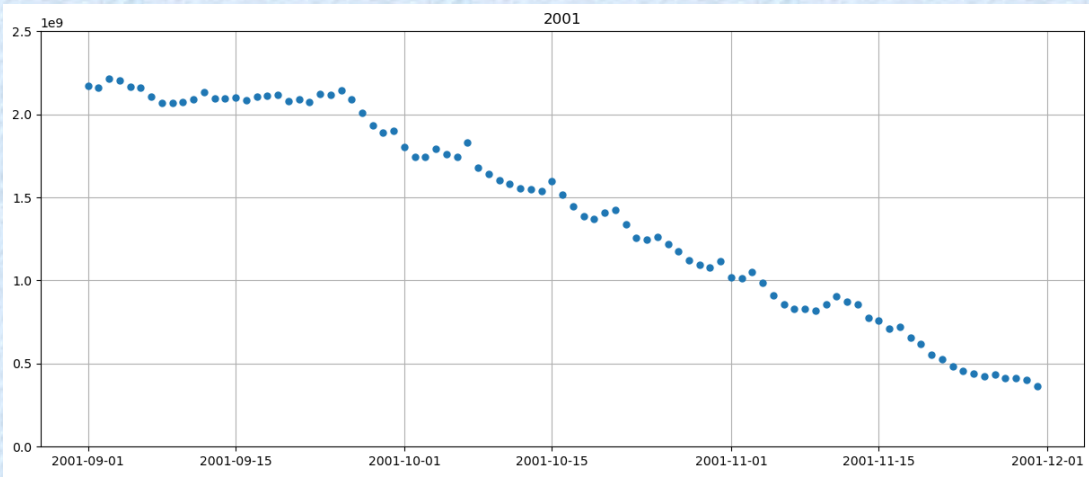
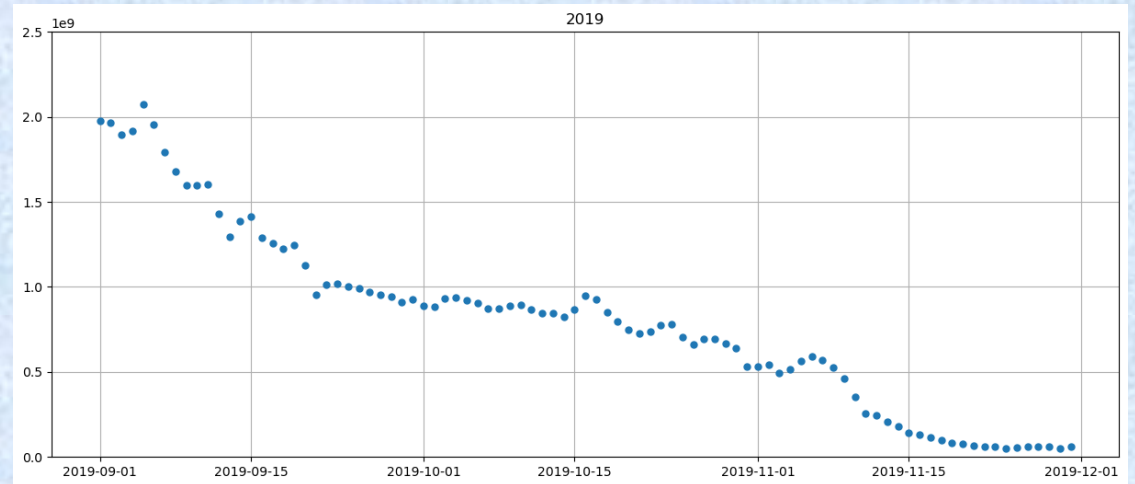
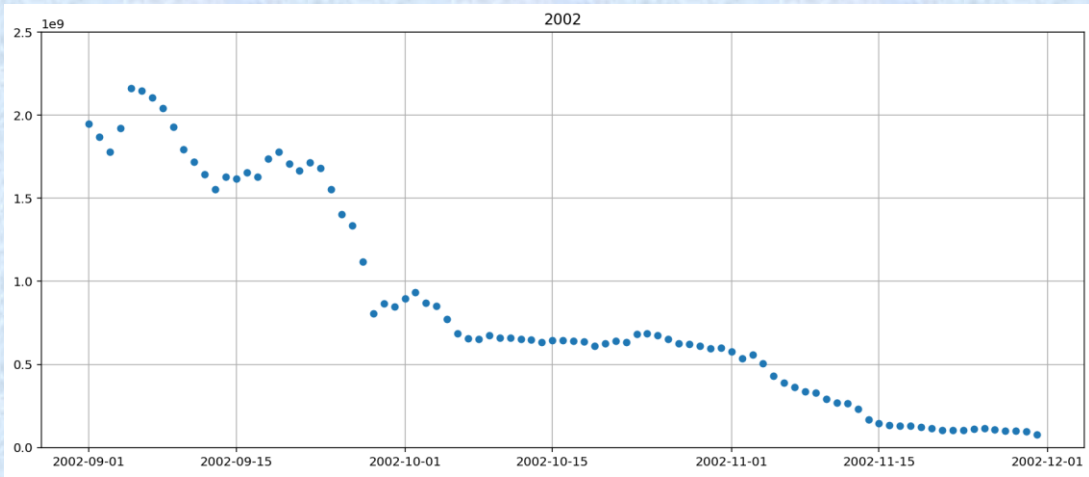
ΔS_i - area of the grid region,
where velocity is \vec{v}_i

$$\iint_{S(L)} \vec{e}_r \cdot \text{rot} \vec{v} dS = \iint_{S(L)} \text{rot} \vec{v} \cdot \vec{n} dS = \oint_L \vec{v} \cdot \vec{\tau} dL$$

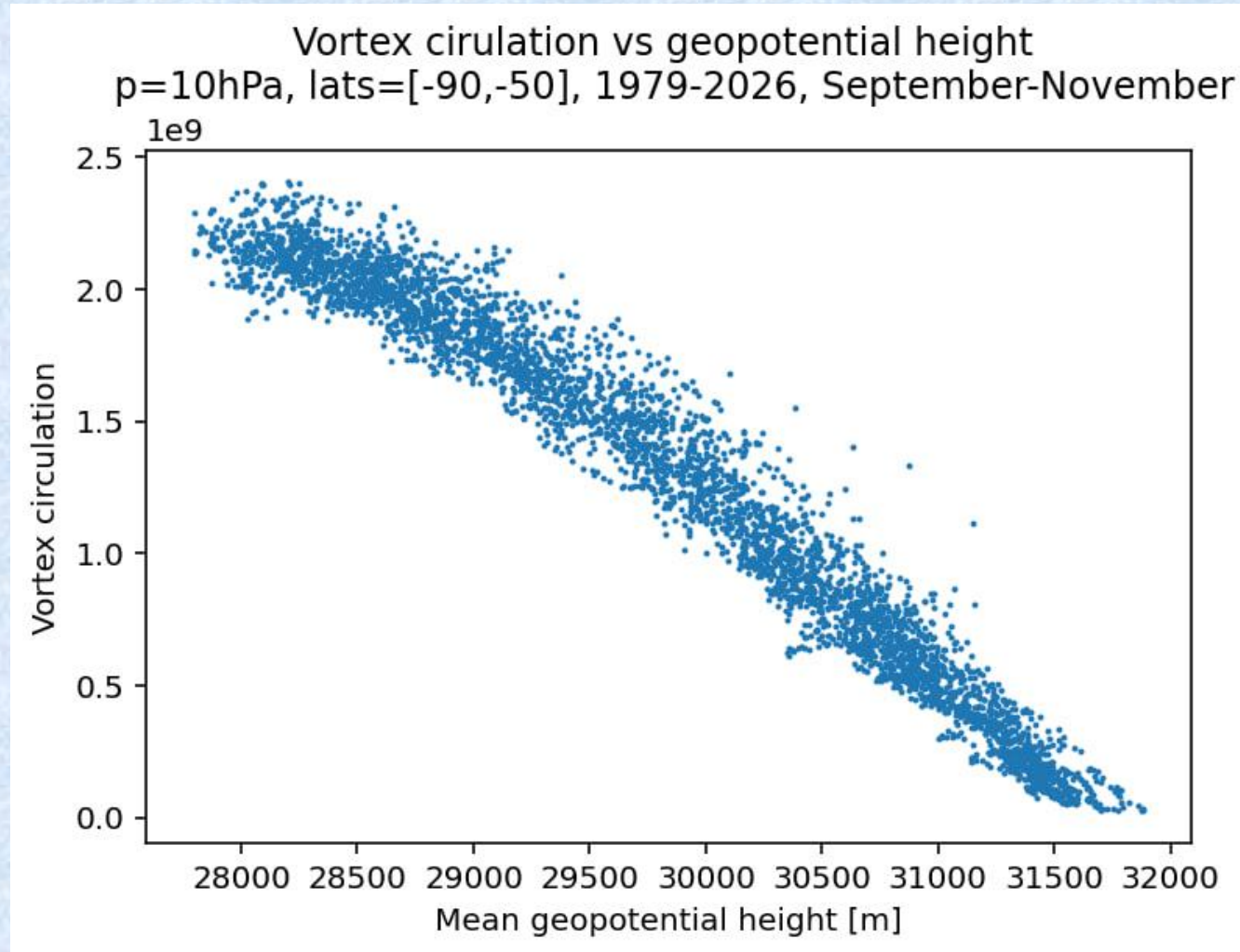
C – analogue of the circulation of vector field \vec{v}

This parameter physically represents the total ‘rotational capacity’ of the vortex, it is proportional to the mass unit angular momentum of the vortex

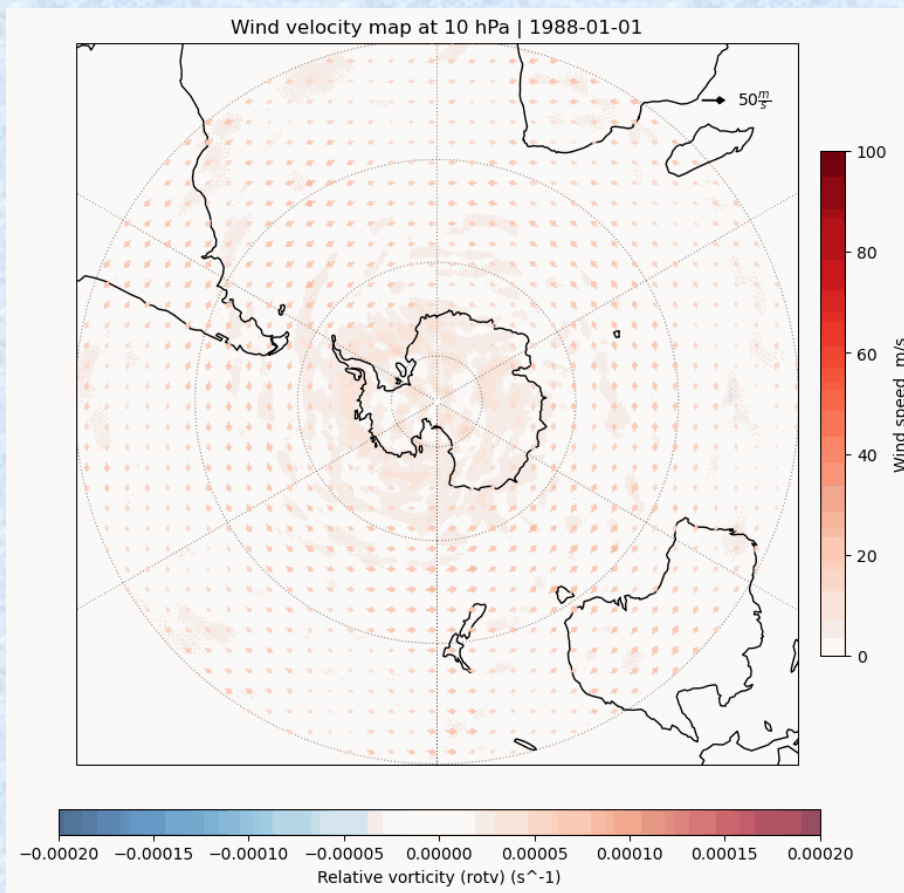
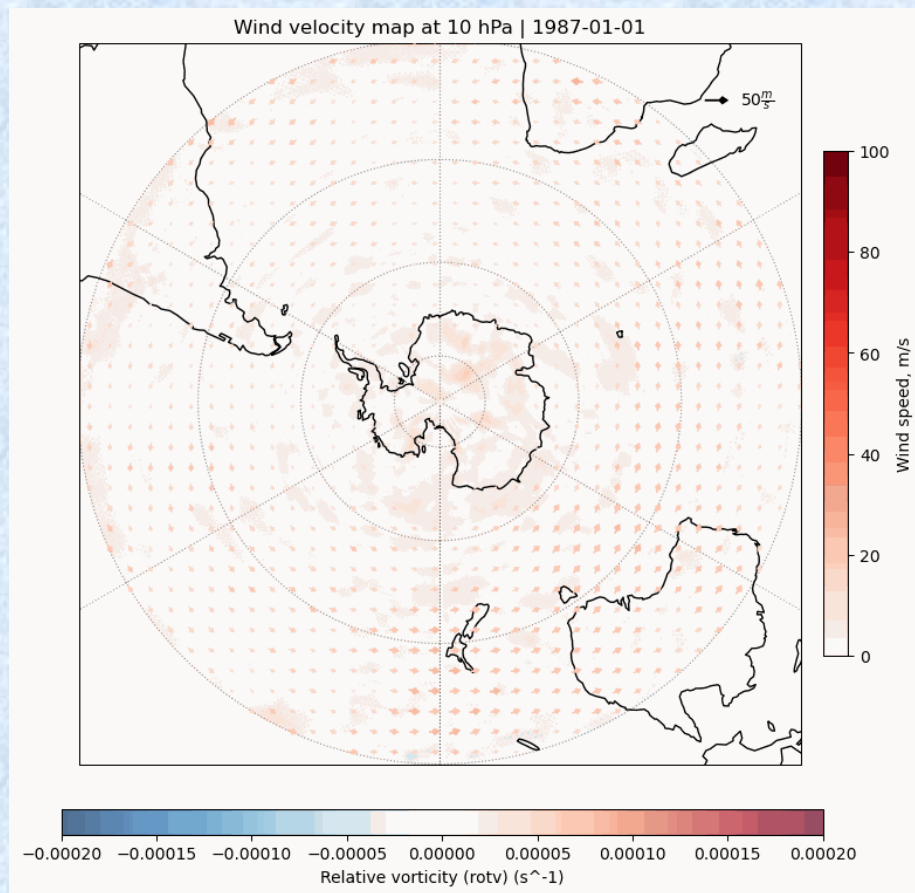
Circulation parameter for warmings years is lower than for years with strong vortex



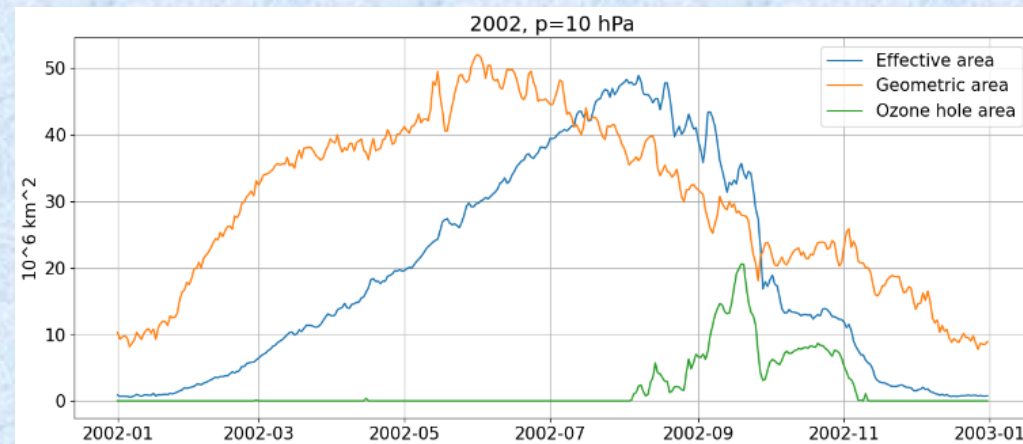
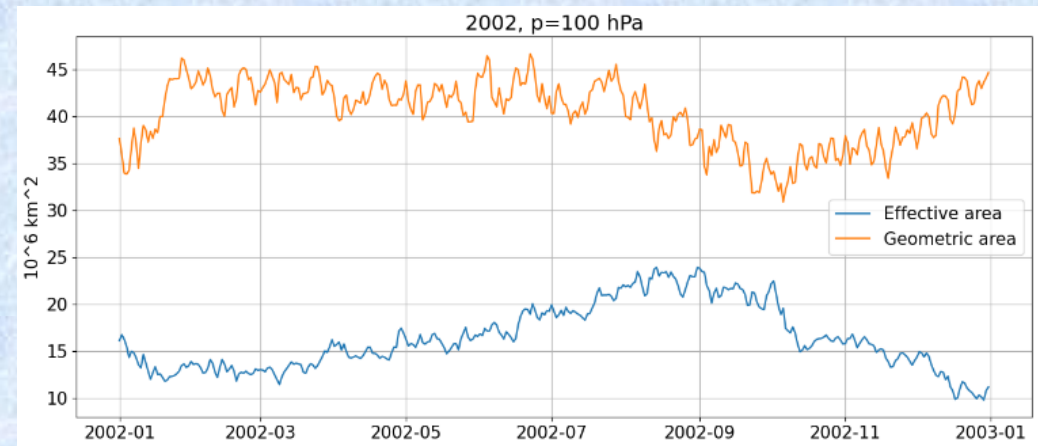
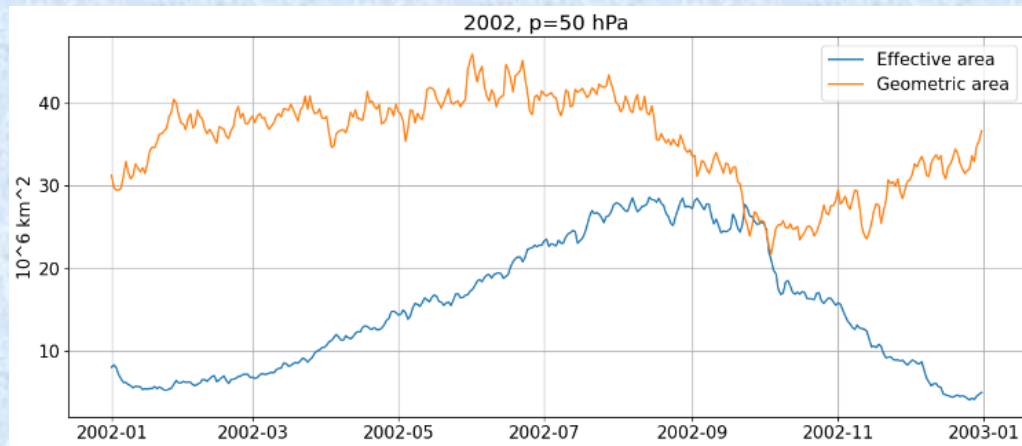
There is strong correlation between circulation and geopotential height



Differences in polar vortex in 1987 and 1988



The effective area shows the higher changes at 10hPa in comparison to 50hPa and 100hPa as well as geometric area



The conservation of circulation in case of potential fields and baroclinic state

In case of baroclinic $\rho = \rho(p)$ frictionless vortex state inside the potential field the circulation parameter is conserved.

$$\frac{d\vec{v}}{dt} = \vec{f} - \frac{1}{\rho} \vec{\nabla} p$$

$$\oint_L \frac{d\vec{v}}{dt} d\vec{L} = \oint_L \left(\vec{f} - \frac{1}{\rho} \vec{\nabla} p \right) d\vec{L} = \oint_L \vec{f} d\vec{L} - \oint_L \frac{1}{\rho(p)} \vec{\nabla} p d\vec{L}$$

$\oint_L \vec{f} d\vec{L} = 0$ in case of potential fields

$\oint_L \frac{1}{\rho(p)} \vec{\nabla} p d\vec{L} = \oint_L \vec{f}(p) d\vec{L} = \oint_L d\Phi(p) = 0$ if the density is the function of the pressure only

Other ways to calculate vortex area

Area, surrounded by constant value of PV^* at the level of constant potential temperature θ_0 (Butchart & Remsberg, 1986)

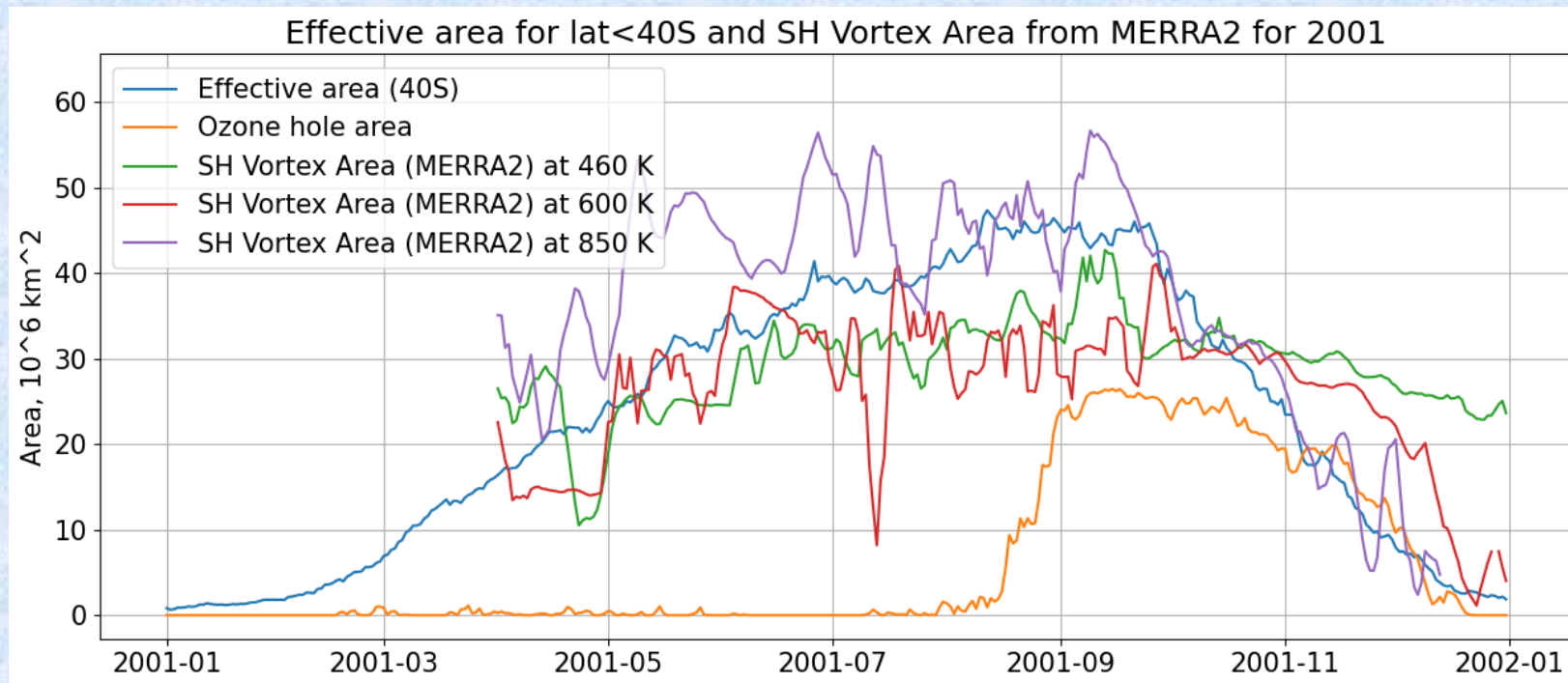
$$\frac{dA(t)_{PV \geq PV^*}}{dt} = \oint_L \frac{F}{|\nabla_{\theta} PV|} dL + \oint_{A(t)_{PV \geq PV^*}} \vec{\nabla}_{\theta} \vec{v}_{\theta} dA$$

$$F = \frac{1}{\rho} (2\vec{\omega} + rot\vec{v}) \cdot \vec{\nabla} \left(\frac{D\theta}{Dt} \right) - \frac{D\theta}{Dt} \frac{\partial PV}{\partial \theta} \Big|_{\theta=\theta_0}$$

Sensitive to the PV^* value.

Other ways to calculate vortex area

The Nash method defines the polar vortex boundary at the equivalent latitude ϕ_{eq} , where $\frac{\partial PV}{\partial \phi_{eq}}$ reaches a local maximum. Strong oscillations of vortex area exists.



We used MERRA-2 data at different potential temperature values and see that such area is oscillating. So it is difficult to say if this change points to SSW event or not. Our effective area parameter (blue line) is definitely more stable.