One Candidate Mechanism of Low-Frequency Oscillation

- Coriolis Parameter Variance Associated with Latitude

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Abstract:

It can be proved that existing probability of absolute geostrophic balance almost reaches zero in real atmosphere, it also can be ascertained that existing probability of non-geostrophic equilibrium is at least close to 70.0% through a new probability density function, this is often referred as quasi-geostrophic equilibrium , which is ubiquitous in real atmosphere, this is possibly one source of low-frequency oscillation; it is inferred that low frequency oscillation (30-70days) initiate from tropical region (low latitude), especially commence from tropical ocean area; relatively, shorter periods of oscillation (one week or so) originate from high latitude; furthermore, there is exist of two weeks-oscillations in mid-latitude. depending on a properties of continuous medium and the total derivative of the vorticity in atmosphere, the so-called "high frequency" of shorter than one week, which embedded in the high-level steering flow, similar to Rossby wave trains, generally propagate possibly from high-latitude toward to low-latitude, and vice versa: low-frequency oscillation also propagate along the high-level steering flow from low-latitude toward to middle−high latitude as well.

1, Introduce

Although there being a notion that the atmosphere is not determinedly predictable beyond a few weeks due to that non-integrable properties of the atmosphere may

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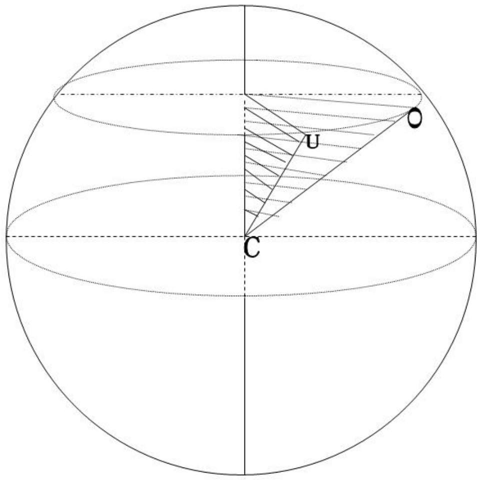
produce chaos, but previous some studies have confirmed the facts of oscillation of low-frequency existing in atmosphere , such as Blocking ;MJO, Kelvin Wave and Monsoon Trough etc ,whose vary periodicities is various from one week to 70 days , some of them especially perform eastward-propagation from tropical ocean, however, what mechanism exactly of this kind of low-frequency vibration still remain unclear hitherto ,at least is not agreed unanimously among scientists, so this issue motivates many researcher to do further studying the cause of atmospheric low-frequency fluctuation, particularly paying attention to large-scale persistent flow patterns. So as to explore and explain low-frequency phenomenon here one model is employed to establish a nonlinear differential equation based on the Non-Equilibrium between geostrophic force and pressure gradient force; furthermore according to quasi- geostrophic equilibrium theory, the one-dimension nonlinear differential equation with air friction has been also established. it is noteworthy that In order to prevent the oscillation equation from no solution, the probability density function has been deduced, this probability density function bring out that the occurrence probability of absolute geostrophic equilibrium is zero. the occurrence probability of quasi- geostrophic equilibrium can get as bigger as 70.0% in real atmosphere when these kinds of quasi- geostrophic equilibrium oscillation generally take place within the interval between $\pm\sigma$ （one standard deviation）.So the frequency spectrum from several days to 70 days associated with latitude variation has been found, since then in theory low-oscillation perhaps come into being and Coriolis Parameter Variance may be possibly one of some mechanisms related to atmospheric Low-Frequency Oscillation. This paper will be arranged as bellow: To obtain oscillation equation and make calculation of oscillation periodicity in section 2. Do analysis of what the occurrence probability regarding absolute geostrophic equilibrium and quasi-geostrophic equilibrium in section 3. Give summary and discussion in section 4.

2, Oscillation Equation and Solution



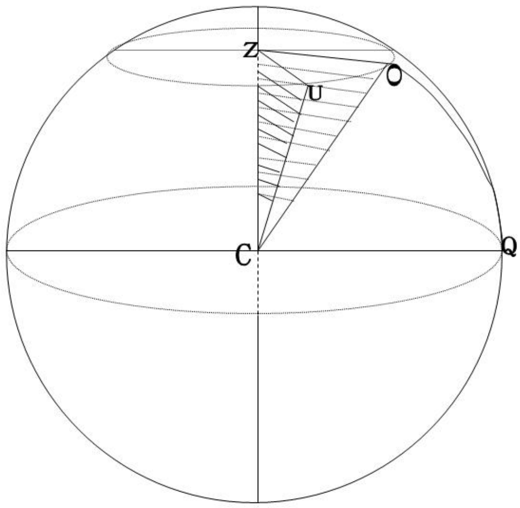


Chart1 ： Oscillation flat **OCU** (triangle) on the cone fixed in the center of earth.

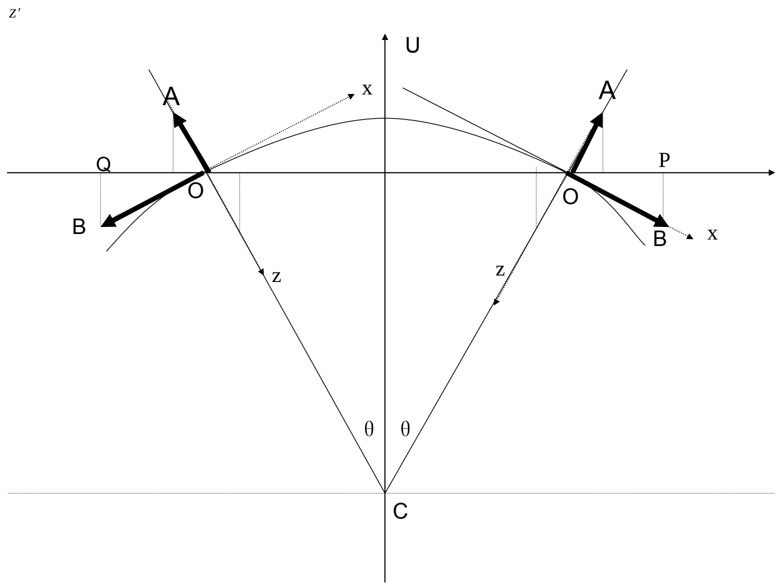


Chart 2: triangle flat **OCU** of oscillation (note: Coriolis force is greater than pressure gradient force). Right (left) part correspond to meridian wind toward north (south). Right (left) indicate west (east) ridge point of anti-cyclone.

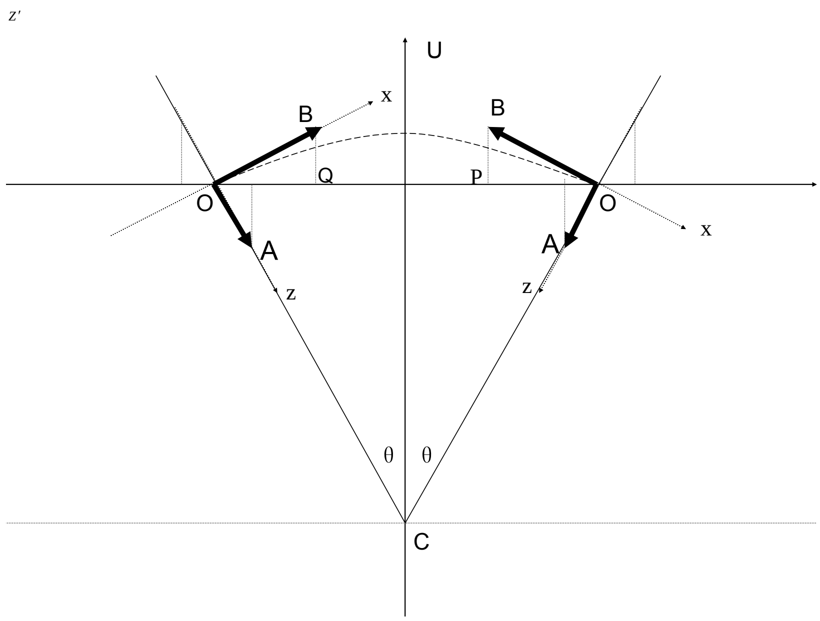


Chart 3: triangle flat **OCU** of oscillation (note: pressure gradient force is greater than Coriolis force) . Right (left) part correspond to meridian wind toward north (south). Right (left) indicate west (east) ridge point of anti-cyclone.

(1), the case 1

The basic component for the case 1 is that Coriolis force is greater than pressure gradient force; meridian wind is toward north, see right part of chart 2.The case 1 is classified as the special situation when the Coriolis force (geostrophic force) is greater than pressure gradient force, which is showed like right part in chart 2. Here θ denote the angle between two line of earth radius of the cone fixed in the center of earth (see chart1), θ is also oscillation angle which is just equal to Δ OCU in chart 1 and in fact θ is spherical angle. So θ variation means spherical oscillation rather than east-west oscillation. At some level in troposphere, Z is the vertical coordinate, here deserve emphasis that the Z direction toward to center of earth is positive, otherwise, negative. x is the zonal coordinate (eastward is positive). right O point denote west point of some anti-cyclone in chart 2 , such as West Pacific Subtropical High ,Blocking high at 500hPa or South-Asian High at 200hPa etc. Due to O point move on one geopotential surface, so it is necessary that gravity and buoyancy should be considered.

It show that the Coriolis force (geostrophic force) is greater than pressure gradient force at right point O in chart 2, meanwhile the pressure at point O will change into smaller, then air will move toward High Pressure (Anti-cyclone) Center, so air at this given point O will ascend (shown in chart 2 ,right part) because of negative anomalous pressure at this reference point.

It is proposed that there is no any internal friction in atmosphere in total duration, so the all force is projection on QP line described as below:

(See chart 2)

$$\begin{aligned} F(\theta) &= OB\cos\theta + OA\sin\theta \\ &= (2\Omega V\sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial x})\cos\theta + (g - \frac{1}{\rho}\frac{\partial p}{\partial z})\sin\theta \end{aligned}$$

Here x the zonal coordinate; z the vertical coordinate; L equal to the earth radius(R_e) added by altitude of middle level in troposphere; undoubtedly L is approximately R_e ; meridian wind is V ; g being the acceleration of gravity ; $2\Omega\sin\varphi$ is just Coriolis parameter (f) ; p is pressure; t is time; ρ is air density; θ denote the angle between two line of earth radius of the cone fixed in the center of earth, θ is also oscillation angle.

It should be also noticeable here that $F(\theta)$ is the projection of vertical direction force added with horizontal direction force on QP line ,including gravity、buoyancy、geostrophic force and pressure gradient force. Meanwhile $F(\theta)$ toward to x coordinate direction (eastward) is positive, otherwise, negative. Owing to in real atmosphere the force on vertical direction is bigger than force on horizontal direction at least by two-three magnitude. For example, in fact $2\Omega V\sin\varphi \rightarrow 5.8\times10^{-4}m\cdot s^{-2}$,And $g = 9.8\times10^{0}m\cdot s^{-2}$ .In order to simplify analysis it is necessary that we should separate force on horizontal direction from the force on vertical direction, then

$$F_H(\theta) = OB\cos\theta = (2\Omega V\sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial x})\cos\theta$$

Here $F_H(\theta)$ means the projection of horizontal direction force on QP line. It is also assumed that $F_H(\theta)$ toward to x coordinate direction (eastward) is positive, otherwise, negative. in this situation, the geostrophic force is greater than pressure gradient force, this situation represent analysis of west ridge point of anti-cyclone (see right part of chart 2).On other words, $F_H(\theta) > 0$,in addition, $V > 0, \frac{\partial p}{\partial x} > 0, \left|2\Omega V\sin\varphi\right| > \left|\frac{1}{\rho}\frac{\partial p}{\partial x}\right|$.

Then

$$(2\Omega V\sin\varphi-\frac{1}{\rho}\frac{\partial p}{\partial x})>0.$$

This result can guarantee that right side of the oscillation equation is positive.

(2), the case 2

The basic component for the case 2 is that Coriolis force is greater than pressure gradient force; meridian wind is toward south, see left part of chart 2.here we only treat the projection of horizontal direction force on QP line, According to that $F_H(\theta)$ toward to x coordinate direction(eastward) is positive, otherwise, negative. in contrast to the case 1, $F_H(\theta)$ is negative as $F_H(\theta)$ is toward to anti-direction of x coordinate, meanwhile, the geostrophic force is bigger than pressure gradient force, this situation represent analysis of east ridge point of anti-cyclone(See left part of chart 2). In a word,

$F_H(\theta)<0$,in addition, $V<0$, $\frac{\partial p}{\partial x}<0$, $\left|2\Omega V\sin\varphi\right|>\left|\frac{1}{\rho}\frac{\partial p}{\partial x}\right|$.

$$-F_H(\theta)=OB\cos\theta=(2\Omega V\sin\varphi-\frac{1}{\rho}\frac{\partial p}{\partial x})\cos\theta$$

Also

$$F_H(\theta)=OB\cos\theta=(-2\Omega V\sin\varphi+\frac{1}{\rho}\frac{\partial p}{\partial x})\cos\theta$$

In the case 2, V <0 (the wind is toward to south), $\partial p/\partial x<0$,moreover, the geostrophic force is also bigger than pressure gradient force, so that for east ridge point of anti-cyclone(see left part of chart 2),

$$(-2\Omega V\sin\varphi+\frac{1}{\rho}\frac{\partial p}{\partial x})>0$$

This result can guarantee that right side of the oscillation equation is positive.

(3), the case 3

The basic component for the case 3 is that Coriolis force is smaller than pressure gradient force; meridian wind is toward north, see right part of chart 3.here we only treat the projection of horizontal direction force on QP line. $F_H(\theta)$ toward to x coordinate anti-direction(westward) is negative. $F_H(\theta)<0$, for west ridge point of anti-cyclone (see left part of chart 3), although V >0 (wind is toward to north), $2\Omega V\sin\varphi>0$,but as the geostrophic force is smaller than pressure gradient force,

this situation represent analysis of west ridge point of anti-cyclone(See right part of chart 3). So that in a word,

$F_H(\theta)<0$,in addition, $V>0$, $\frac{\partial p}{\partial x}>0$, $\left|\frac{1}{\rho}\frac{\partial p}{\partial x}\right|>\left|2\Omega V\sin\varphi\right|$.

The oscillation equation

$$-F_H(\theta)=OB\cos\theta=(2\Omega V\sin\varphi-\frac{1}{\rho}\frac{\partial p}{\partial x})\cos\theta$$

$$F_H(\theta)=(-2\Omega V\sin\varphi+\frac{1}{\rho}\frac{\partial p}{\partial x})\cos\theta$$

Finally

$$(-2\Omega V\sin\varphi+\frac{1}{\rho}\frac{\partial p}{\partial x})>0$$

This result can also guarantee that right side of the oscillation equation is positive.

(4), the case 4

The basic component for the case 4 is that Coriolis force is smaller than pressure gradient force; meridian wind is toward south, for east ridge point of anti-cyclone, $F_H(\theta) > 0$, see left part of chart 3.Here we only treat the projection of horizontal direction force on QP line. $F_H(\theta)$ toward to x coordinate direction(eastward) is positive.

But In the case 4

V <0 (the wind is toward to south), $\partial p/\partial x < 0$.

However, Coriolis force is smaller than pressure gradient force,

$$F_H(\theta) > 0$$

So that in a word,

$F_H(\theta) > 0$,in addition, $V < 0$, $\frac{\partial p}{\partial x} < 0$, $\left|\frac{1}{\rho}\frac{\partial p}{\partial x}\right| > \left|2\Omega V \sin\varphi\right|$.

$$F_H(\theta) = (2\Omega V \sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial x})\cos\theta$$

Finally

$$(2\Omega V \sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial x}) > 0$$

This result can also guarantee that right side of the oscillation equation is positive.

Solution of equation

Owing to for four cases above right sides of the oscillation equation is always positive,The rest are also same for all four cases, so here only solution of the oscillation equation of case 1 will be analyzed, the results of the rest equations can be obtained by analogy.

(1) Solution of equation for geostrophic equilibrium in case1

The oscillation equation

$$F_H(\theta) = (2\Omega V \sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial x})\cos\theta$$

According to the second law of Newton

$$L\frac{d^2\theta}{dt^2} = (2\Omega V \sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial x})\cos\theta$$

Here L=CO, approximately, equal to the earth radius (R_e) added by altitude of middle level in troposphere.

Using the approach of linearization

$$\theta = \overline{\theta} + \theta'$$

$\overline{\theta}$ is mean state, θ' is small perturbation.

$$\cos\theta \approx 1 - \overline{\theta}\theta' - \frac{\overline{\theta}^2}{2}$$

$$L\frac{d^2\theta'}{dt^2} = (2\Omega V \sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial x})(1 - \overline{\theta}\theta' - \frac{\overline{\theta}^2}{2})$$

$$\frac{d^2\theta'}{dt^2} + \frac{\overline{\theta}}{L}(2\Omega V \sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial x})\theta' + \frac{1}{L}(\frac{\overline{\theta}^2}{2} - 1)(2\Omega V \sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial x}) = 0$$

Make the equation above to be homogeneous equation.

$$\omega_0^2 = \frac{\overline{\theta}}{L}(2\Omega V \sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial x})$$

$$\omega_1^2 = \frac{1}{L}(\frac{\overline{\theta}^2}{2} - 1)(2\Omega V \sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial x})$$

Then

$$\frac{d^2\theta'}{dt^2} + \omega_0^2\theta' + \omega_1^2 = 0$$

If continue homogeneous equation

And $$\psi = \omega_0^2\theta' + \omega_1^2$$

$$\theta' = \frac{\psi}{\omega_0^2} - \frac{\omega_1^2}{\omega_0^2}$$

$$\frac{d^2\theta'}{dt^2} = \frac{1}{\omega_0^2}\frac{d^2\psi}{dt^2}$$

So

$$\frac{d^2\psi}{dt^2} + \omega_0^2\psi = 0$$

Then solution is expressed as follow because the equation above is standard ordinary differential equation

$$\Psi(t) = \Psi_0 \sin(\omega_0 t + \alpha)$$

In detail this process is oscillation movement with periodicities T . Ψ_0 is the initial amplitude, α the initial phase, T is the oscillation periodicities, and

$$T = \frac{2\pi}{\omega_0}$$

To retrieve

$$\psi = \omega_0^2\theta' + \omega_1^2$$

Then

$$\theta' = \theta_0' \sin(\omega_0 t + \alpha) + \theta_1$$

And

$$\theta_0' = \frac{\psi_0}{\omega_0^2} \qquad \theta_1 = -\frac{\omega_1^2}{\omega_0^2}$$

If initial conditions

$$\theta'(t \to \infty) = 0; \theta'(t = 0) = Maximum = \theta'_M$$

Then

$$\theta_0' = \frac{\psi_0}{\omega_0^2} = \theta_M'$$

$$\theta_1 = 0$$

$$\alpha = \frac{\pi}{2}$$

Finally

$$\theta' = \theta_M' \cos(\omega_0 t)$$

The physics process is explained as that (1)the meridian V will change into smaller due to longitudinal(east-west) temperature gradient attenuating when longest time passes away so that longitudinal(east-west) pressure gradient will be bigger than the Coriolis force, this atmosphere oscillation finish.(2)the initial phase is when atmosphere oscillation just commence and at this time actually longitudinal(east-west) temperature gradient is biggest, the extent to which the oscillation system deviates from its balance position is also greatest, similarly, Coriolis force is biggest at this point.

From

$$\theta = \overline{\theta} + \theta'$$

The periodicity of θ' oscillation is just same as the periodicity of θ oscillation.

$$T_H = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{\frac{\overline{\theta}}{L}(2\Omega V \sin\varphi - \frac{1}{\rho}\frac{\partial P}{\partial x})}}$$

If $\partial p/\partial x = 1hPa/300Km$, $\rho = 1.3kg \cdot m^{-3}$ (conditioned $0^\circ C$ and $1000hPa$),

$\frac{1}{\rho}\frac{\partial p}{\partial x}$ approximately $2.5\times10^{-4} m \cdot s^{-2}$

And $\varphi = 30^\circ N$, $V = 8m \cdot s^{-1}$, $\Omega = 7.3\times10^{-5} s^{-1}$,

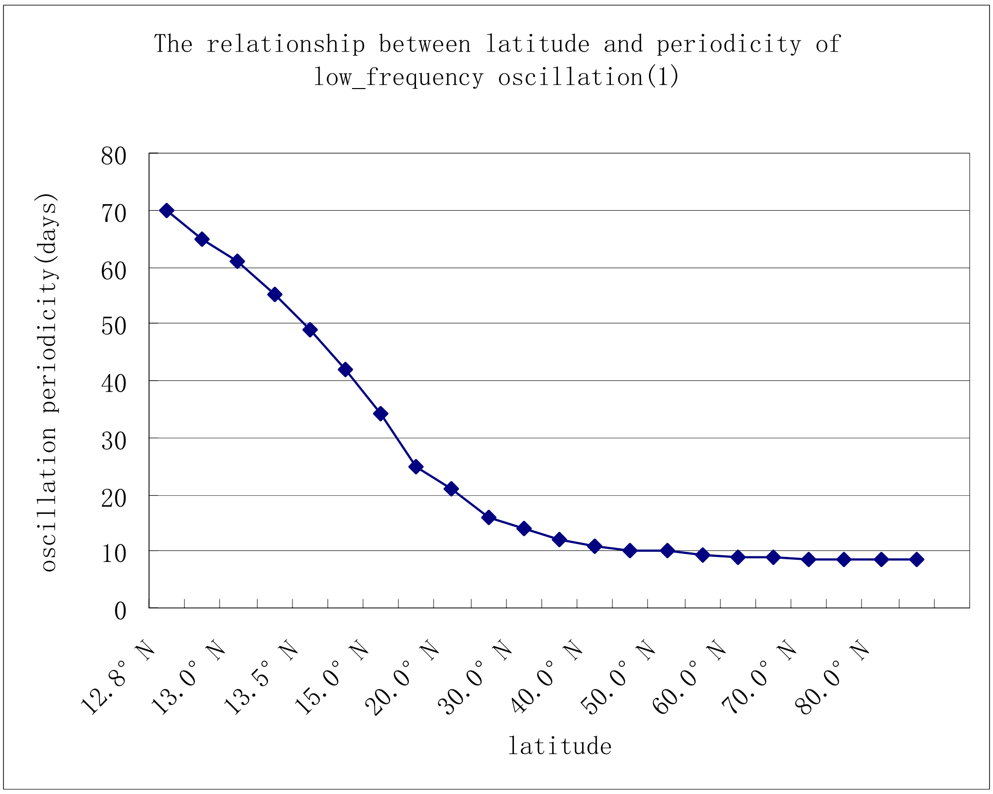


Chart 4 the periodic low-frequency oscillation associated with latitudes

then $2\Omega V \sin\varphi$ approximately $5.8\times10^{-4} m\cdot s^{-2}$, $\overline{\theta}=1.0$ (unit is radian), $L \approx R_e = 6.378\times10^{6} m$. In $\sqrt[2]{\ }$ the magnitude is near to $10^{-11} s^{-2}$. Finally the results calculated of periodicity of θ oscillation (spherical angle oscillation on earth surface) when latitudes various from 12.8 degree to 80 degree is showed in table 1 and chart 4.

Table1 the periodic oscillation associated with latitudes

Latitude(°N)	12.8	14	15	20	30	40	50	60	70	80
Periodicity (days)	70	42	34	21	14	11	10	9	8.7	8.4
∂p/∂x		1hPa/300km								

Those conclusion demonstrate that the oscillation of earth spherical angle possess quality of low-frequency wave from one-week or so to 70 days, at same time, low-frequency oscillation (50-70days) originates from low latitude region(tropical area

); two-week oscillation comes from middle latitude, less shorter than one-week oscillation firstly appears in high-latitude area.

(2) solution of equation for hydrostatic equilibrium in case1

Oscillation equation

$$F_g(\theta) = OA\sin\theta = (g - \frac{1}{\rho}\frac{\partial p}{\partial z})\sin\theta$$

Owing to Z is the vertical coordinate whose direction is toward to center of earth(toward earth center is positive), so here $(g - \frac{1}{\rho}\frac{\partial p}{\partial z})$ <0 ,or is negative.(see right part of chart 2)

Similarly using the approach of linearization

$$\theta = \overline{\theta} + \theta'$$

$\overline{\theta}$ is mean state, θ' is small perturbation.

$$\sin\theta \approx \overline{\theta} + \theta'$$

$$\frac{d^2\theta'}{dt^2} + \frac{1}{L}(\frac{1}{\rho}\frac{\partial p}{\partial z} - g)\theta' + \frac{\overline{\theta}}{L}(\frac{1}{\rho}\frac{\partial p}{\partial z} - g) = 0$$

Homogeneous equation.

$$\omega_0^2 = \frac{1}{L}(\frac{1}{\rho}\frac{\partial p}{\partial z} - g)$$

$$\omega_1^2 = \frac{\overline{\theta}}{L}(\frac{1}{\rho}\frac{\partial p}{\partial z} - g)$$

Then

$$\frac{d^2\theta'}{dt^2} + \omega_0^2\theta' + \omega_1^2 = 0$$

Way of solution is as same as the solution used in the geostrophic oscillation, In addition to, here is the state equation.

$$P = \rho RT$$

Then

$$T_g = \frac{2\pi}{\sqrt{\frac{R}{L}\left(\frac{\partial T}{\partial Z} - \frac{g}{R}\right)}}$$

Here $\frac{g}{R}$ is （3.45k/100m） and also is recognized as "automatic convection constant" in atmosphere. In addition,here

$$g = 9.8\times10^0\, m\cdot s^{-2}, R = 287m^2\cdot s^{-2}\cdot k^{-1},$$

$$\partial T/\partial z = 1k/100m, \frac{1}{\rho}\frac{\partial p}{\partial z} = R\times\frac{\partial T}{\partial z} = 2.87\times10^0\, m\cdot s^{-2},$$

$L \approx R_e = 6.378\times10^6\, m$, in $\sqrt[2]{\ }$ the magnitude is approaching to $10^{-6}s^{-2}$.

The outcome estimated is showed in Table2 bellow.

Table 2　　the gravity oscillation periodicities with Temperature lapse rate

Temperature lapse rate ∂T/∂z(℃ /100m)	0.5	1	2	2.5	3	3.45
Periodicities Tg (minute)	4	5	7	8	12	100

Physical significant is that the more the atmospheric stratification is not stability the more time is needed to finish one vertical oscillation.In addition, analysis of the gravity oscillation may be categorized into four same parts, just like that of the geostrophic oscillation from West Ridge Point to East Ridge Point and distinct south wind from north wind. The rest process is left out.

Those different outcomes of fluctuation periodicities regarding geostrophic balance oscillation and static balance oscillation reflect in geostrophic adjustments process being slow-process ($10^5 - 10^6$s), oppositely, gravity (static) adjustments process being quick-process ($10^2 - 10^3$s).

3，Analysis of How Probability on Quasi-Geostrophic Equilibrium

See oscillation periodicities formula

$$T_H = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{\frac{\bar{\theta}}{L}(2\Omega V \sin\varphi - \frac{1}{\rho}\frac{\partial P}{\partial x})}}$$

If Coriolis force is equal to pressure gradient force in equation above as well as

$$2\Omega V \sin\varphi = \frac{1}{\rho}\frac{\partial p}{\partial x}$$

If so indeed

$$T_H \to \infty$$

Therefore oscillation periodicities formula discussed above will be invalidated. So that if geostrophic equilibrium really happens; there is no any solution about equation above. This question motivate us search for how many probability on state of the geostrophic equilibrium, and how many probability at near to the state of the geostrophic equilibrium, the latter is also regarded as quasi-geostrophic equilibrium. It is postulated that the equation should be **available** If there could be smallest probability on absolute geostrophic equilibrium, and there could be relatively bigger probability on quasi-geostrophic equilibrium (very adjacent to geostrophic equilibrium). So as to reach this purpose the internal friction in atmosphere should be considered in investigation, so oscillation equation is bellow:

$$L\frac{d^2\theta}{dt^2} = (2\Omega V\sin\varphi - \frac{1}{\rho}\frac{\partial p}{\partial x})\cos\theta - \frac{\mu s L}{\Delta L}\frac{d\theta}{dt}$$

Here μ is the coefficient of dynamic viscosity of air, s is unit area by which air touch the boundary, $\frac{L}{\Delta L}\frac{d\theta}{dt}$ is horizontal velocity gradient from bottom to some height level.

Finally

$$\theta = \theta_M e^{-\beta t}\cos(\omega_0 t)$$

Proof process is omitted.

Here $\beta = \frac{\mu s}{2\Delta L}$.

If θ_M , β , ω_0 are supposed as random variables, then θ will become also random variables.

So envelope equation is as follow

$$\theta = \theta_M e^{-\beta t}$$

Developing

$$t = \frac{1}{\beta}\ln\frac{\theta_M}{\theta}$$

Here is a hypothesis that direction of time is just direction of probability, on other words, the longer time is the bigger is probability, t time is proportional to the probability of system occurrence, so that

$$t \propto f(\theta)$$

Here $f(\theta)$ is probability density function, then

$$f(\theta) = \frac{c_1}{\beta}\ln\frac{\theta_M}{\theta}$$

c_1 is a scale factor.

$$f(\theta)=\frac{c_1}{2\beta}\ln(\frac{\theta_M}{\theta})^2$$

Continue

$$f(\theta)=c\ln(\frac{\theta_M}{\theta})^2,(-\theta_M<\theta<\theta_M)$$

c is also a whole scale factor.

$$c=\frac{c_1}{2\beta}$$

Now we solve what is scale factor c, depending on the property of probability density function, so

$$\int_{-\theta_M}^{\theta_M} f(\theta)=c\int_{-\theta_{M<}}^{\theta_M}\ln(\frac{\theta_M}{\theta})^2 d\theta=1$$

Below could be proved

$$c=\frac{1}{4\theta_M}$$

Certainly, probability density function is just bellow

$$f(\theta)=\frac{1}{4\theta_M}\ln(\frac{\theta_M}{\theta})^2 \qquad (-\theta_M<\theta<\theta_M)$$

m-th moment of the function

When m is denoted even, then

$$\nu_m=\frac{1}{4\theta_M}\int_{-\theta_M}^{\theta_M}\theta^m\ln(\frac{\theta_M}{\theta})^2 d\theta=\frac{\theta_M^m}{(m+1)^2}$$

m=2（variance）,m=4（fourth moment）

So variance

$$\nu_2 = D(\theta) = \frac{1}{4\theta_M}\int_{-\theta_M}^{\theta_M} \theta^2 \ln(\frac{\theta_M}{\theta})^2 \, d\theta = (\frac{\theta_M}{3})^2$$

Standard deviation

$$\sigma = \frac{\theta_M}{3}$$

When m is denoted odd

$$\nu_m = \frac{1}{4\theta_M}\int_{-\theta_M}^{\theta_M} \theta^m \ln(\frac{\theta_M}{\theta})^2 \, d\theta = 0$$

m=1（expected value）, m=3（third moment）

During process proved above, below can be certified firstly by using L'Hopital's Rule

$$\lim_{\theta \to 0} \frac{2}{m+1}\theta^{m+1} \ln \theta \to$$

$$\frac{2}{(m+1)} \frac{\lim\limits_{\theta \to 0} \ln \theta}{\lim\limits_{\theta \to 0} \frac{1}{\theta^{m+1}}} \to 0$$

The distribution function

We integrate probability density function, So that distribution function is verified below

$$F(\theta) = \int f(\theta) d\theta = \frac{1}{4\theta_M}\{[\theta \ln(\frac{\theta_M}{\theta})^2 + 2\theta] + 2\theta_M\}$$

$$(-\theta_M < \theta < \theta_M)$$

Certainly $F'(\theta) = f(\theta)$

The probability within any interval（$\theta_1 < \theta < \theta_2$）

According to the probability theorem

$$P(a < Y < b) = F(b) - F(a), -\infty < a \leq b < \infty$$

Therefore within any interval the probability is calculated as below formula

$$\begin{aligned} & P(\theta_1 < \theta < \theta_2) = F(\theta_2) - F(\theta_1) \\ & = \frac{1}{4\theta_M} \int_{\theta_1}^{\theta_2} \ln(\frac{\theta_M}{\theta})^2 \, d\theta \\ & = \frac{1}{4\theta_M} \{[\theta_2 \ln(\frac{\theta_M}{\theta_2})^2 + 2\theta_2] - [\theta_1 \ln(\frac{\theta_M}{\theta_1})^2 + 2\theta_1]\} \\ & \qquad (-\theta_M < \theta < \theta_M) \end{aligned}$$

for example if $\theta_1 = -\frac{\theta_M}{e}$, $\theta_2 = \frac{\theta_M}{e}$, thus P(θ_1 <θ< θ_2)= $\frac{2}{e}$ (approximately 74.04%), if random variables lies in the interval between ±σ（standard deviation）, or

$$\theta_1 = -\frac{\theta_M}{3}, \theta_2 = \frac{\theta_M}{3}$$

Therefore

$$P(-\frac{\theta_M}{3} < \theta < \frac{\theta_M}{3}) \approx 70.0\%$$

The probability of distribution nearly is equal to 70.0%(bigger probability event) within the interval ± standard deviation($\pm\frac{\theta_M}{3}$). The area within the interval ± standard deviation may be proposed as state of quasi-geostrophic equilibrium in atmosphere.

In addition

If $\theta_1 = -\frac{\theta_M}{e}, \theta_2 = \frac{\theta_M}{e}$

Then

$$P(-\frac{\theta_M}{e} < \theta < \frac{\theta_M}{e}) \approx 74.0\%$$

See chart 5

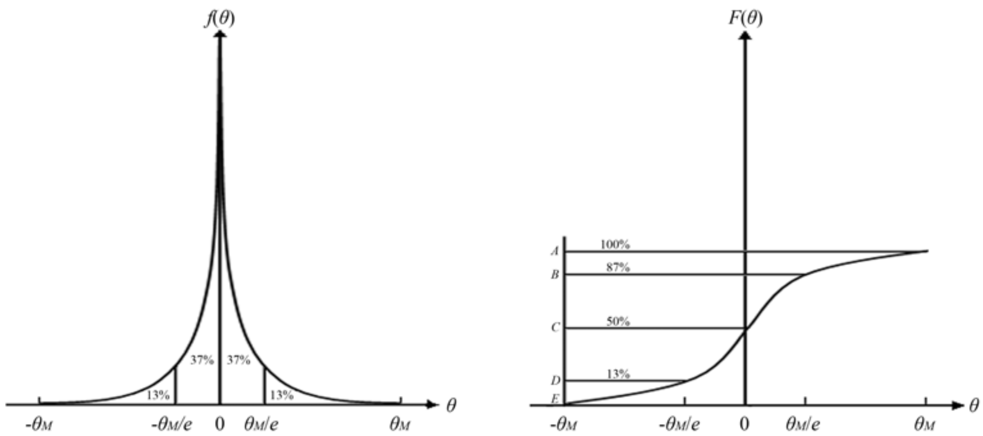


Chart 5: probability density function (left) and probability distribution function (right)

Next step is that the probability of absolute geostrophic balance exists and being smallest will be evolved follow.

Distribution function

$$F(\theta) = \int f(\theta) d\theta = \frac{1}{4\theta_M}\{[\theta \ln(\frac{\theta_M}{\theta})^2 + 2\theta] + 2\theta_M\}$$
$$(-\theta_M < \theta < \theta_M)$$

When $\theta_1 \to 0, \theta_2 \to 0$, the limit of formula above existing will be proved bellow,

$$\lim_{\theta\to 0} F(\theta) = \lim_{\theta\to 0} \ln(\frac{\theta_M}{\theta})^{\frac{\theta}{2\theta_M}} + \frac{\theta}{2\theta_M} \Big|_{\theta\to 0} + \frac{1}{2}$$

$$= \lim_{\theta\to 0} \ln(\theta_M)^{\frac{\theta}{2\theta_M}} - \lim_{\theta\to 0} \ln(\theta)^{\frac{\theta}{2\theta_M}} + \frac{\theta}{2\theta_M}\Big|_{\theta\to 0} + \frac{1}{2}$$

$$= \frac{1}{2}$$

Similarly through L'Hopital's Rule

$$\lim_{\theta\to 0} \ln(\theta)^{\frac{\theta}{2\theta_M}} = 0$$

So

$$P(\theta_1 \to 0 < \theta < \theta_2 \to 0) = P(\theta \to 0) = 1/2 - 1/2 = 0$$

And

$$P(\theta = 0) = 0$$

Physical meaning: when $\theta \to 0$, the Coriolis force is just equal to pressure gradient force, which is called as absolutely geostrophic balance, therefore, the existing probability of absolute geostrophic balance almost reaches zero in real atmosphere, this result mean that absolute geostrophic equilibrium perhaps never occur in the real atmosphere. However, there is special small area within the interval($-\frac{\theta_M}{3} \le \theta \le \frac{\theta_M}{3}$) ,this area is very closest to absolute geostrophic equilibrium but not equal to absolute geostrophic equilibrium, within which the existing probability nearly reach 70.0%,the oscillation system vary within this area is called quasi- geostrophic equilibrium, which is a event of bigger probability. So in principle the special case ($2\Omega V \sin\varphi = \frac{1}{\rho}\frac{\partial p}{\partial x}$) may be avoided to happen as using equations above.

Certainly

$$T_H = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{\frac{\bar{\theta}}{L}(2\Omega V \sin\varphi - \frac{1}{\rho}\frac{\partial P}{\partial x})}}$$

This equation (formula) above is almost available in most condition.

4，Summary and Discussion

It can be evidenced that existing probability of absolute geostrophic balance almost is equal to zero in real atmosphere, but it is ascertained that existing probability of non-geostrophic equilibrium is at least close to 70.0% by analysis of a new probability density function, this interval with 70.0% probability can be referred as quasi-geostrophic equilibrium , which is ubiquitous in real atmosphere, this is possibly one source of low-frequency oscillation; in addition, it is inferred that low frequency oscillation (30-70days) originate from tropical region (low latitude), especially commence from tropical ocean area; relatively, shorter periods of oscillation (one week or so) initiate from high latitude; furthermore, there is two weeks-oscillations existing in mid-latitude. On other hand, according to a properties of continuous medium and the total derivative of the vorticity in atmosphere, it can be envisaged that the so-called "high frequency" of shorter than one week, which embedded in the high-level steering flow, similar to Rossby wave trains, generally propagate possibly from high-latitude toward to low-latitude, and complete differently low-frequency oscillation also similarly propagate along with the high-level steering flow from low-latitude toward to middle-high latitude as well. Finally, it can be illustrated that the oscillation of spherical angle associated with latitude variation perhaps is one alternative mechnism of atmospheric low-frequency fluctuation.

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