

# Learning the Kalman Update End-to-End for Hydrological Forecasting

*From Noise-Covariance Optimization to Neural Gain Learning*

A unified differentiable Kalman framework for learning hydrological state updates

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Yiqun Sun, Xin Tian, Qiongfang Li, Zheng Fang, Sida Liu, Hamid Moradkhani

Hohai University | Deltares | IGRAC | Wageningen University & Research | University of Alabama

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# The Problem and the Gap

## Classical Kalman filtering

*UKF, EnKF*

- ✓ **Sequential predict-update loop**  
Online correction at every time step
- ✗ **Q/R have no ground truth in hydrology**  
Hand-tuned, not reproducible

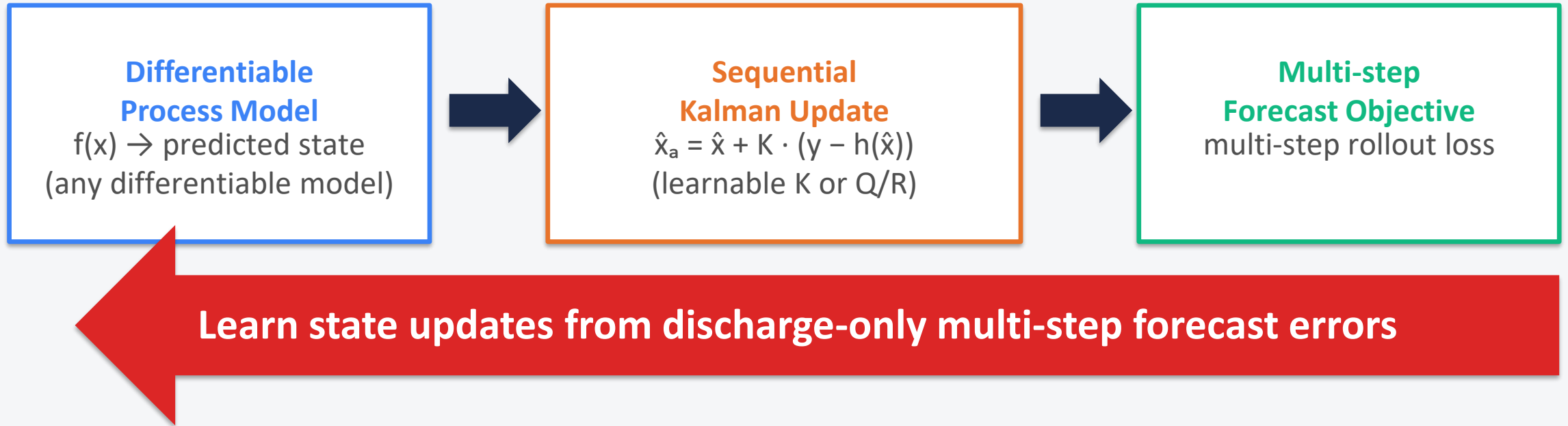
## Recent deep-learning DA

*variational, autoregressive, ...*

- ✓ **Learns the update from data**  
No more hand-tuning
- ✗ **Abandons the sequential loop**  
Offline — loses online correction

**Can we keep the sequential Kalman loop AND learn the update from data?**

# Our Proposal: A Unified Differentiable Kalman Framework



- ▶ After each update, train with a 12-hour discharge forecast loss
- ▶ Self-supervised: discharge observations only — no state labels
- ▶ Extensible to differentiable Kalman-type filters

# Two Learnable Update Paradigms Within the Framework

## Learned-noise UKF

*Indirect update learning*

- ▶ Retain the classical covariance-based UKF update
- ▶ Learn  $\text{diag}(Q) + R$  via backpropagation
- ▶ **6 parameters only**
- ▶ Interpretable Q/R parameters

## LSTM-NGF

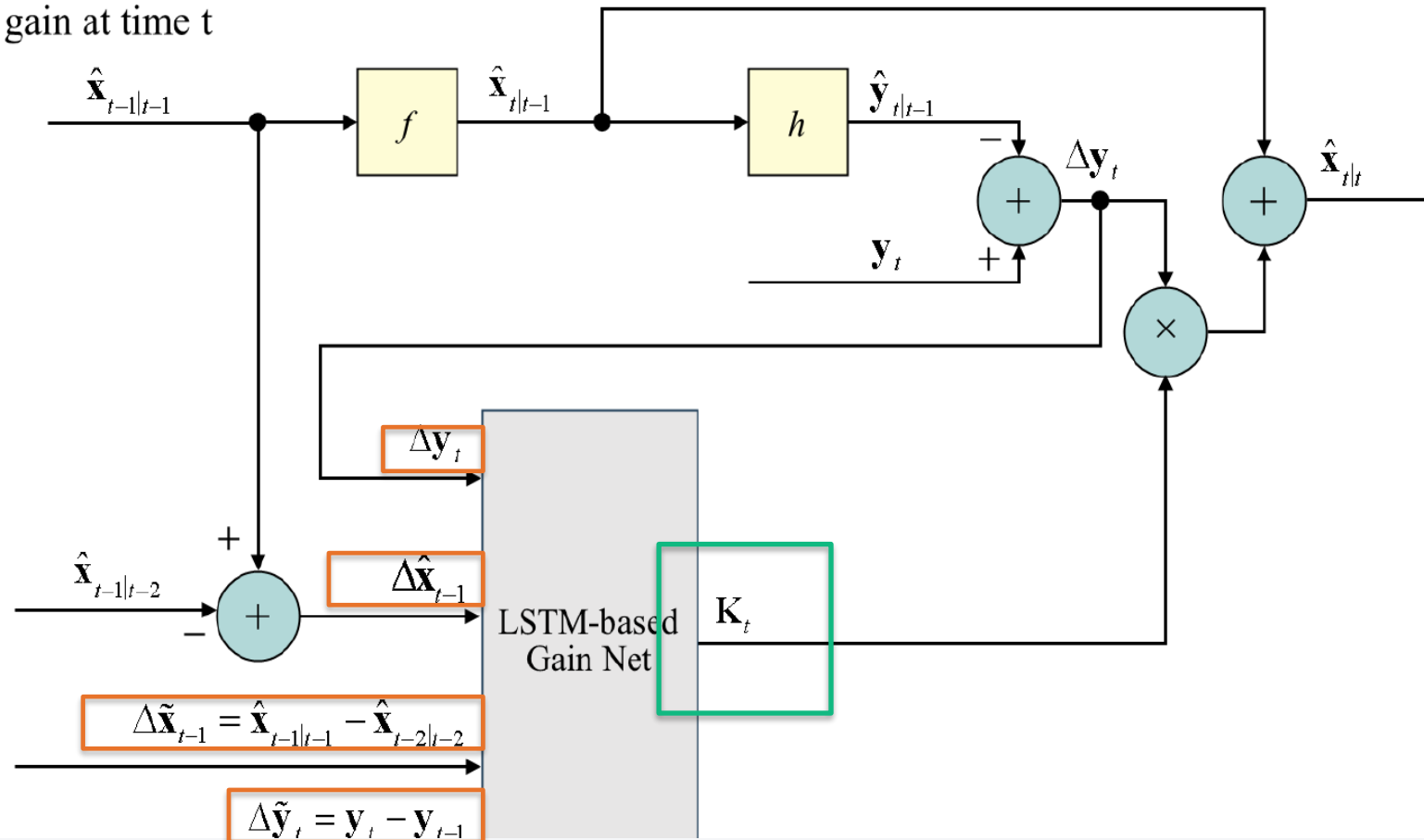
*Direct update learning*

- ▶ LSTM replaces covariance-based gain computation entirely
- ▶ Input: innovation + state evolution
- ▶ **~49,000 parameters**
- ▶ Time-varying, context-aware gain

Same framework · Same process model · Same objective · **Only the update-learning strategy differs**

# LSTM-NGF: How the Neural Gain Is Computed

$y_t$  Observation at time  $t$        $f$ : state transition function  
 $\hat{x}_{t|t}$  Updated state at time  $t$        $h$ : measurement function  
 $\hat{x}_{t|t-1}$  Predicted state at time  $t$   
 $\hat{y}_{t|t-1}$  Predicted output at time  $t$   
 $K_t$  Kalman gain at time  $t$



## Key idea

LSTM learns  $K$  from innovation sequences directly

## Inputs

- ▶ Innovation
- ▶ Observation diff
- ▶ Forward state evolution
- ▶ Update difference

## Why it matters

Time-varying gain can help when covariance updates weaken

# Controlled Comparison Within the Proposed Framework

## Study Area

### Regge Catchment

The Netherlands · 957 km<sup>2</sup> · Lowland

### Data

Hourly P, ETpot, Q (1999–2011)

12 years · ~105,000 time steps

### Process Model

WALRUS (5 state variables, 1 output)

## Experiment Design

### Data Split

70 / 15 / 15 (train / val / test)

Chronological (2009–2011 = test)

### Training

800 h sliding windows + augmentation

Training horizon  $T = 12$  h

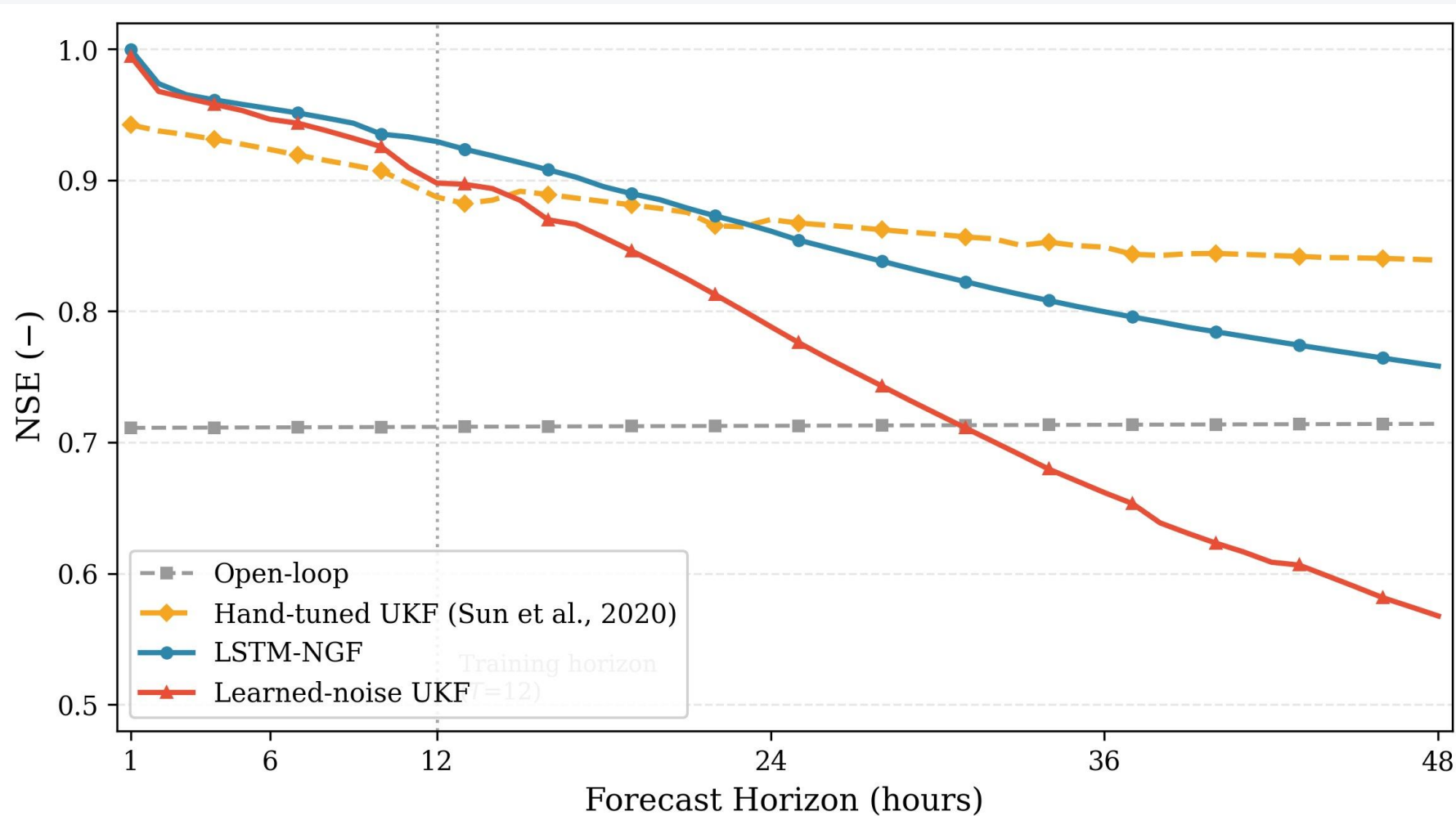
Loss: composite multi-step objective  
(weighted MSE-based)

### Evaluation

Dense rolling forecasts (main focus: H1–H24)

*Training uses a 12-hour rollout objective; results beyond H12 therefore evaluate out-of-training-horizon generalization.  
Same process model, same data split, same training objective — only the update-learning strategy changes.*

# Direct Gain Learning Gives the Best Short-Range Skill



At H12 (*training boundary*)

**0.929 LSTM-NGF**

**0.898 Learned-noise UKF**

**0.887 Hand-tuned UKF**

At H24 (*beyond training*)

**0.861 LSTM-NGF**

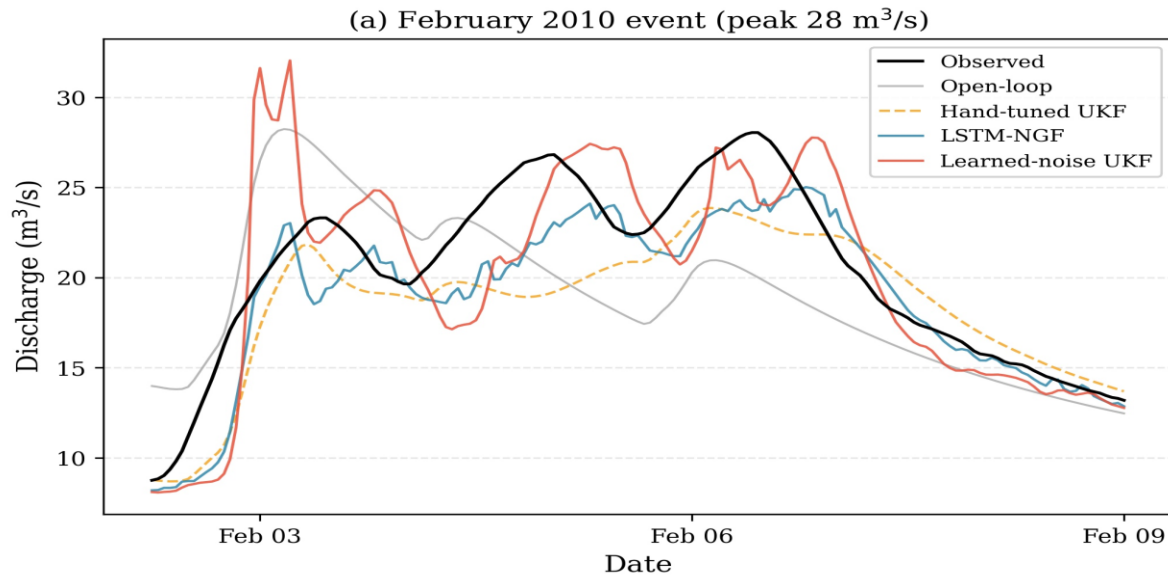
**0.870 Hand-tuned UKF**

**0.788 Learned-noise UKF**

LSTM-NGF gives the strongest short-range skill and shows the strongest H12 performance across all three flow regimes, while the hand-tuned UKF stays more conservative beyond the training horizon.

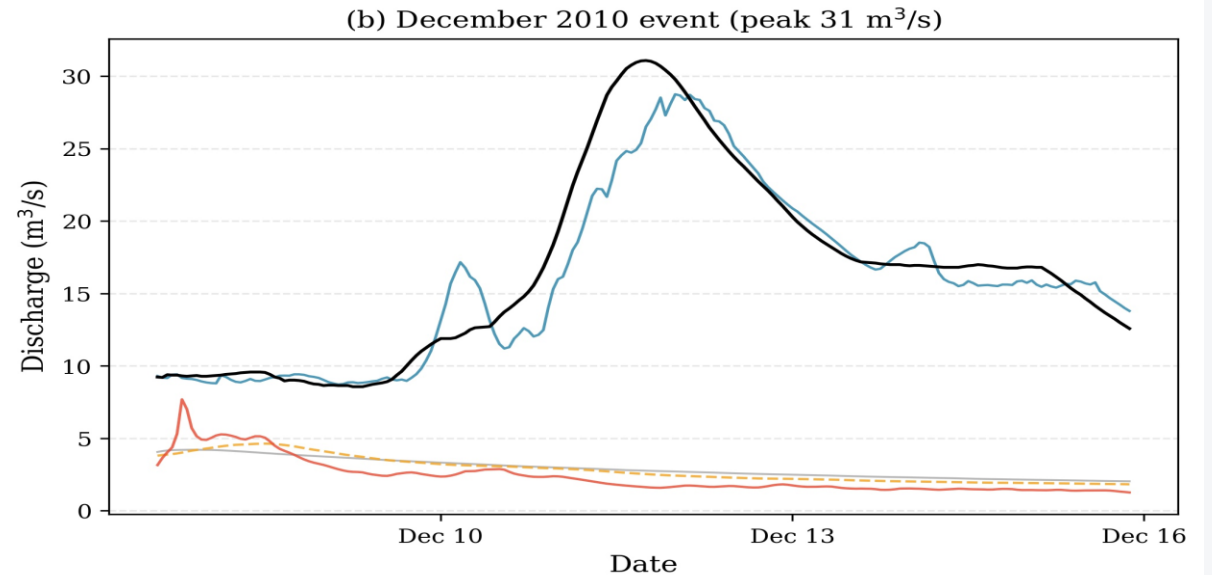
# Why the Methods Differ: Flood-Event Behavior

12-hour-ahead (H12) forecasts on independent test set (2009–2011)



## (a) Multi-peak event — Feb 2010

- ▶ LSTM-NGF tracks the observed hydrograph most closely
- ▶ Learned-noise UKF oscillates (near-zero R)
- ▶ Hand-tuned UKF underestimates peaks

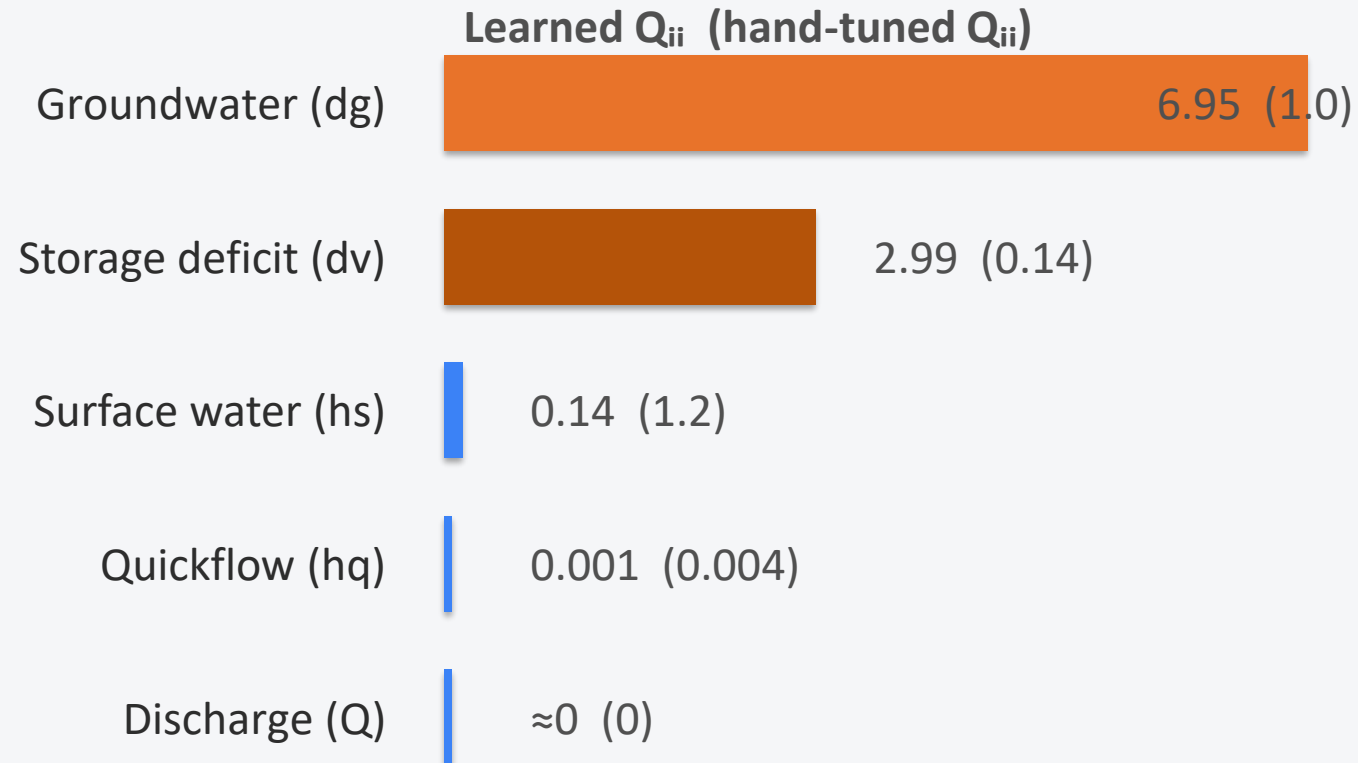


## (b) Sensitivity-collapse-like case — Dec 2010

- ▶ Both UKF variants give weak corrections in this sensitivity-collapse-like regime
- ▶ LSTM-NGF still captures ~92.5% of the flood peak, consistent with its gain not being directly tied to local covariance sensitivity

# Learned Q/R Provide Forecast-Oriented Diagnostics

What indirect learning adds: transparent Q/R diagnostics



## Key Findings

- ▶ Keeps covariance-based Kalman structure
- ▶ Learns only 6 parameters:  $\text{diag}(Q) + R$
- ▶ Largest learned  $Q$ : groundwater + storage deficit
- ▶ Learned  $R \approx 0$ : observations dominate
- ▶ Shows where forecast loss allows stronger corrections

***Forecast-oriented effective parameters — not physical noise estimates.***

# Take-Home Messages

1

**A unified differentiable Kalman framework makes end-to-end update learning possible**

Self-supervised with discharge only; retains the sequential predict-update loop.

2

**Direct gain learning gives the strongest short-range forecast skill**

Strongest H12 performance across all three flow regimes; shows more robust flood-event behavior in this case study.

3

**Learning Q/R remains useful**

Preserves Kalman structure and exposes forecast-oriented Q/R diagnostics.

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**Outlook:** Multi-catchment validation · Hybrid short-range / long-range strategy

# Thank You!

I'd be happy to discuss both the methodological side  
and the hydrological implications.

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Manuscript under review

**Contact**

[yiqun.sun@hotmail.com](mailto:yiqun.sun@hotmail.com)