Quantifying differences in circulation patterns with probabilistic methods

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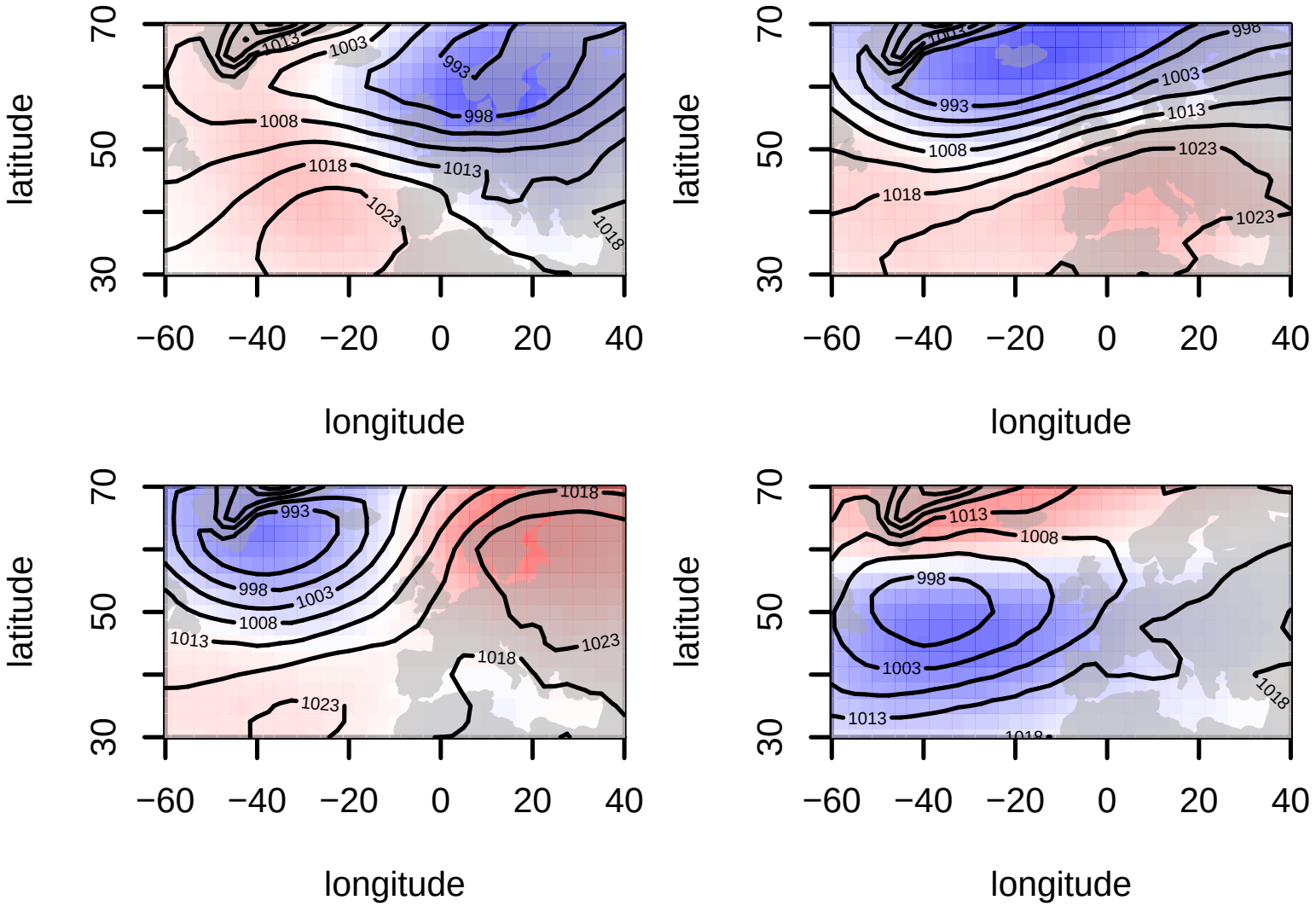
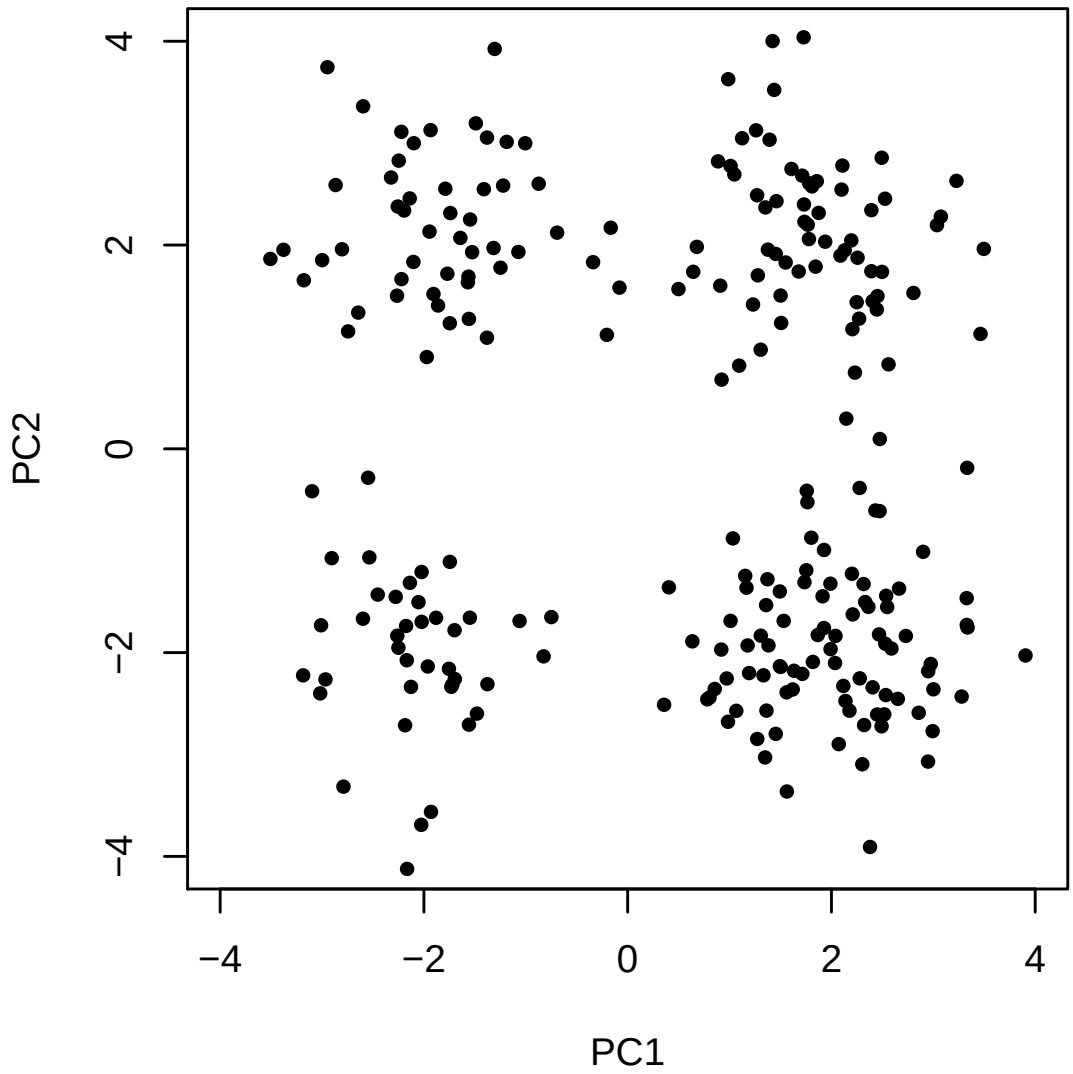
EMS2011, Berlin

Motivation


latitude
longitude
70
50
30
−60
−40
−20
0
20
40
1013
1003
993
998
1008
1018
1023

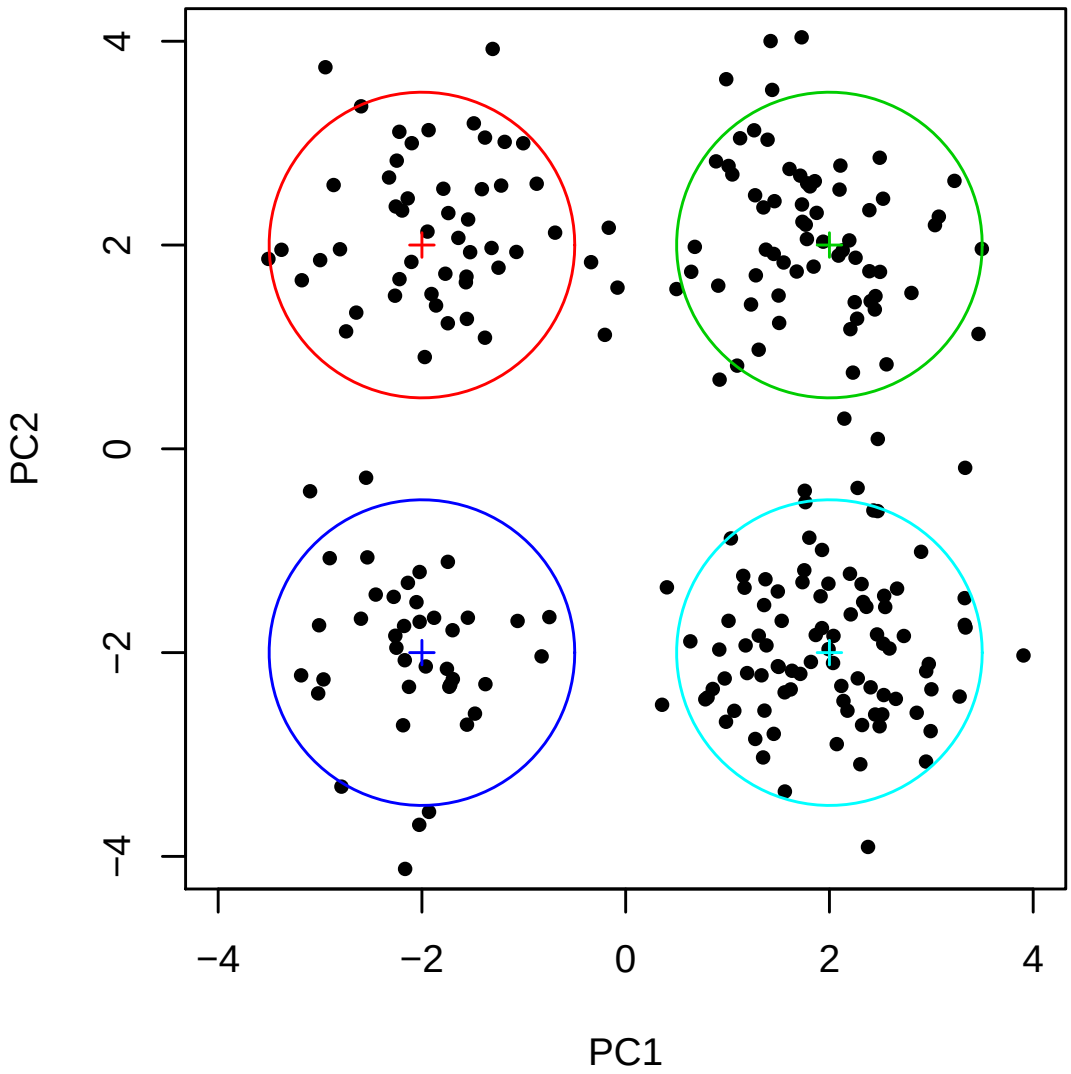
Motivation



Procedure

- reanalysis

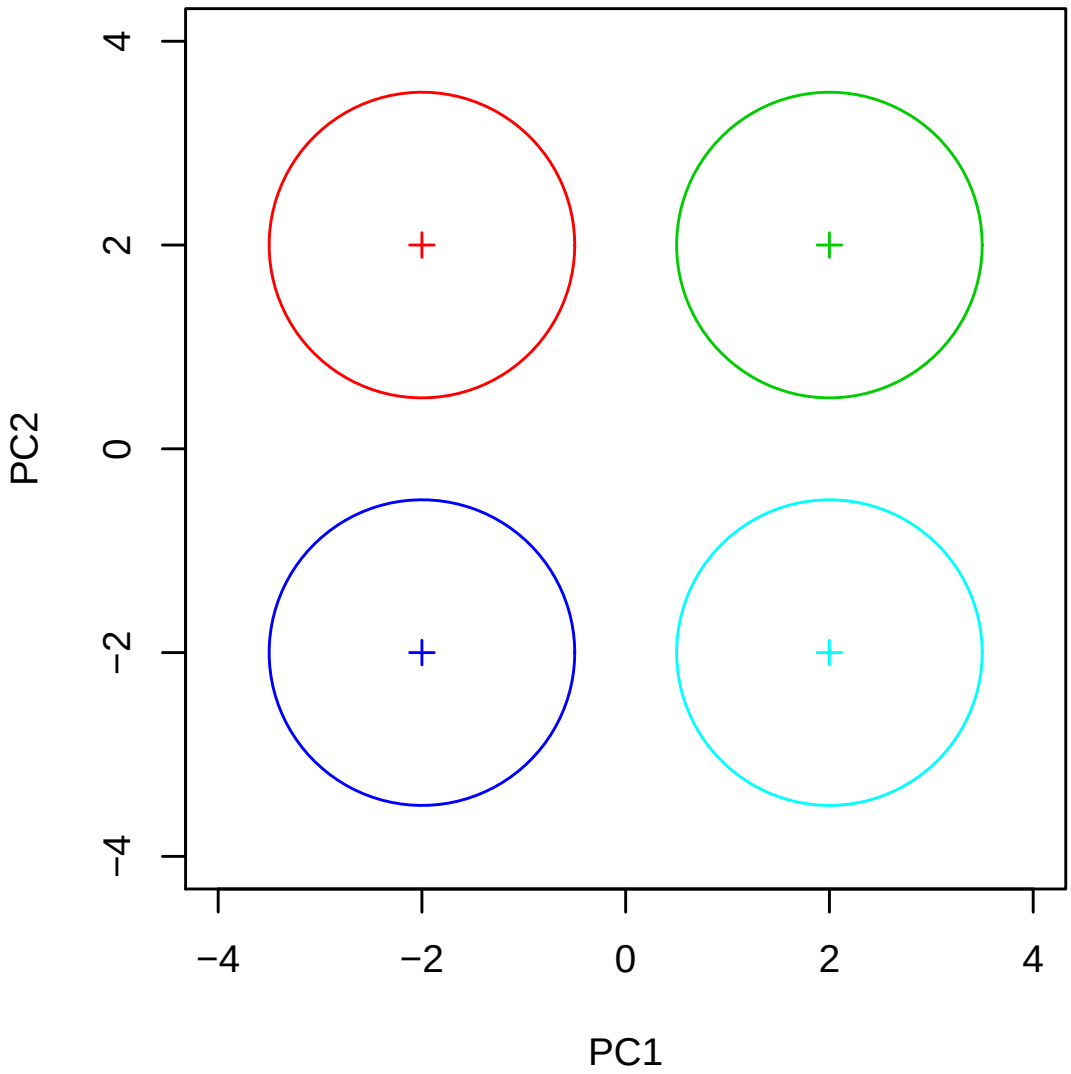
Motivation



Procedure

- reanalysis
- clusters

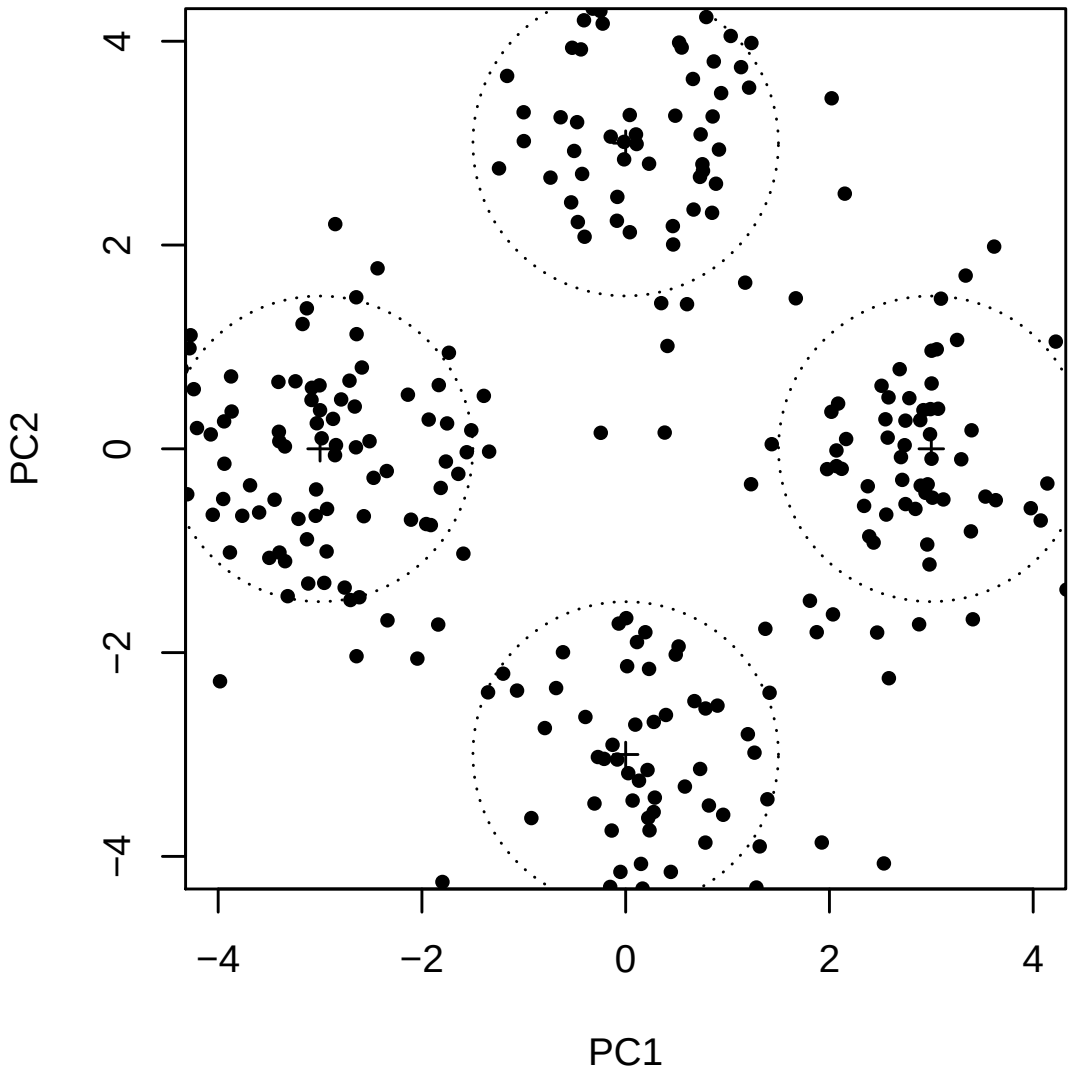
Motivation



Procedure

- reanalysis
- clusters
- CPs!

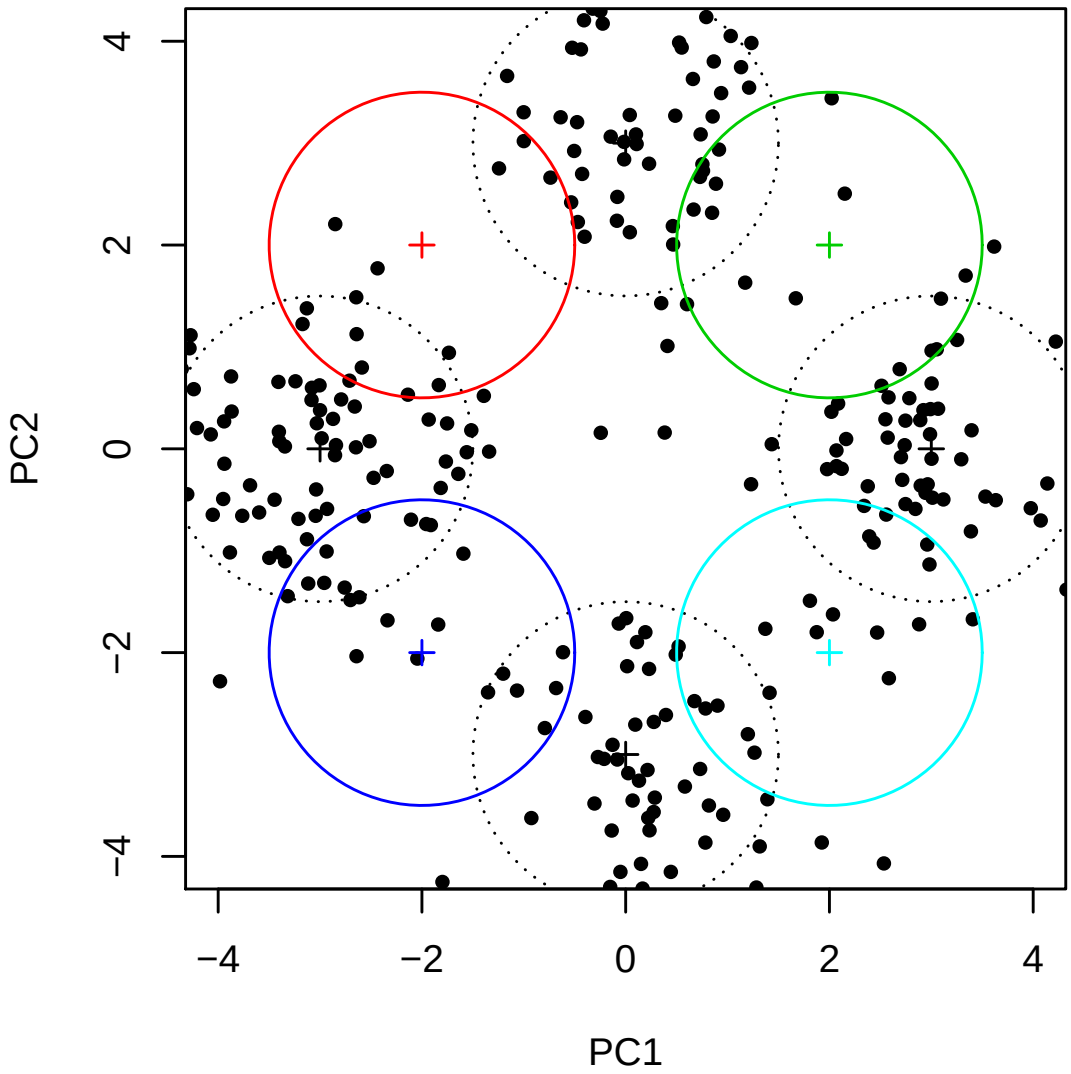
Motivation



Procedure

- reanalysis
- clusters
- CPs!
- model

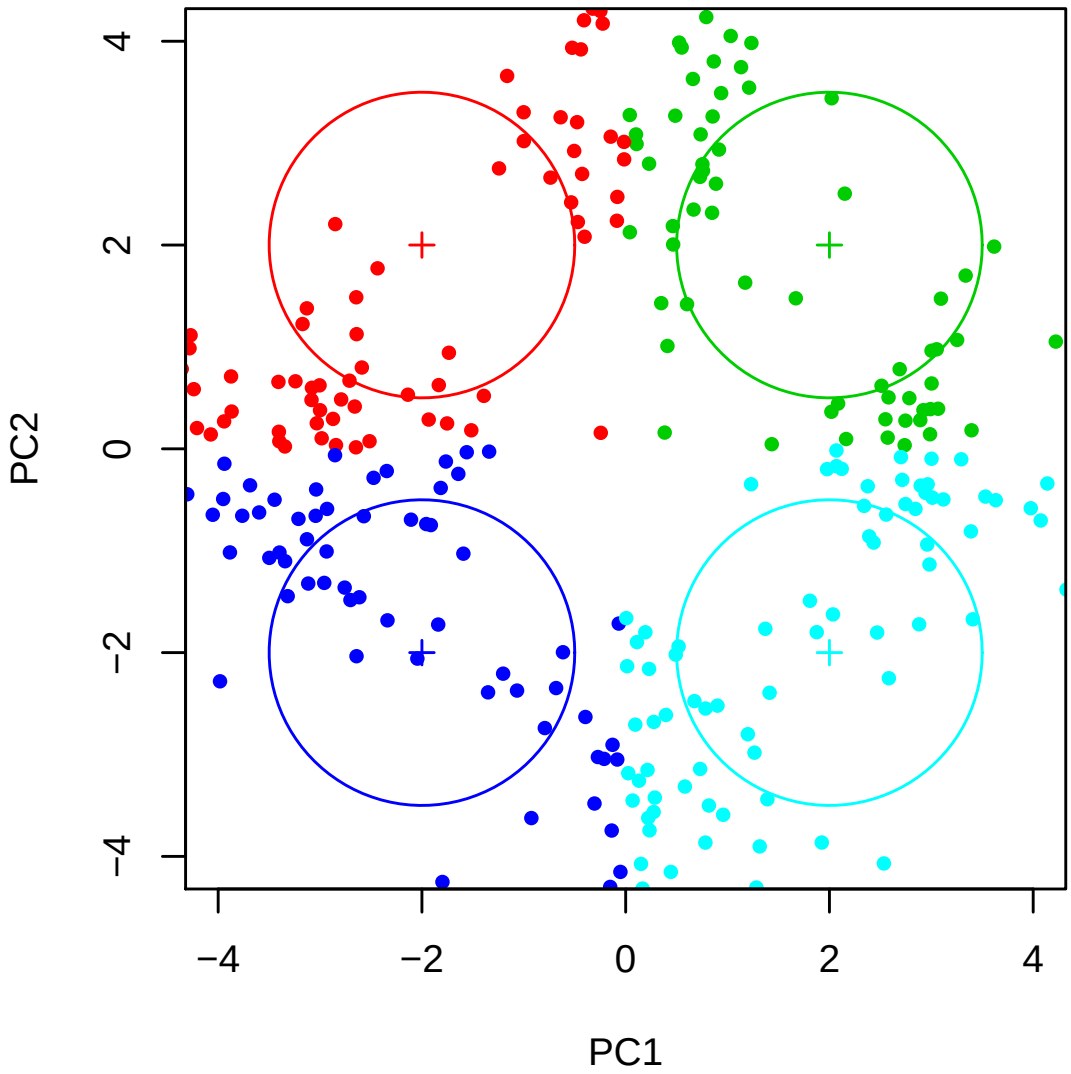
Motivation



Procedure

- reanalysis
- clusters
- CPs!
- model
- project

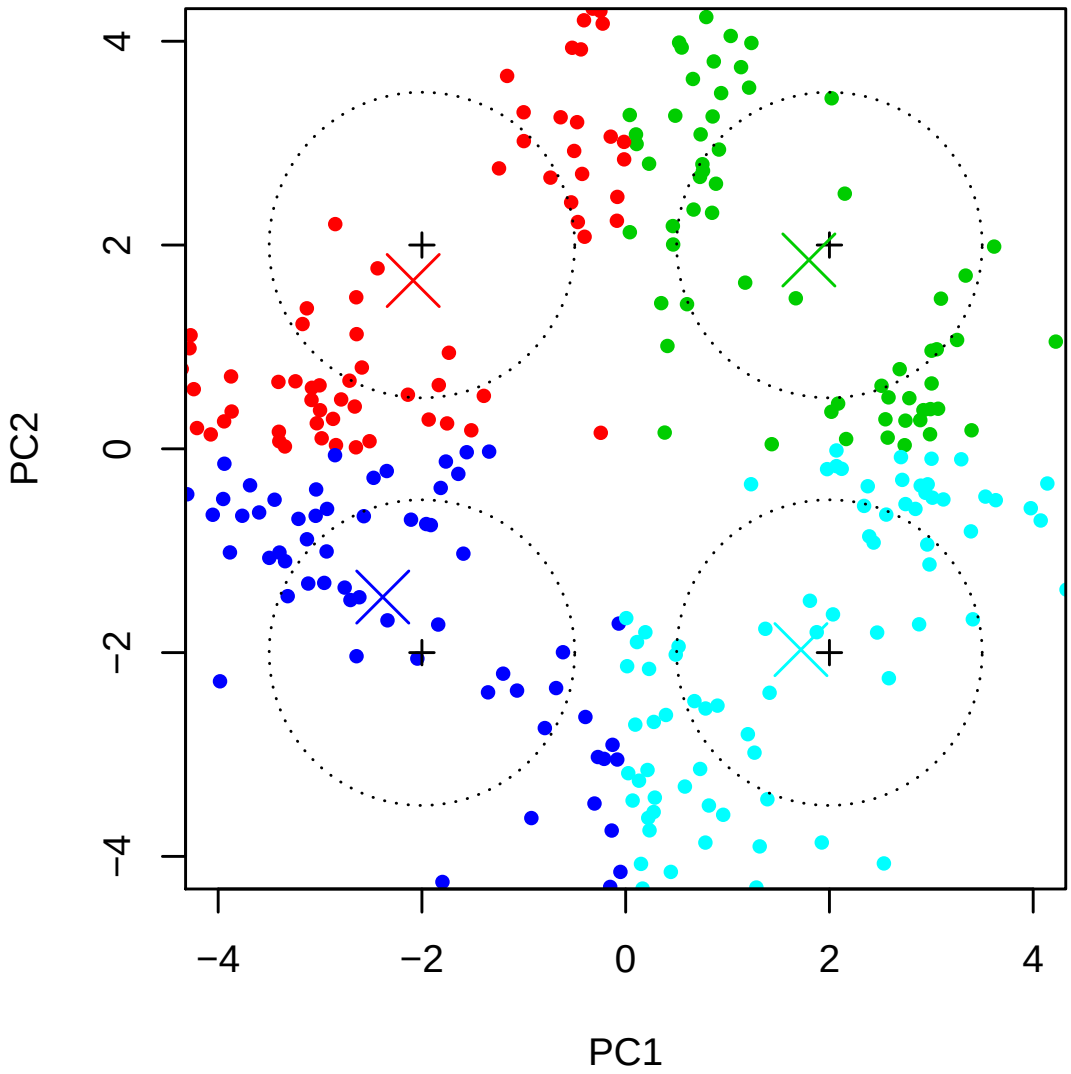
Motivation



Procedure

- reanalysis
- clusters
- CPs!
- model
- project
- associate

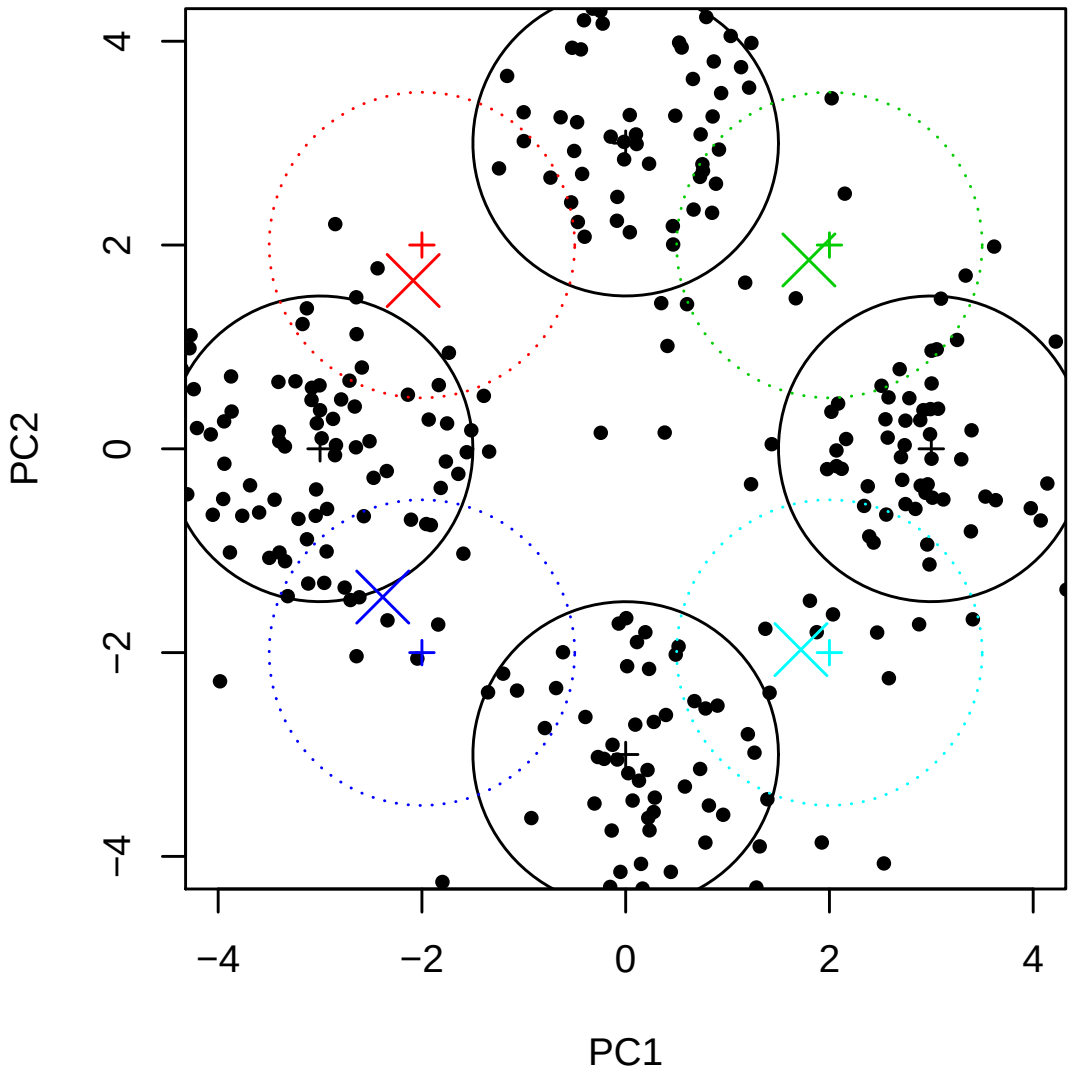
Motivation



Procedure

- reanalysis
- clusters
- CPs!
- model
- project
- associate
- centroids

Motivation



Procedure

- reanalysis
- clusters
- CPs!
- model
- project
- associate
- centroids
- meaning?

Motivation

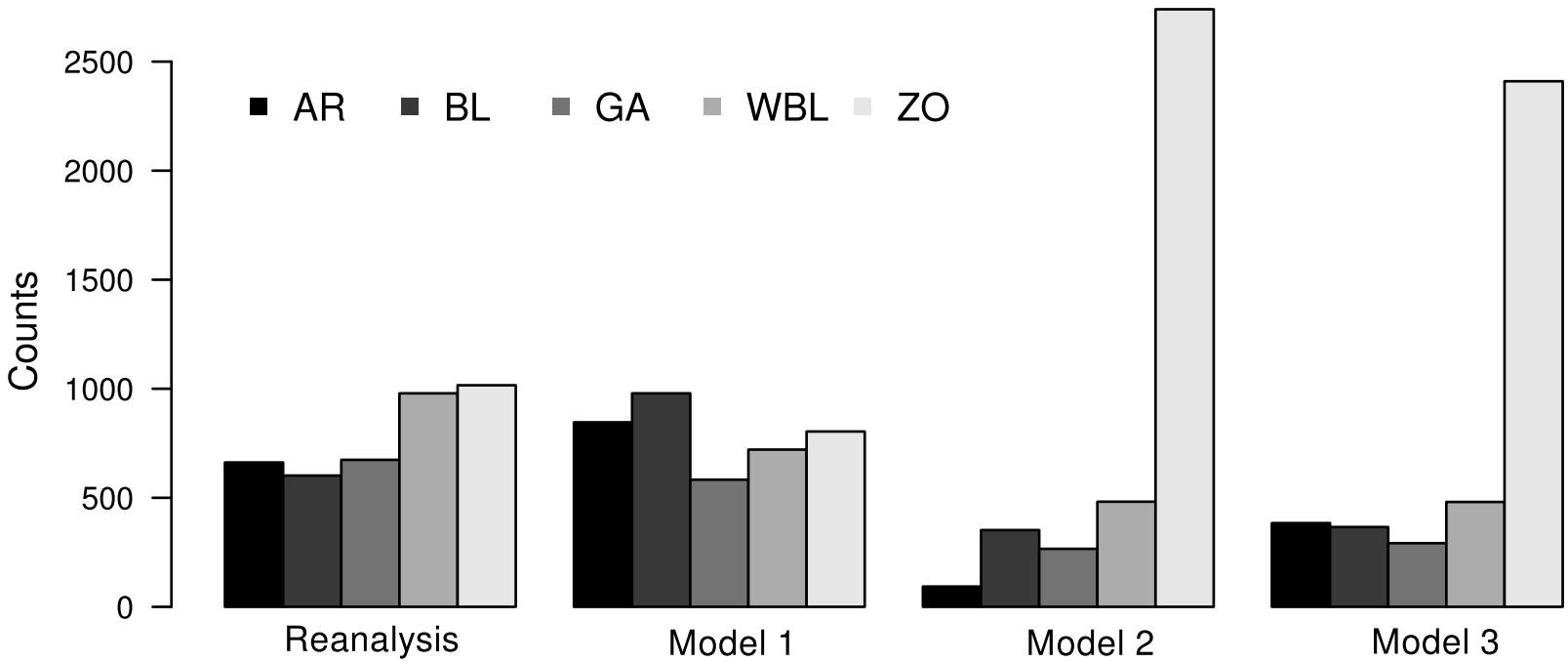
Comparing circulation patterns

- projecting onto reanalysis patterns
- comparing centroids

Motivation

Comparing circulation patterns

- projecting onto reanalysis patterns
- comparing centroids
- counting frequencies



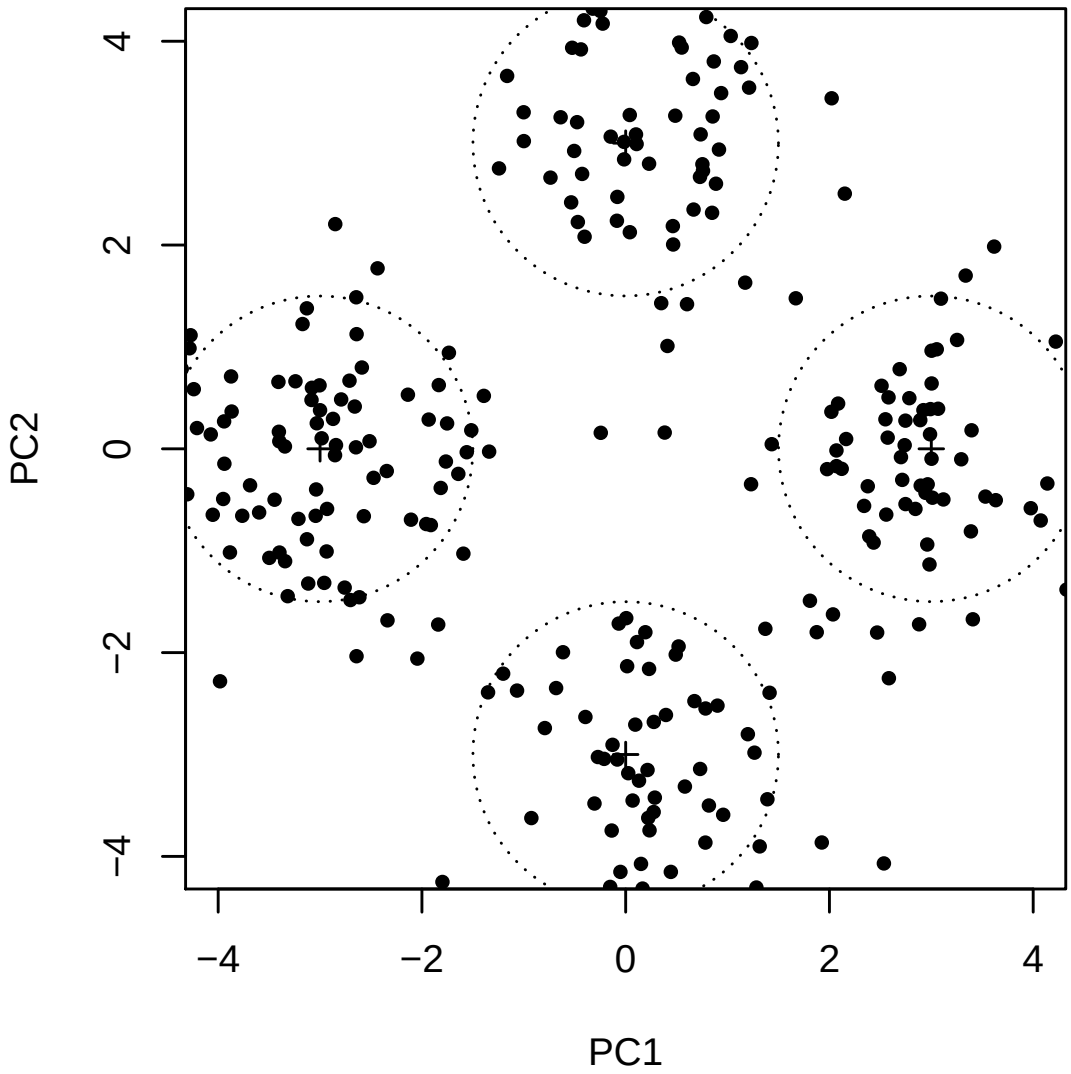
Motivation

Comparing circulation patterns

- projecting onto reanalysis patterns
- comparing centroids
- counting frequencies

other possibilities?

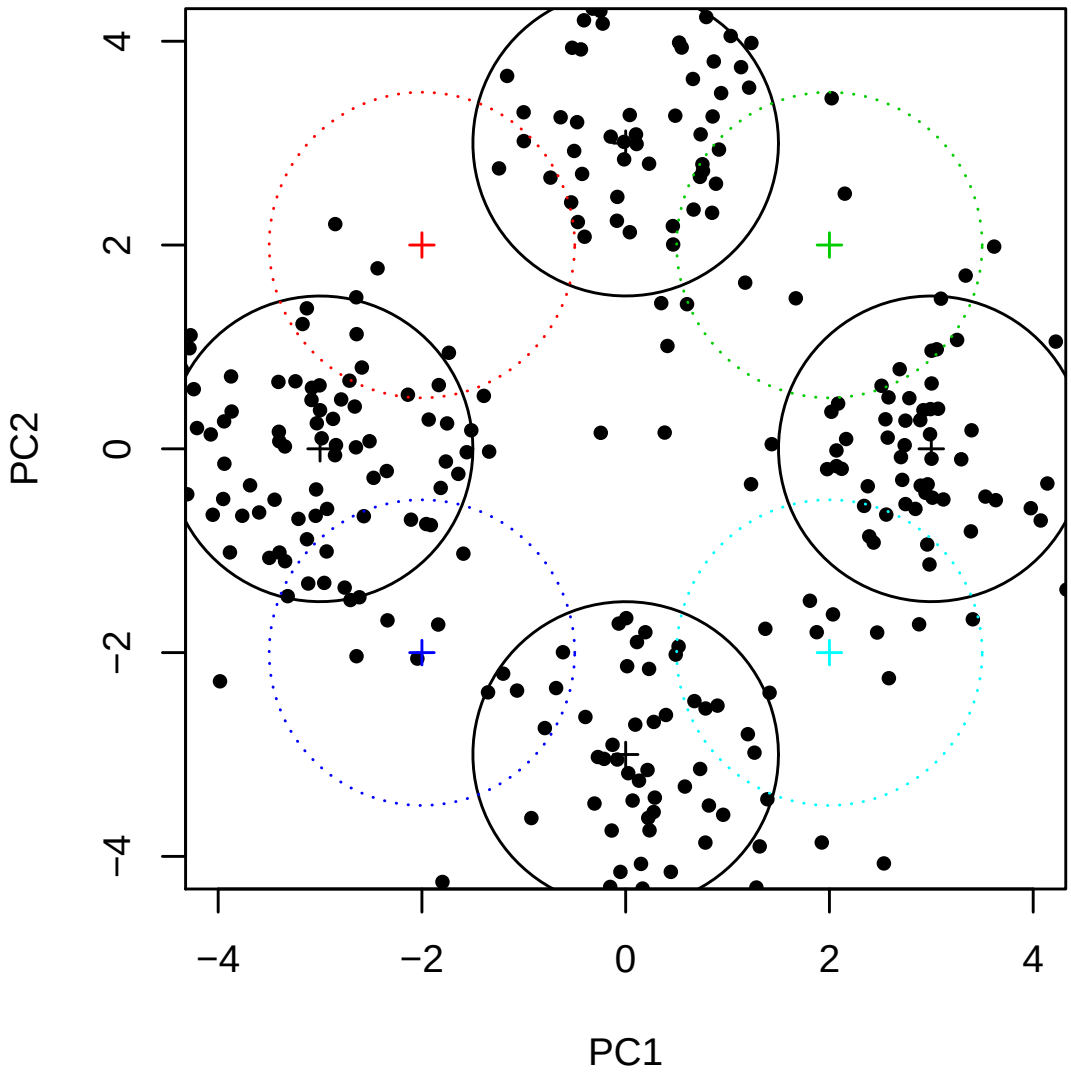
Motivation



Procedure

- cluster separately

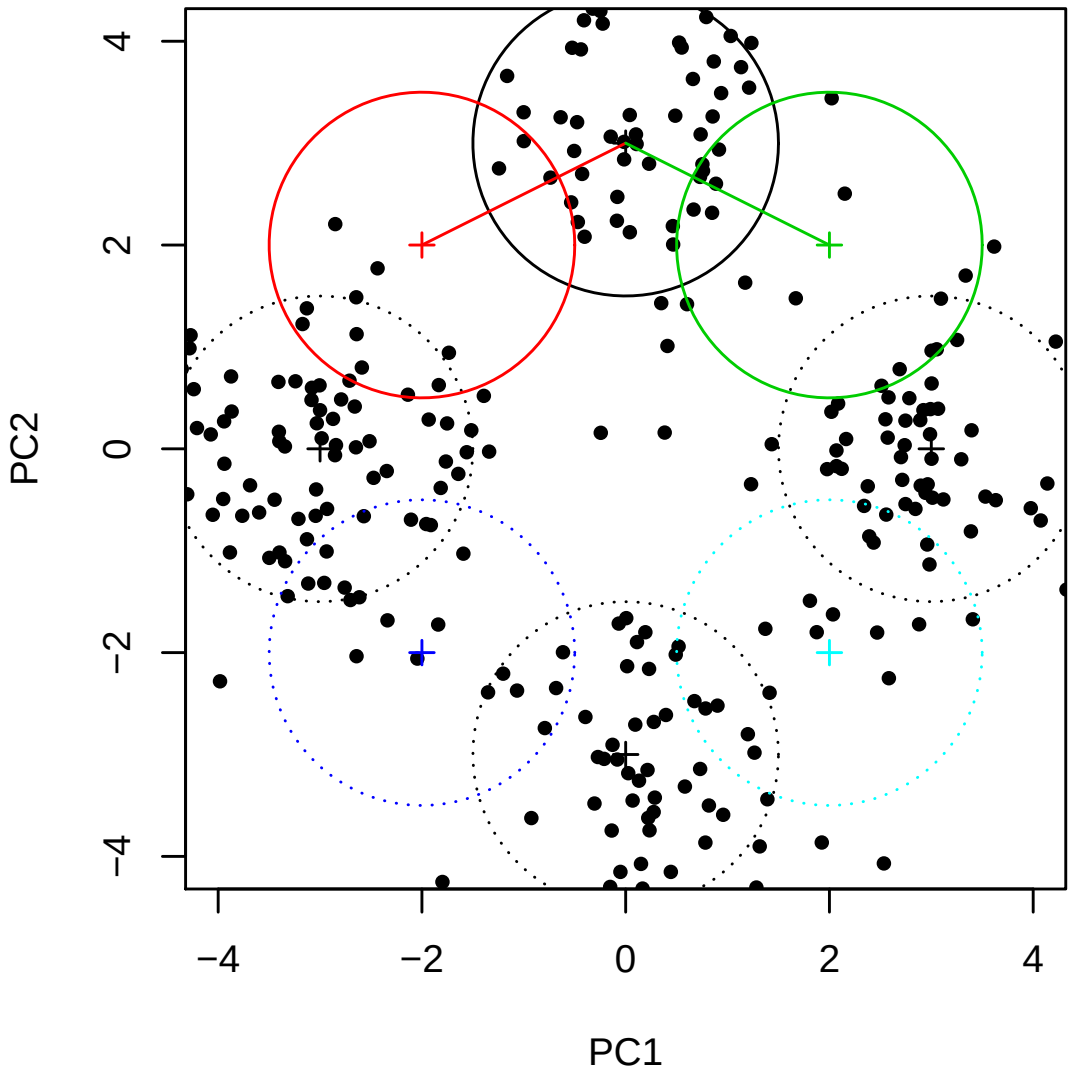
Motivation



Procedure

- cluster separately
- compare clusters

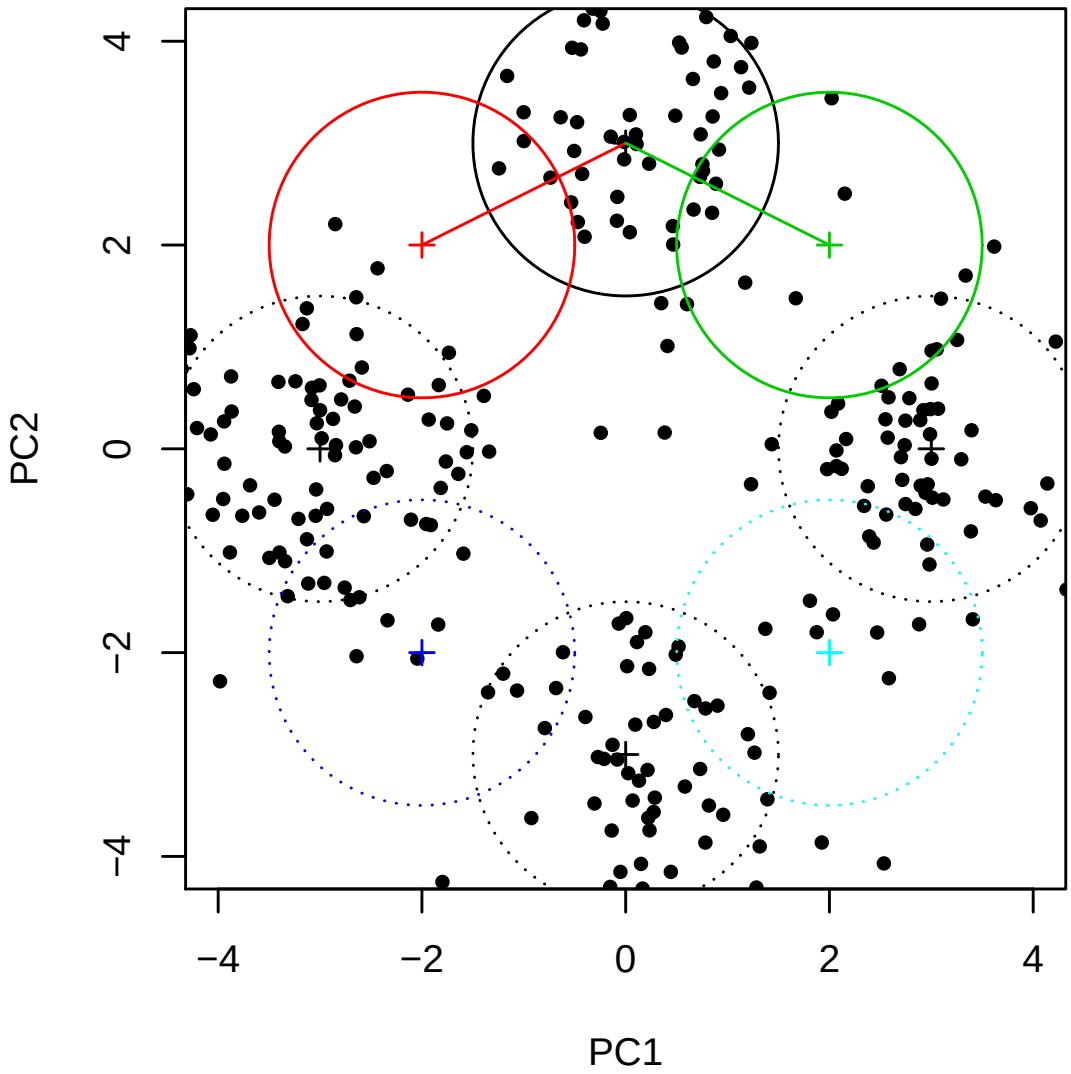
Motivation



Procedure

- cluster separately
- compare clusters
- e.g. means

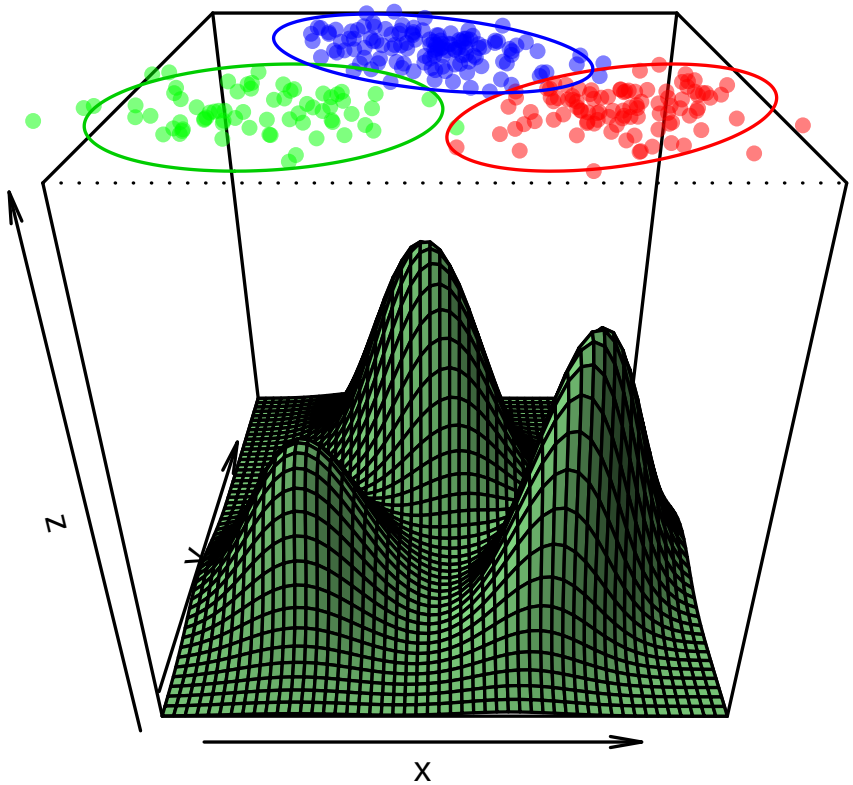
Motivation



Procedure

- cluster separately
- compare clusters
- e.g. means
- allows for other ways

Cluster analysis: Gaussian mixture models

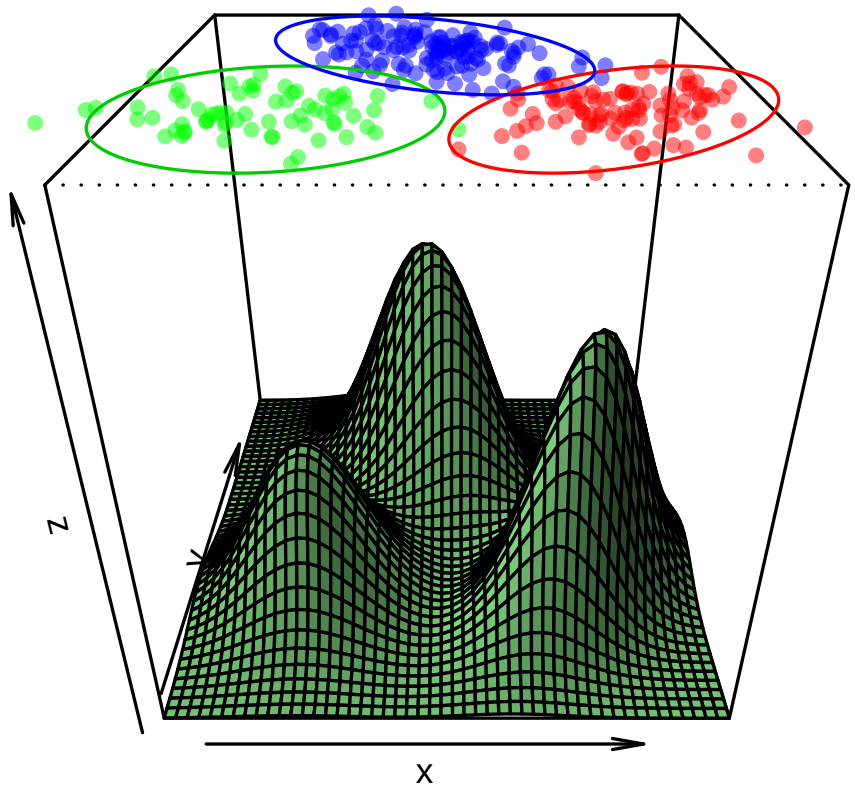


Modelling

- mixture of Gaussians

$$p(x \mid \theta) = \sum_{k=1}^{3} a_k f(\mathbf{x}; \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k),$$

Cluster analysis: Gaussian mixture models



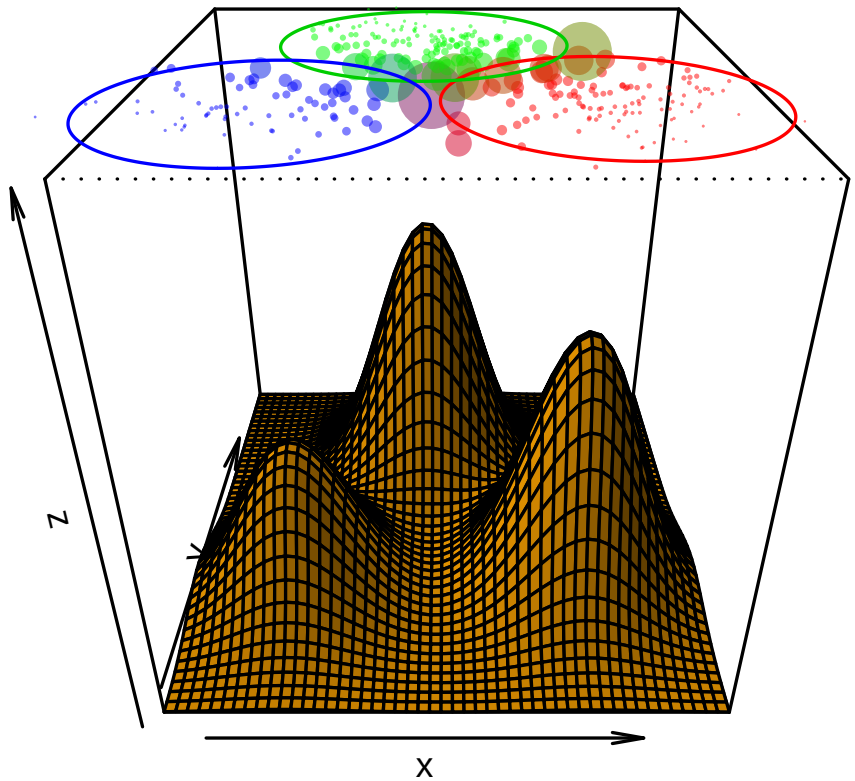
Modelling

- mixture of Gaussians
- parameter estimation

Parameter estimates (EM):

$$\hat{\boldsymbol{\theta}} = (\hat{\boldsymbol{\mu}}_1, \hat{\boldsymbol{\mu}}_2, \hat{\boldsymbol{\mu}}_3, \hat{\boldsymbol{\Sigma}}_1, \hat{\boldsymbol{\Sigma}}_2, \hat{\boldsymbol{\Sigma}}_3, \hat{a}_1, \hat{a}_2, \hat{a}_3)$$

Cluster analysis: Gaussian mixture models



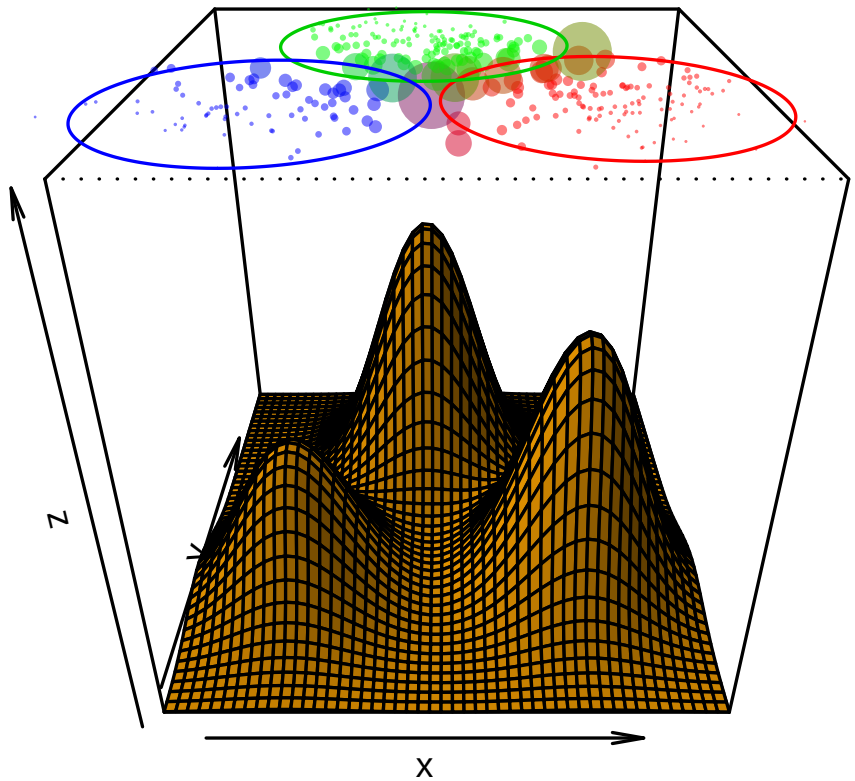
Modelling

- mixture of Gaussians
- parameter estimation
- classification with uncertainty

Classification uncertainty for $\mathbf{x_i}$:

$$\bar{P}(\mathbf{x}_i) = 1 - P(\mathbf{x_i} \in \mathrm{Cl}_{\mathbf{x_i}} \mid \mathbf{x_i})$$

Cluster analysis: Gaussian mixture models



Modelling

- mixture of Gaussians
- parameter estimation
- classification with uncertainty
- ellipsoidal shapes

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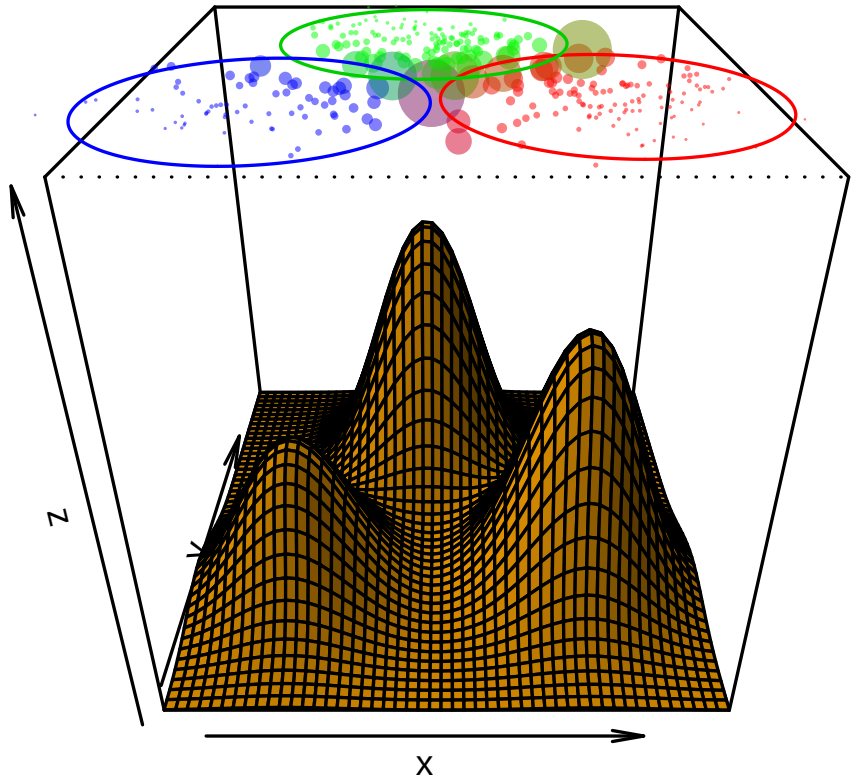
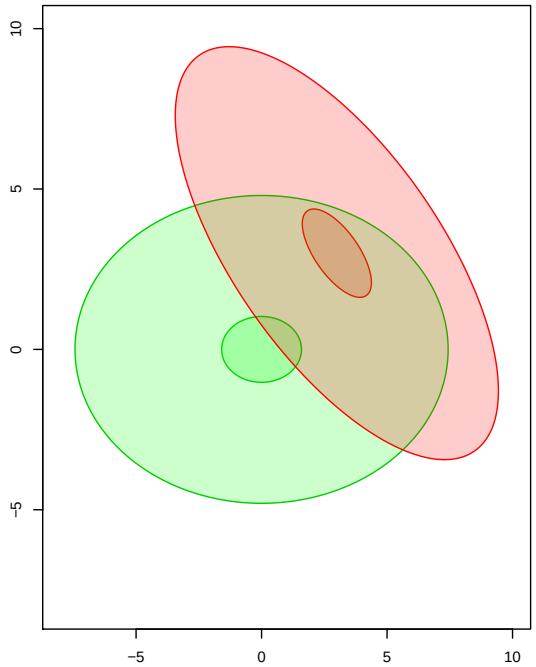


Modelling

- mixture of Gaussians
- parameter estimation

- classification with uncertainty
- ellipsoidal shapes
- other means of comparison

Difference/Similarity Measures for (Gaussian) pdfs



Difference/Similarity Measures for (Gaussian) pdfs

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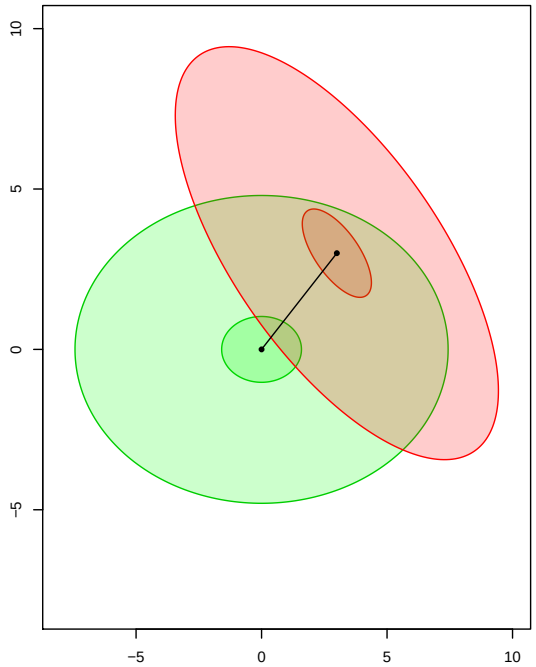
   $d^2_{\mathrm{Eucl}}(P, Q) = \|\boldsymbol{\mu}_p - \boldsymbol{\mu}_q\|^2$



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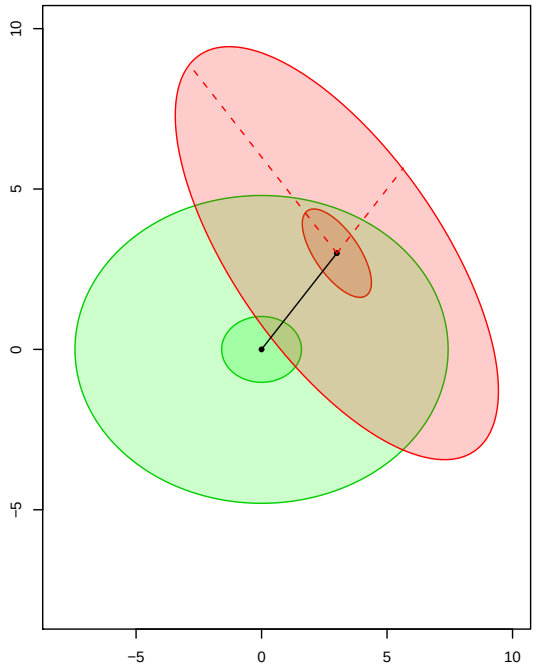

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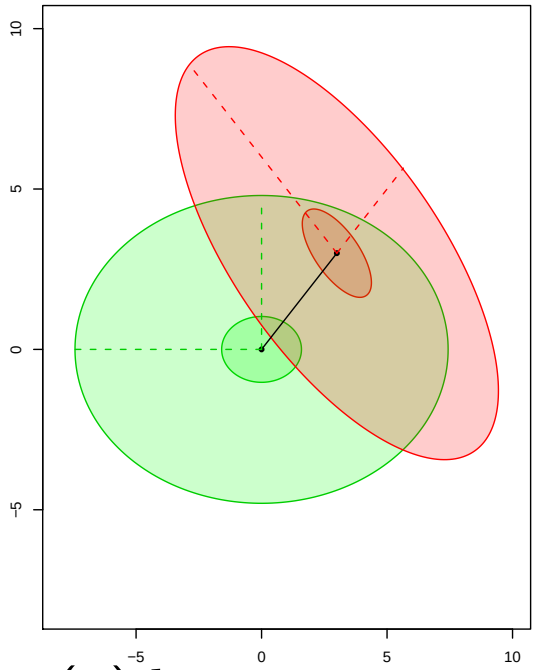

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3. Kullback-Leibler discrimination (KL):
   $d_{\mathrm{KL}}(P, Q) = I(P \mid Q) = \int_{\mathbb{R}} \log\left(\frac{q(x)}{p(x)}\right) q(z)\mathrm{d}x$

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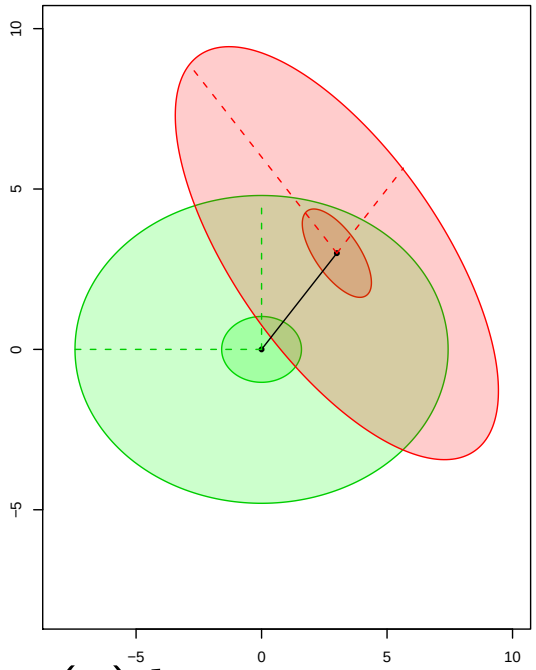

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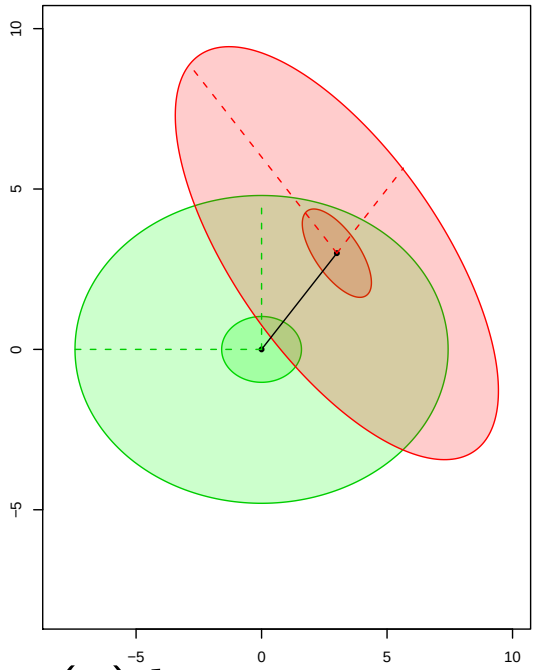

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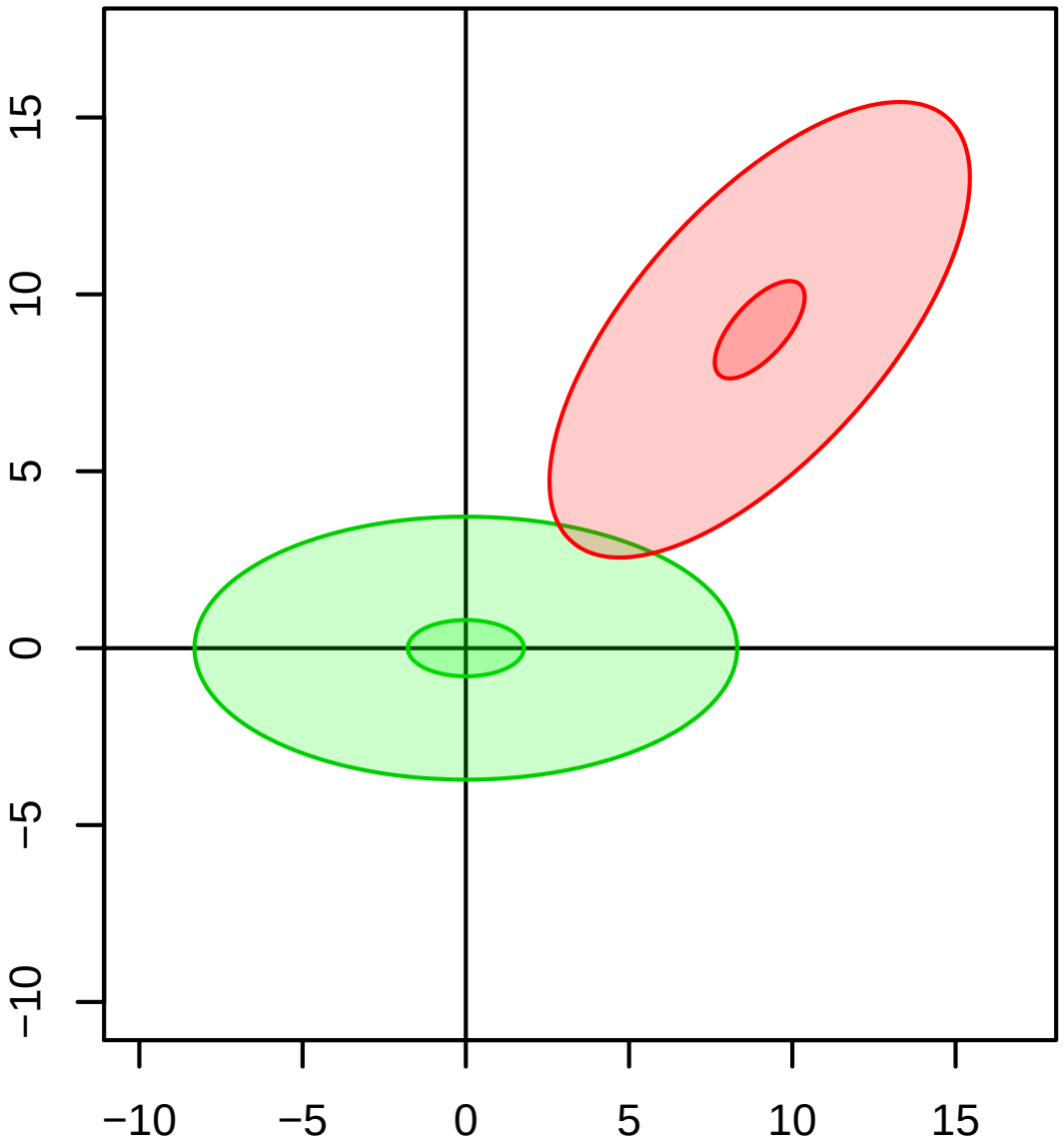
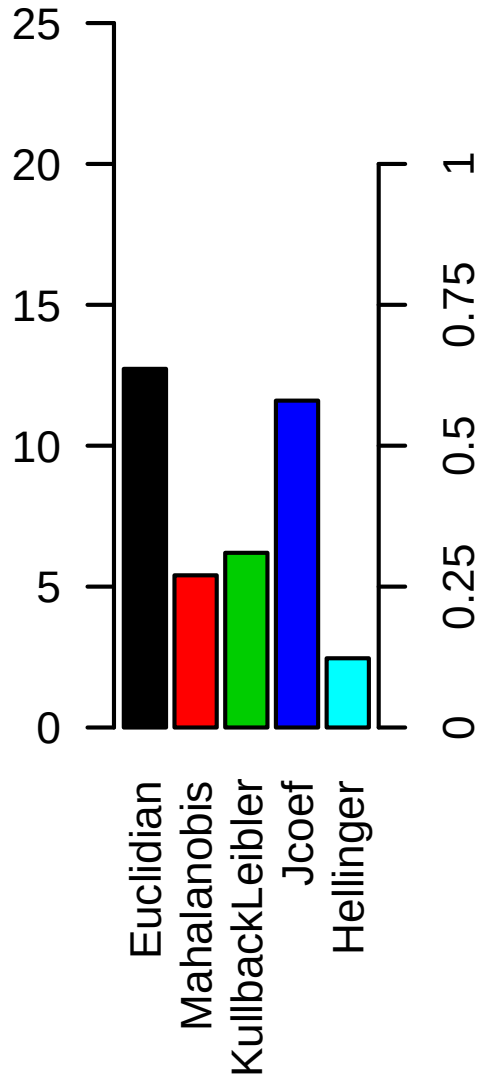
5. Hellinger coefficient (s=1/2):
   $d_{\mathrm{H}}(P,Q) = \int_{\mathbb{R}} q(x)^s p(x)^{(1-s)}\mathrm{d}x, \quad d_{\mathrm{H}} \in [0,1]$

Difference and similarity measures for pdfs





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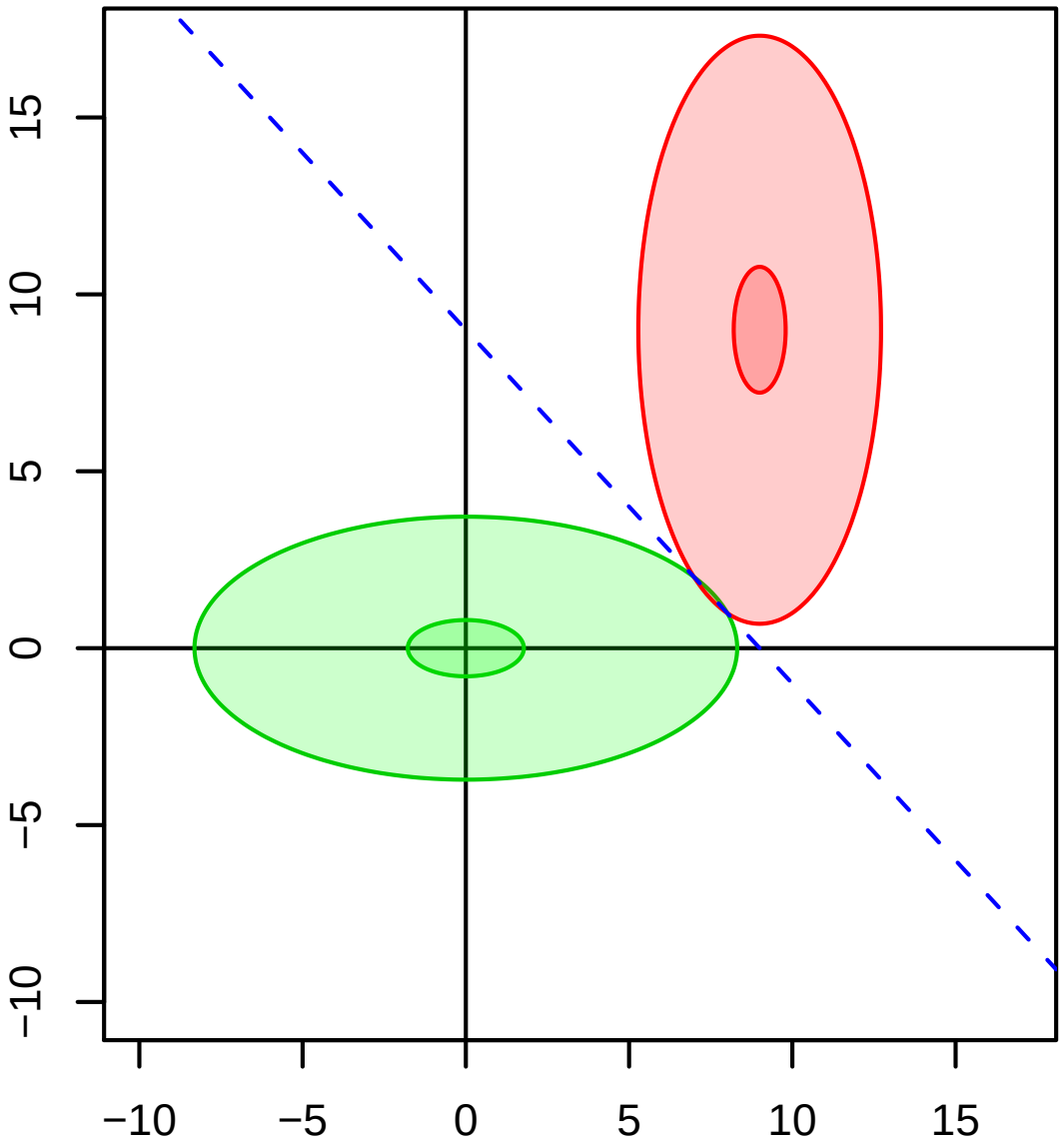

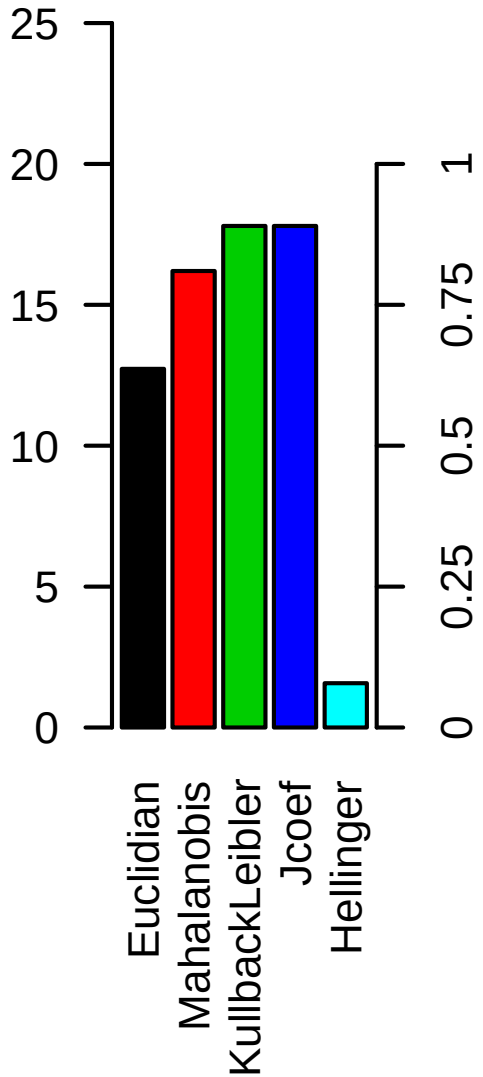

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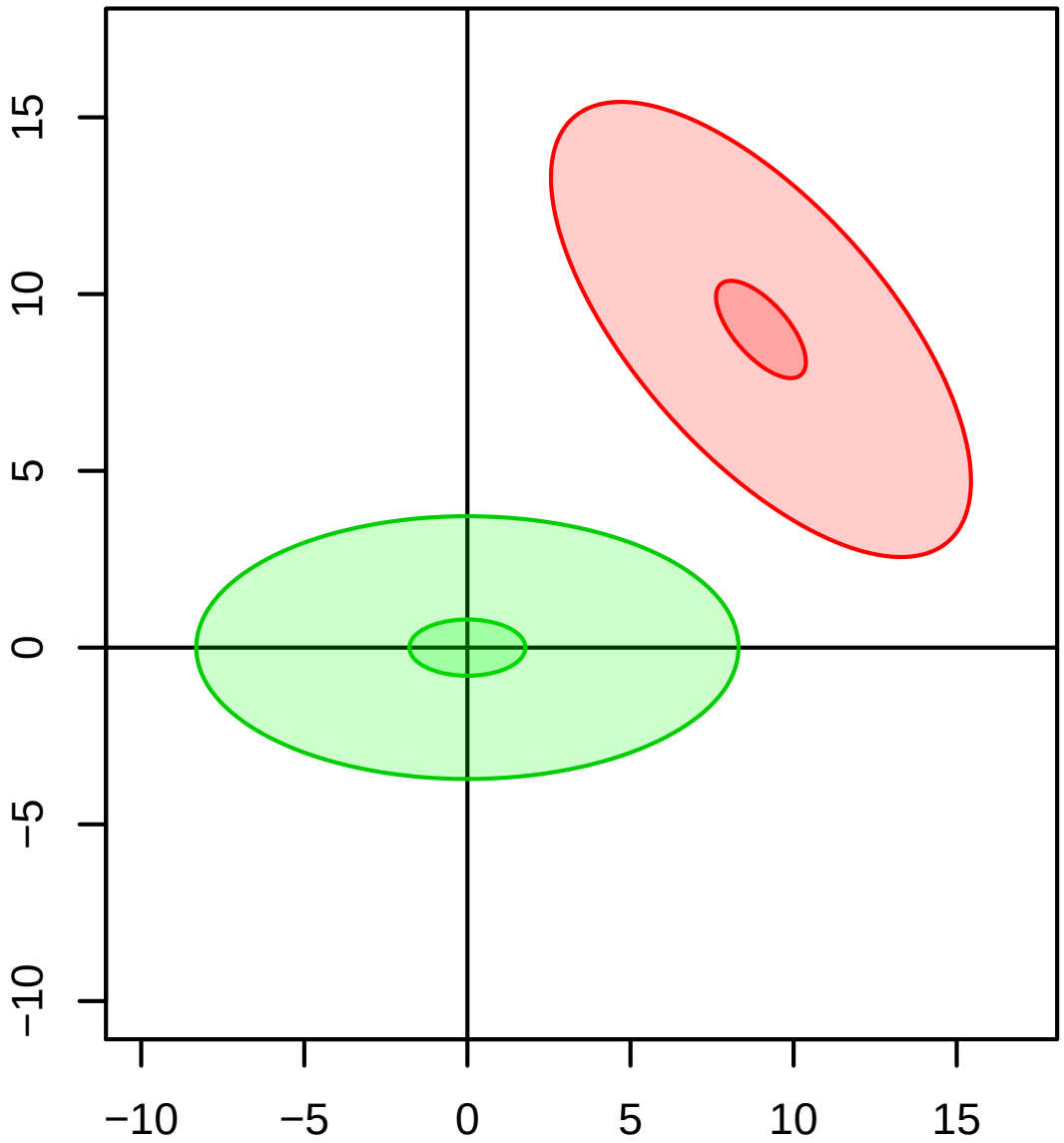

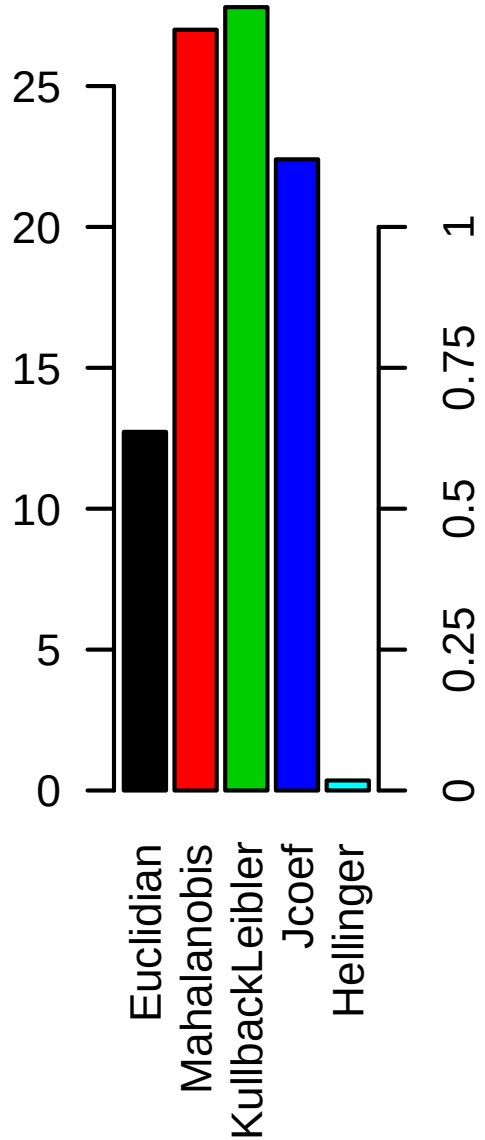

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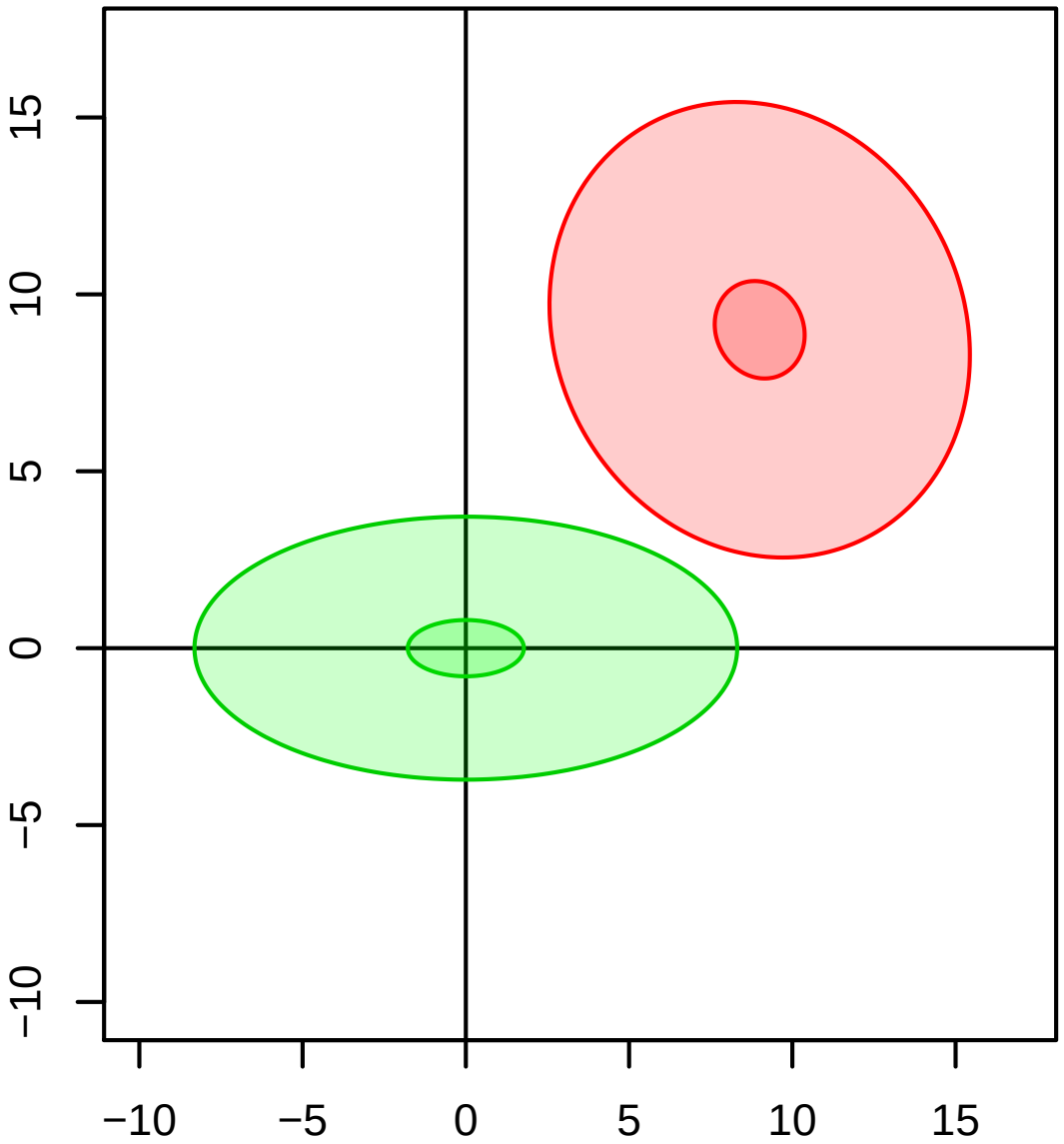

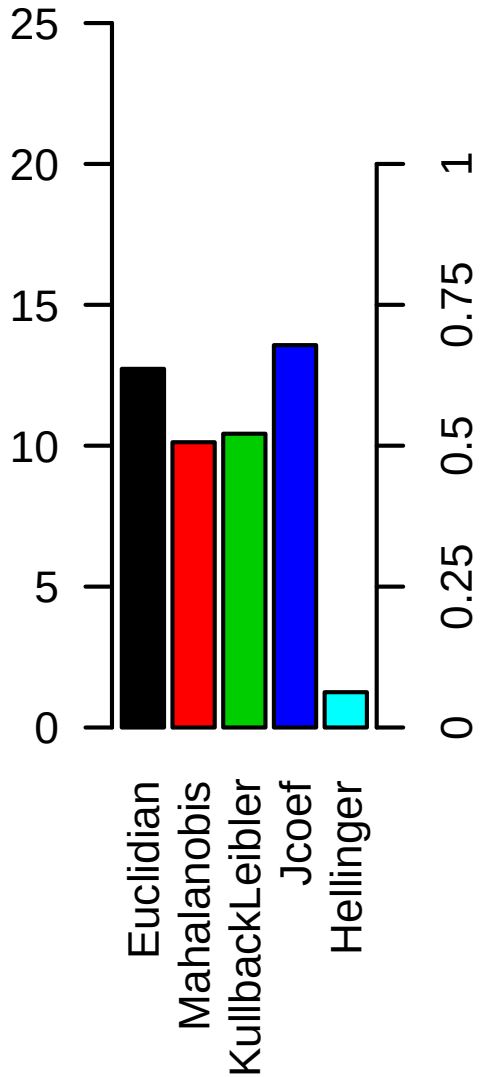

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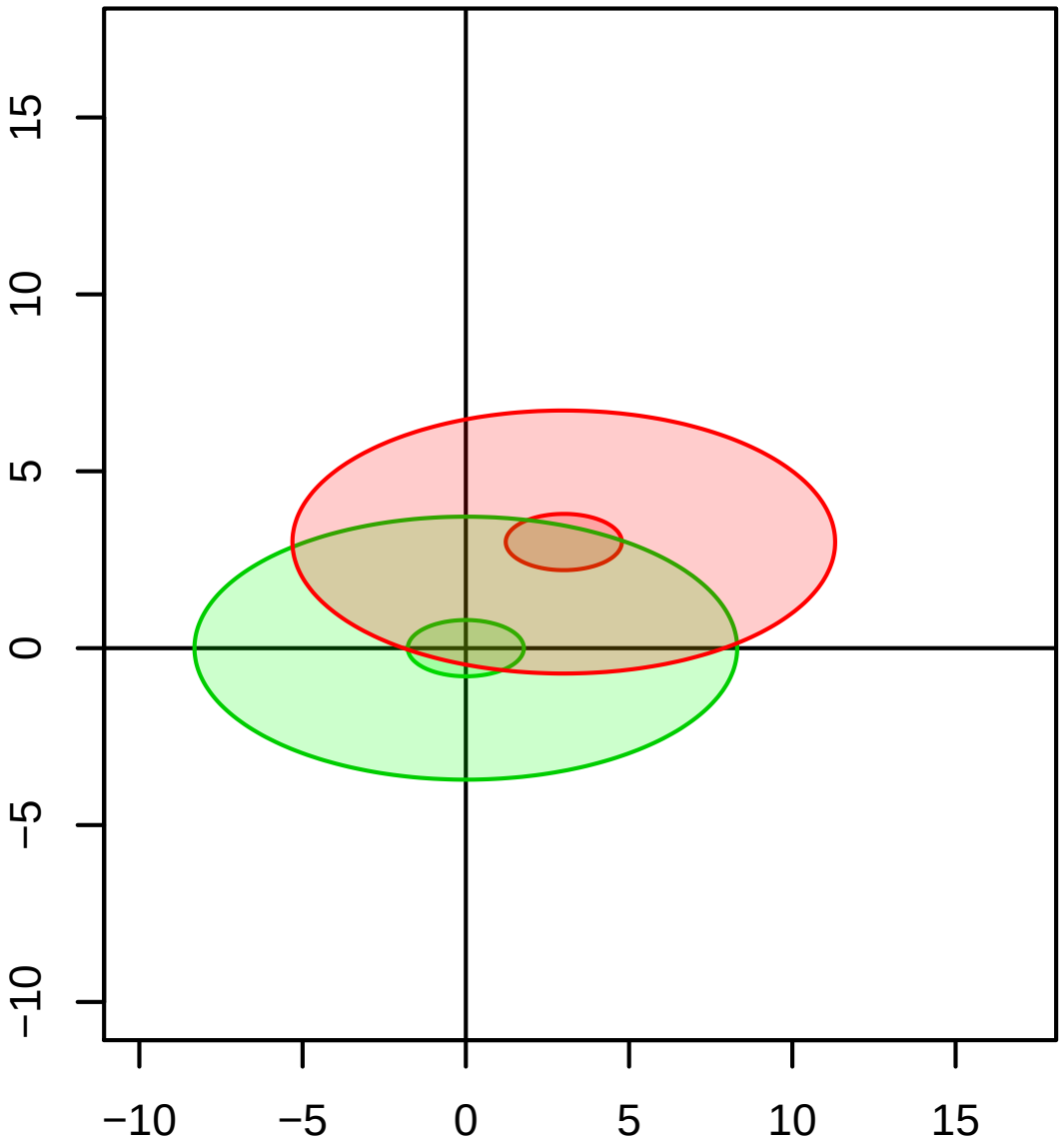

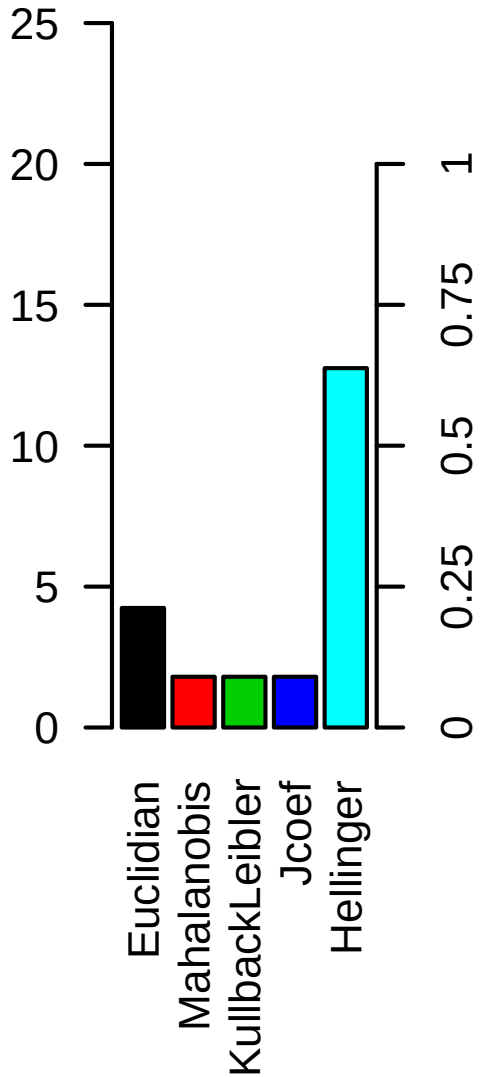

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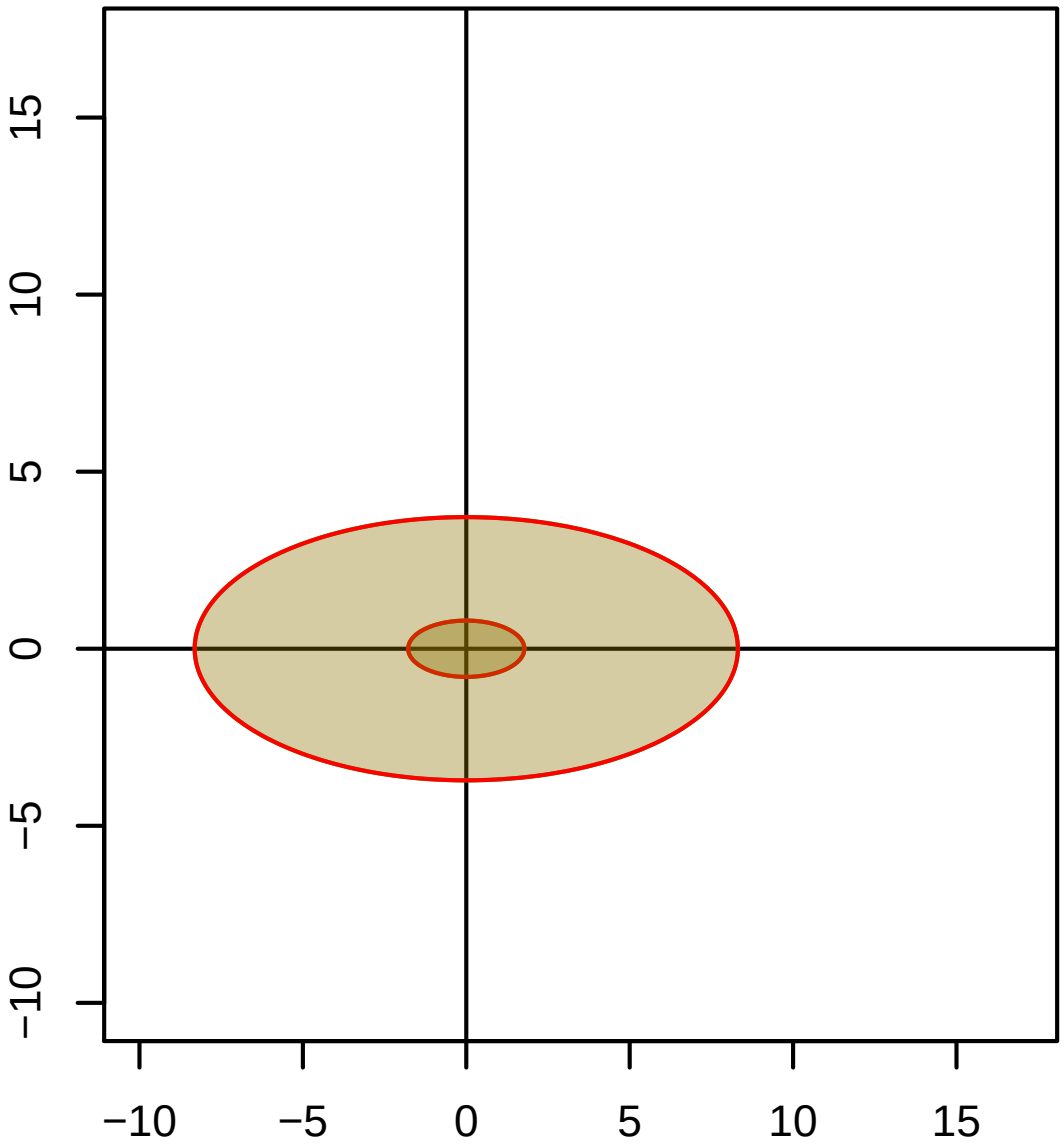

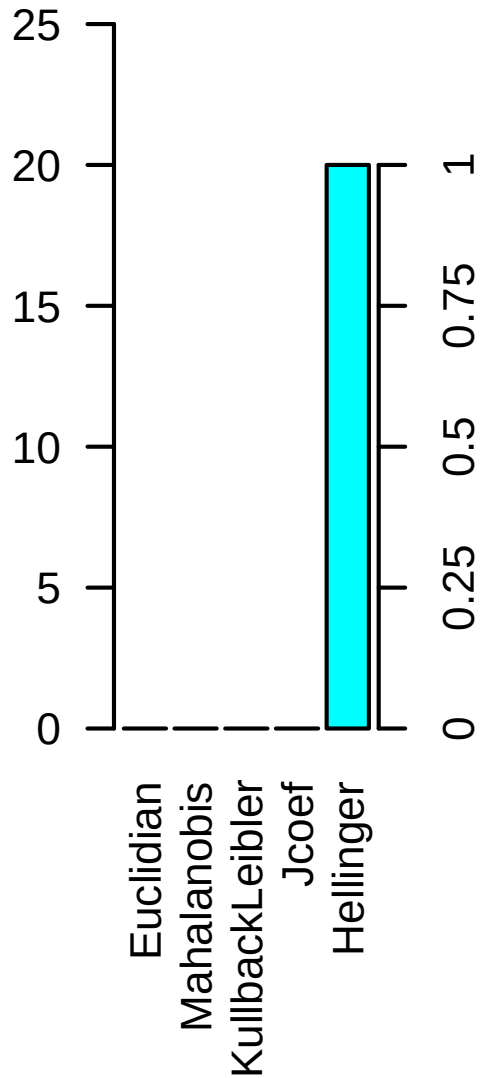

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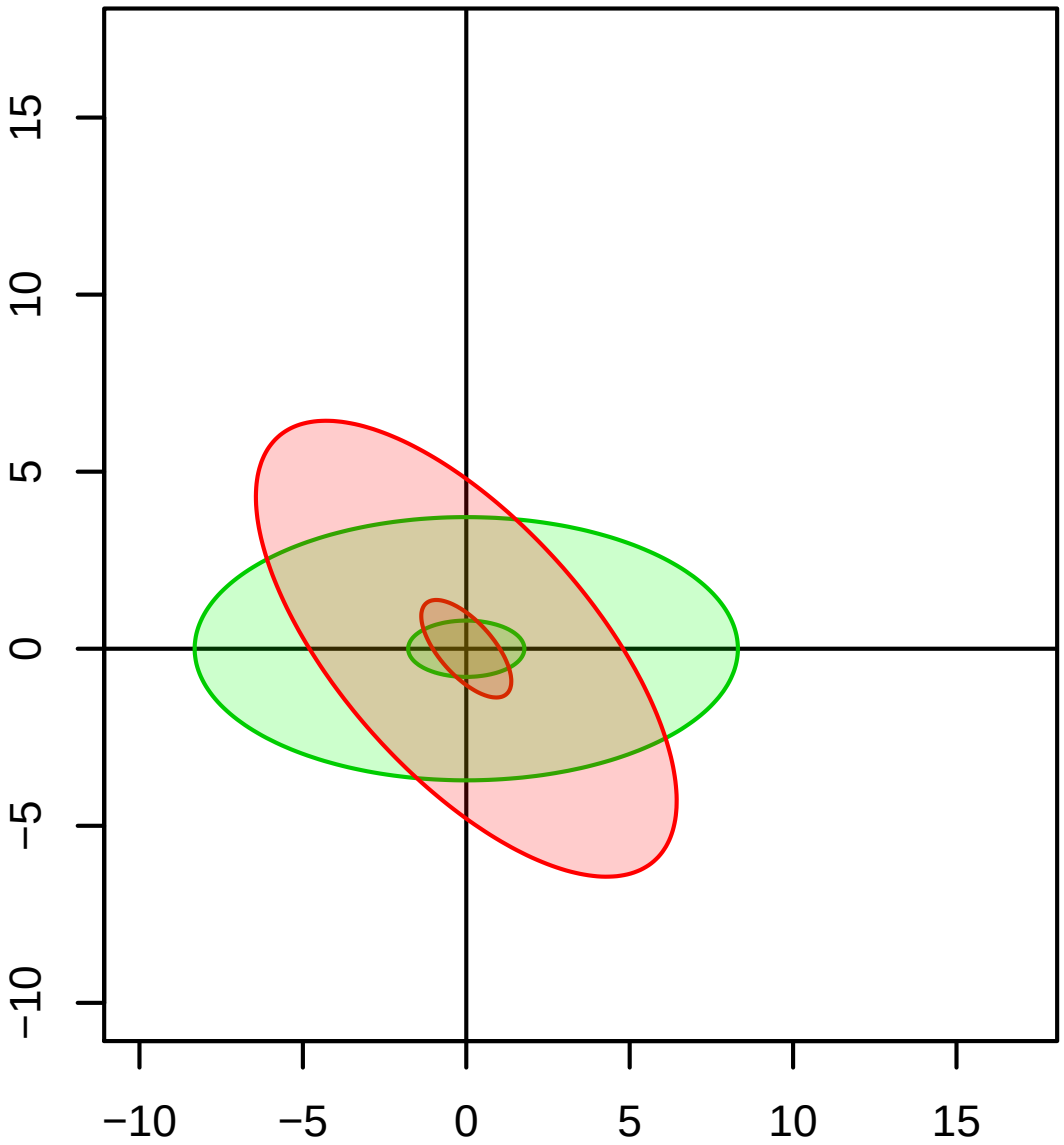
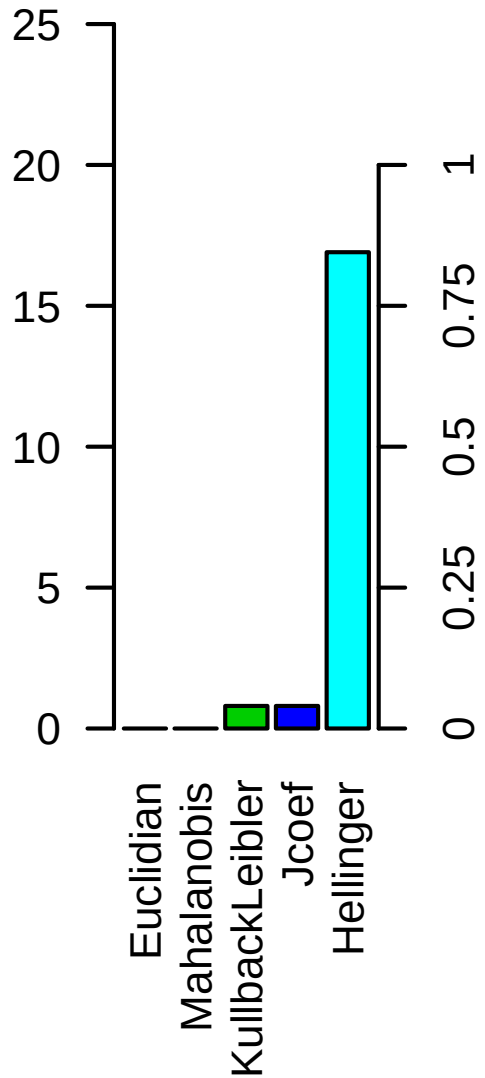
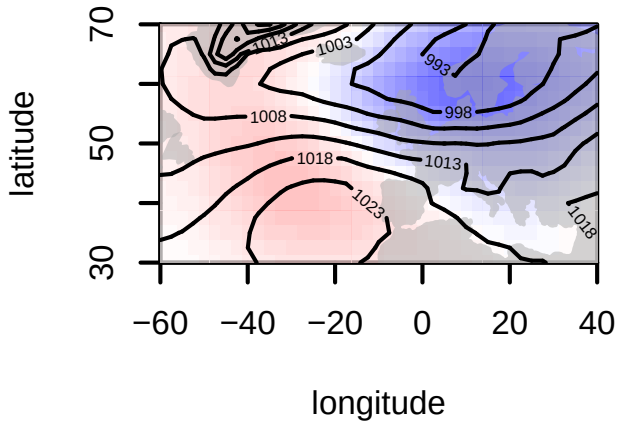




North Atlantic region

Setting

- North Atlantic region
- daily SLP anomalies
- 1975 – 2000, NDJFM



North Atlantic region

Setting

- North Atlantic region
- daily SLP anomalies
- 1975 – 2000, NDJFM
- data sets, interpolated to NCEP/NCAR grid

Reanalyses

- NCEP/NCAR
- ERA-40

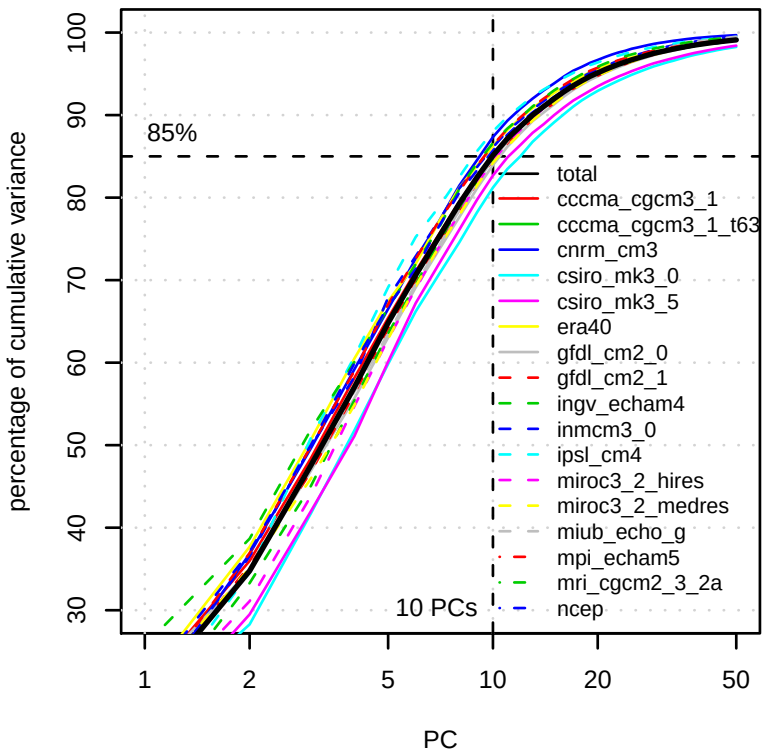
14 IPCC Models (AR4)

- CCCMA CGCM3.1, T47
- CNRM CM3.0
- CSIRO MK3.0 and MK3.5
- GFDL CM2.0 and CM2.1
- INGV ECHAM4
- INM CM3.0
- IPSL CM4
- MIROC 3.2 high/medium resolution
- MIUB ECHO.G
- MPI ECHAM5
- MRI CGCM 2.3.2a

North Atlantic region

Setting

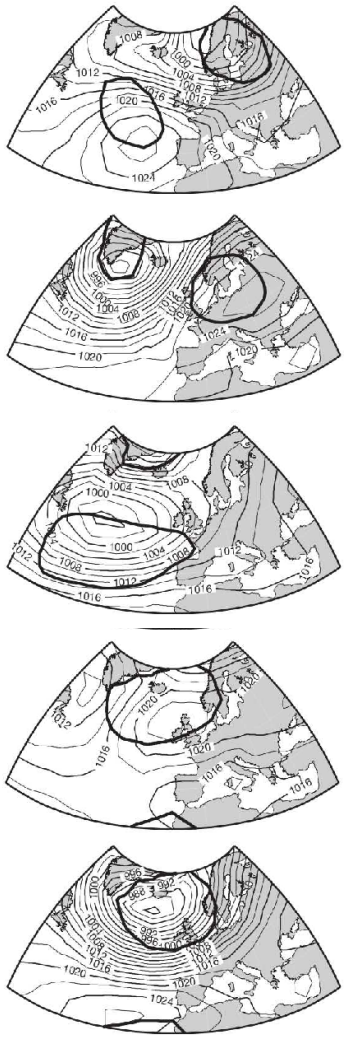
- North Atlantic region
- daily SLP anomalies
- 1975 – 2000, NDJFM
- data sets, interpolated to NCEP/NCAR grid
- common PCA 85% (10PCs)



North Atlantic region

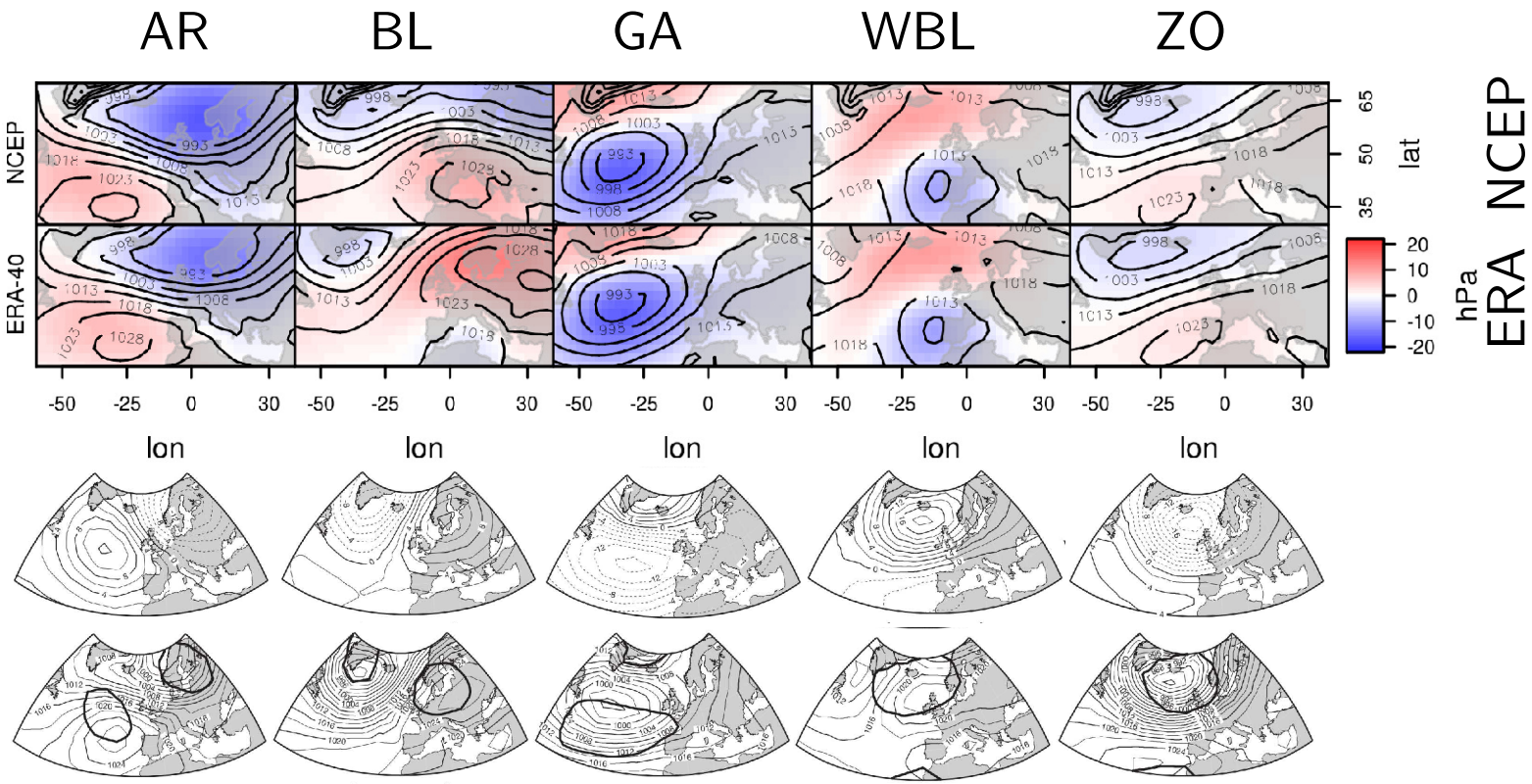
Setting

- North Atlantic region
- daily SLP anomalies
- 1975 – 2000, NDJFM
- data sets, interpolated to NCEP/NCAR grid
- common PCA 85% (10PCs)
- Clusteranalysis use 5 Clusters for comparison



Plaut, Simonnet (2001)

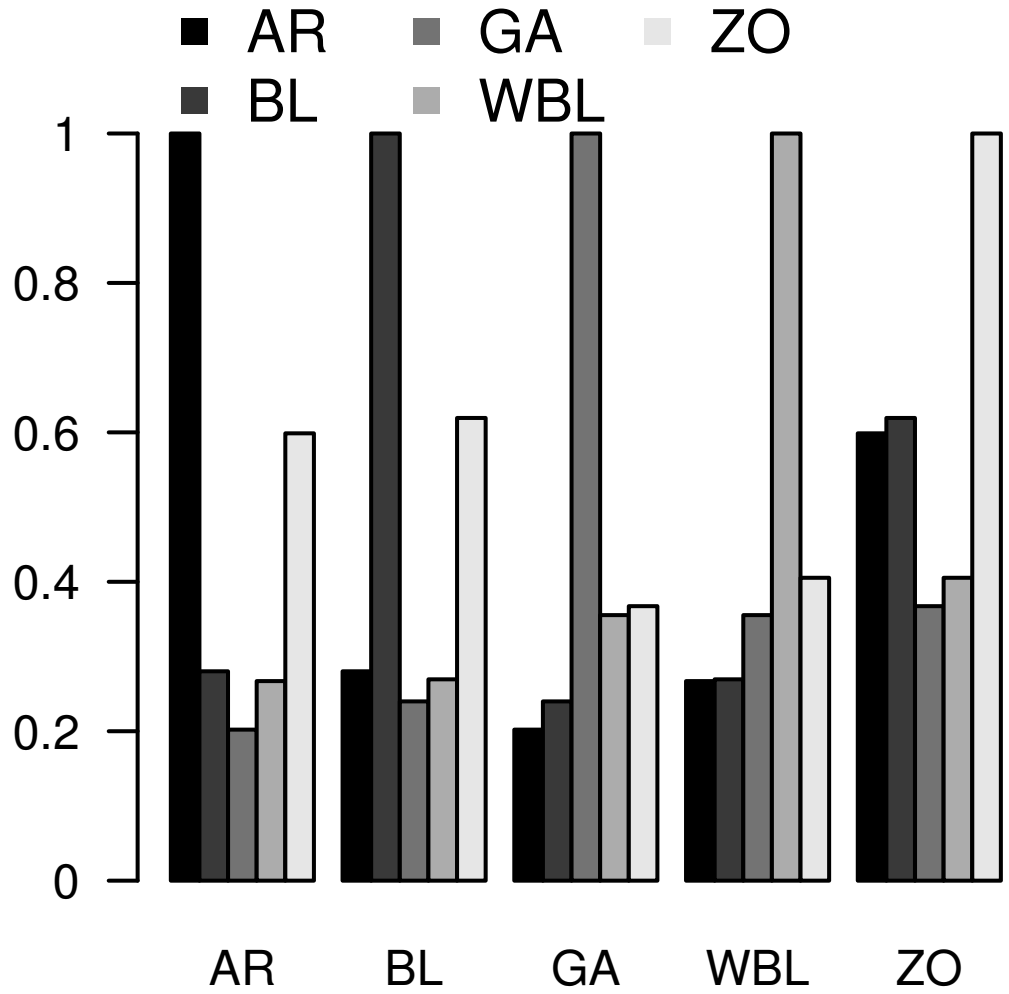
Comparison with PS01, centroids



from Plaut and Simonnet, *Clim.Res.* (2001)

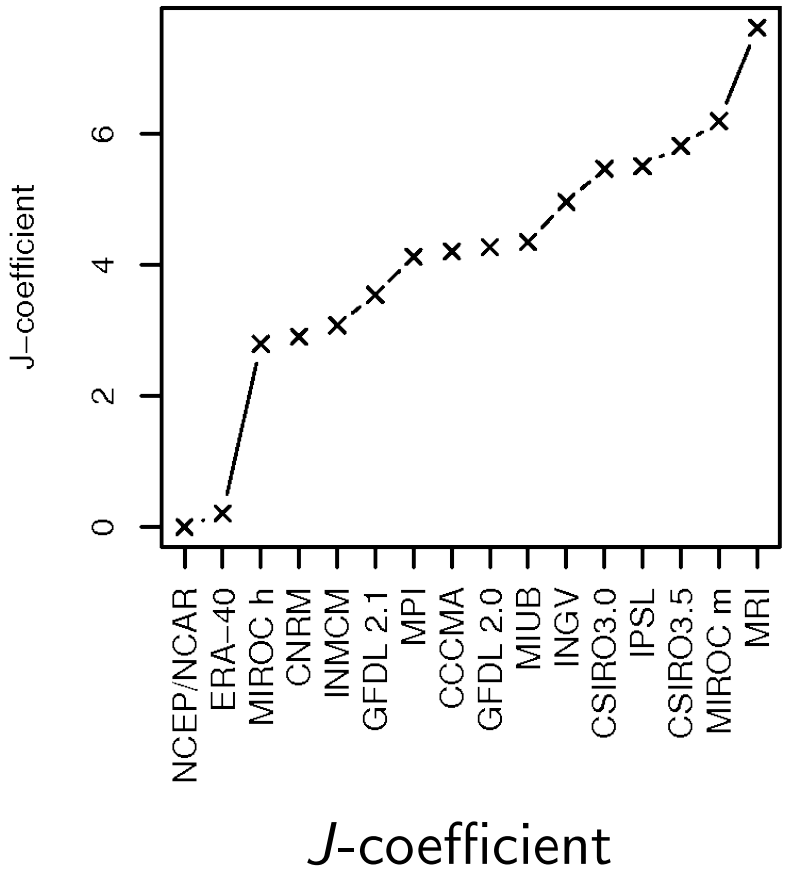
Results

Cluster configuration NCEP/NCAR (Hellinger)



Results

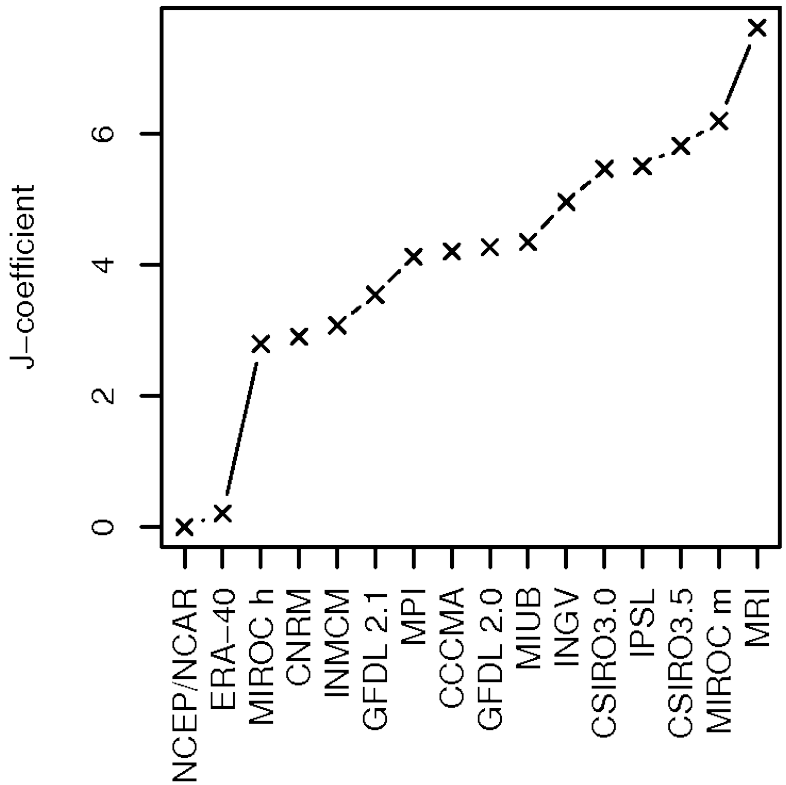
GA: many GCMs with NCEP/NCAR



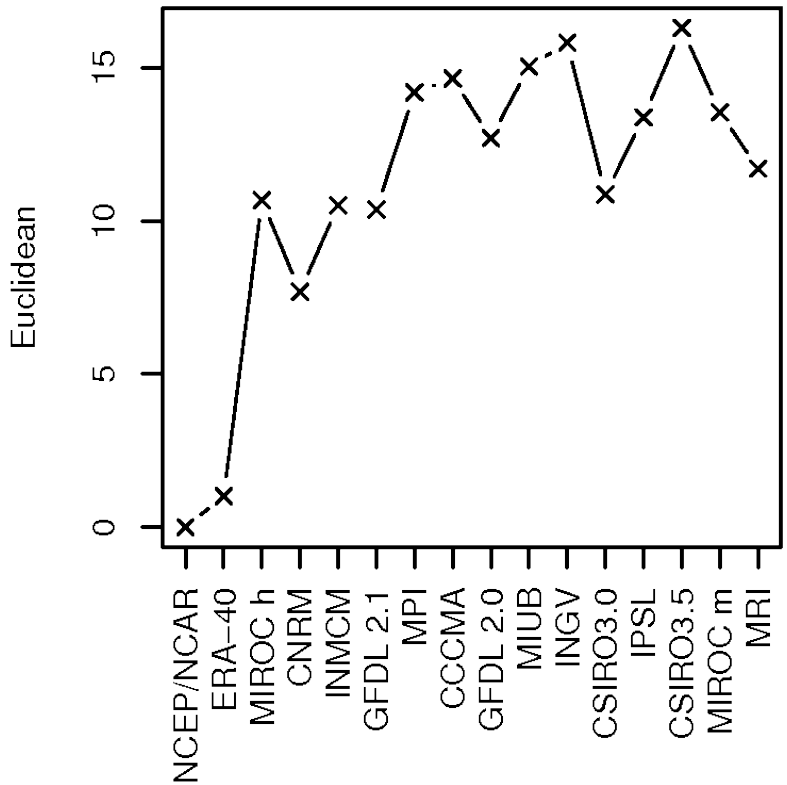
Ellipsoidal GCM

Results

GA: many GCMs with NCEP/NCAR



J-coefficient



Euclidian distance

Ellipsoidal GCM

Summary

- Gaussian mixture model clustering
  - non-spherical clusters
  - pdf-based

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  - explore state space
  - means, variances (size, shape)
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Summary

- Gaussian mixture model clustering
  - non-spherical clusters
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- quantitative cluster differences
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  - means, variances (size, shape)
  - compare many models
- North Atlantic region
  - ZO overlap with AR/BL, GA isolated
  - MIROC h, smallest distance (difference is CP dependent)
  - GA: J-coeff $\neq$ Euclidian

- Mahalanobis
- Kullback-Leibler
- J-coefficient
- Hellinger coeff.

References

- H. W. Rust et al.
  Quantifying differences in circulation patterns based on probabilistic models: AR4 multimodel comparison for the North Atlantic
  J.Climate 23:6573-6589 (2010)

- R-packages `gaussDiff` and `mclust` on CRAN
  `http://cran.r-project.org`