Sea Ice Thickness Measurement in Polar Environments: An Electromagnetic Detection Approach Using a Dual-Receiver Coil System

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Highlights

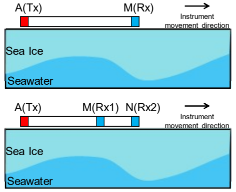
- ➢ Improved the single electromagnetic sensor architecture of traditional electromagnetic sea ice detection instruments by adopting dual sensors for reception, enabling the acquisition of richer electromagnetic information, which lays a physical foundation for more fine-scale ice thickness characterization.
- ➢ Conducted inversion research on the data characteristics measured by the dual receivers, and further developed a three-feature weighted inversion algorithm for ice thickness retrieval. To validate its effectiveness, inversion was performed using both synthetic data and measured data, and the results were compared with ice thickness measurements obtained from the EM31 instrument, verifying the feasibility of the developed algorithm.

1 Motivation

Sea Ice Thickness (SIT) is a critical parameter for polar climate modeling, marine navigation, and ocean-atmosphere energy exchange.Electromagnetic (EM) induction is the most effective non-destructive method for SIT measurement, relying on the extreme conductivity contrast between highly resistive sea ice and highly conductive seawater.

Problems

- The existing EM31 instrument relies on single electromagnetic sensor reception, providing only a single data point per measurement, which exacerbates the ill-posedness of the sea ice thickness inversion problem.
- The currently widely used method for obtaining ice thickness still employs measured seawater salinity combined with empirical formula fitting. In the empirical fitting formula, a single conductivity value is often used to represent the conductivity of the entire region; therefore, results obtained through this approach are generally considered relatively coarse.



Our Proposed Solution

Designed a dual-receiver coil system to acquire dual-offset electromagnetic responses, enriching the collected electromagnetic information.

2 Forward modeling theory and finite element solution

Based on Maxwell' s equations under the quasi-static approximation (ignoring displacement currents), the frequency-domain electromagnetic diffusion equation for the magnetic field $\boldsymbol{H}$ is:

$$\nabla \times \left(\frac{1}{\sigma} \nabla \times \mathbf{H} \right) + i\omega\mu_0 \mathbf{H} = \mathbf{J}_s$$

To avoid the source singularity of the transmitter (Tx), the total field $\boldsymbol{H}_{tot}$ is decomposed into a primary field $\boldsymbol{H}_p$ and a secondary field $\boldsymbol{H}_s$:

$$\mathbf{H}_{tot} = \mathbf{H}_p + \mathbf{H}_s$$

We utilize a 2D FEM to accurately simulate complex sea-ice morphologies.

Following the low-induction-number approximation, the quadrature (out-of-phase) component of the secondary magnetic field is extracted. For our Dual-Receiver system, we obtain two independent apparent conductivities with different coil separations

$$\sigma_{a,j} \approx \frac{4}{\omega\mu_0 s_j^2} \operatorname{Im}\left(\frac{H_{s,j}}{H_{p,j}} \right), \quad j \in \{1,2\}$$

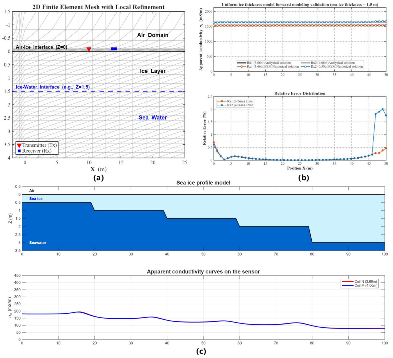


Fig.(a) Mesh discretization adopted in this study

Fig.(b) To verify the effectiveness of the forward modeling algorithm, we established a model with uniform ice thickness, compared the numerical solution with the analytical solution, and calculated the error.

Fig.(c) This figure presents a stepped sea ice model and shows the apparent conductivity curves on the two receiving sensors.

3 Three-feature weighted Gauss-Newton inversion

To fully exploit the dual-receiver system and mitigate the ill-posedness of 2D inversion, we extract three distinct physical features to constrain the ice thickness model ($\boldsymbol{m}$):

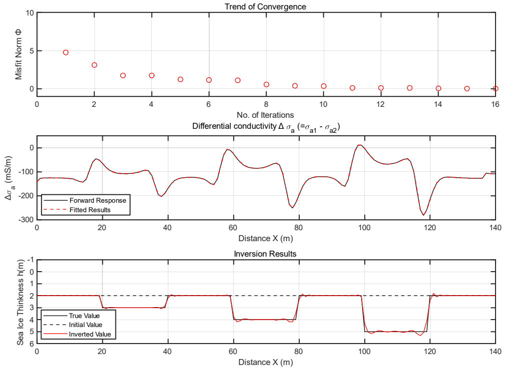
- **Feature 1**($\boldsymbol{d_1}$): Quadrature response of Rx1→High resolution for shallow level ice and thin ice.
- **Feature 2**($\boldsymbol{d_2}$):Quadrature response of Rx2→Deeper penetration capability for thicker sea ice.
- **Feature 3**($\boldsymbol{d_3}$):Differential response $\Delta = \boldsymbol{d_1} - \boldsymbol{d_2}$→Provides finer ice thickness gradient information, enabling the inversion algorithm to respond more sensitively to ice thickness variations.

For the three features, we constructed the data weighting matrix:

$$\Phi(\mathbf{m}) = \| \mathbf{W}_d (\mathbf{d}_{obs} - \mathbf{d}_{pred}(\mathbf{m})) \|_2^2 + \lambda \| \mathbf{W}_m \mathbf{m} \|_2^2$$

The non-linear inversion is iteratively solved using a damped Gauss-Newton method. At the $\boldsymbol{k}$-th iteration, the model update Δ is obtained by solving:

$$\left(\mathbf{J}^T \mathbf{W}_d^T \mathbf{W}_d \mathbf{J} + \beta_k \mathbf{I} + \lambda \mathbf{W}_m^T \mathbf{W}_m \right) \Delta\mathbf{m} = \mathbf{J}^T \mathbf{W}_d^T \mathbf{W}_d \Delta\mathbf{d} - \lambda \mathbf{W}_m^T \mathbf{W}_m \mathbf{m}_k$$

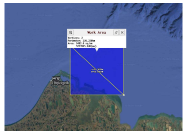


(Top) Convergence Curve: The weighted objective function decreases rapidly and stabilizes, proving the robustness of the multi-feature GN algorithm.

(Middle) Differential Feature Fitting: Excellent data fitting of the differential feature （ Δd ）, demonstrating the algorithm's ability to capture high-frequency morphological variations.

(Bottom) Ice Thickness Reconstruction: The results indicate that the algorithm achieves favorable inversion outcomes for sea ice of varying thicknesses in the model, and the positions of abrupt ice thickness transitions align well with those in the true model.

4 Application of ice thickness inversion using measured data



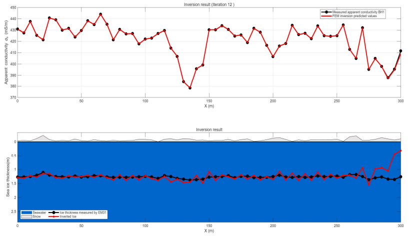
To validate the core 2D FEM inversion using real-world scenarios, we utilized the public dataset from the NASA AMSRIce03 campaign (Barrow, Alaska).

- **Instrument:** EM-31
- **Provided parameters:** sea ice thickness, snow depth, and seawater salinity measured by the EM-31 built-in module, etc.

Table 1.Parameter or Variable

Dist (m)	BH1 (Raw measurement of conductivity)	E (Ocean salinity)	Snow (cm)	Ice+Snow (cm)
0	430.9	33.34	3.7	130.9
5	427.56	33.38	5.5	132
10	437.62	33.384	3.7	128.9
15	425.42	33.388	11.8	132.6
20	421.44	33.39	22.8	133.9
25	440.68	33.39	7.8	128
30	439.14	33.391	3.2	128.5
35	430	33.393	5.4	131.2
...	...	...	...	...

Although our proposed hardware is a dual-receiver system, our 2D Gauss-Newton inversion framework is fully backward-compatible with legacy single-coil data. We adapted the algorithm to perform a single-feature 2D inversion on the EM31 apparent conductivity data, aiming to overcome the inherent limitations of the traditionally used 1D empirical transformation.



The two show relatively small differences in ice thickness at the starting and middle segments, falling within the same numerical range, but exhibit significant divergence at the tail end, which may be caused by insufficient mesh discretization accuracy at the end of the survey line.

5 Conclusion

- The proposed dual-receiver coil system effectively overcomes the under-determined nature of traditional single-coil EM instruments. By acquiring multi-offset apparent conductivity data, it provides richer EM information, which is physically essential for resolving complex sea ice morphometry.
- We developed a three-feature weighted Gauss-Newton inversion based on the 2D Finite Element Method. By introducing differential response and regularization operations, the algorithm possesses the capability to jointly fit ice thickness using dual-sensor and their differential information, preliminarily achieving profile reconstruction of simple sea ice thickness models.
- The application of the 2D inversion framework to legacy EM31 field data has demonstrated the feasibility of the inversion method to a certain extent. Compared with conventional 1D empirical fitting, our 2D approach can incorporate seawater salinity at different data points, thereby enabling more precise inversion of sea ice thickness during underway surveys.

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