Estimating Deformations of random processes for correlation modelling in variational data assimilation

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Météo-France & CNRS, CNRM-GAME/GMAP/RECYF

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Background Error Modeling: Spatial Deformations

Change of spatial coordinate (also called deformation, warping, distortion) is a way of achieving anisotropic correlations [Sampson and Guttorp 1992].

The idea is to find a grid deformation that reduces the correlation model to an isotropic and/or stationary one (easy to model).

Physical space *Deformed space*

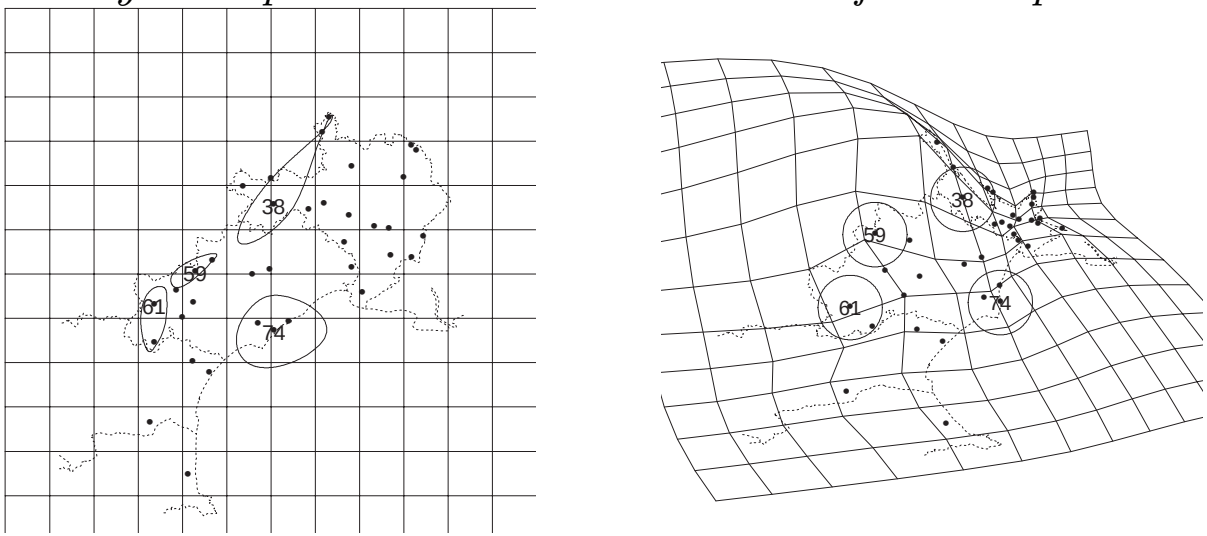


Figure: Geographic map of rainfall monitoring sites in the Languedoc-Roussillon region (Southern France) with 0.9 correlation contours [Sampson et al. 2003].

Here, the deformation is computed using a non-metric multi-dimensional scaling algorithm, only suitable in low dimensions (sparse stations)

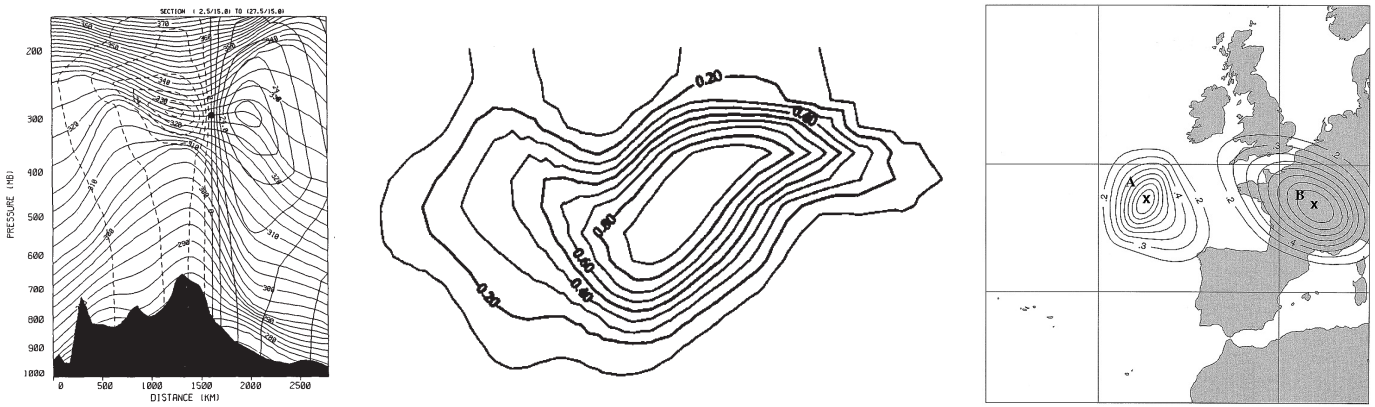
Why use this in geophysical data assimilation?

The Kalman Filter Propagation Equations evolves $\mathbf{A}$ into $\mathbf{B}$:

$$\mathbf{B} = \mathbf{M}\mathbf{A}\mathbf{M}^{\star} + \mathbf{Q}$$

where $\mathbf{M}$ is the linearized model and includes advection as a key processes. Correlations in $\mathbf{A}$ are known to be more isotropic/stationnary especially in data rich areas [Bouttier 1994].

Former use of spatial deformation in data assimilation



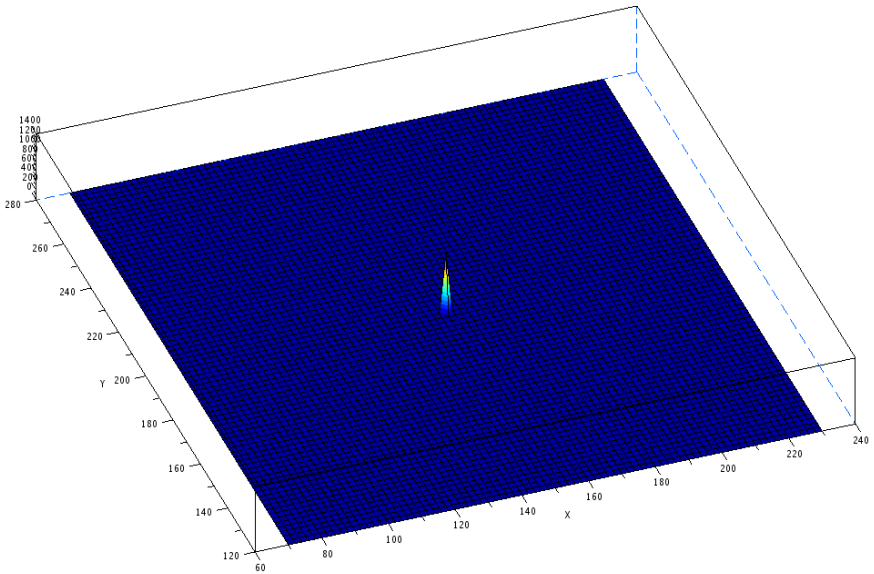
(a) [Benjamin 1989] (b) [Weaver and Courtier 2001] (c) [Desroziers 1997]

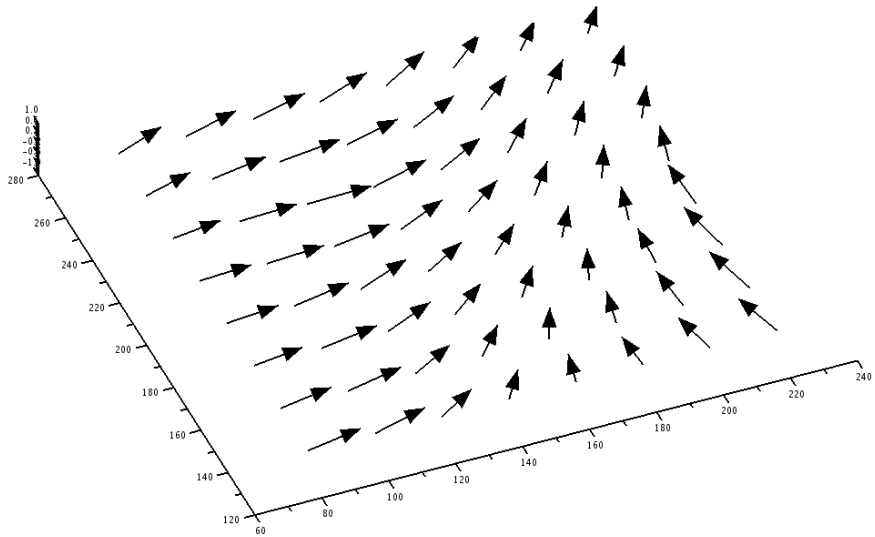
Figure: Some structure functions in $\mathbf{B}$ using (a) an isentropic, (b) an isopycnal and (c) a semi-geostrophic coordinate transform

Deformations used in litterature are mostly arbitrary and depend on background state only.

Deformed correlations from a sequence of operators

$$\mathbf{B} = \mathbf{D}\mathbf{L_x}\mathbf{L_y}\mathbf{L_y}^{\star}\mathbf{L_x}^{\star}\mathbf{D}^{\star}$$





Variational Data Assimilation

- square root form for Control Variable Transform $\mathbf{v} = \mathbf{B}^{1/2}\mathbf{u}$
- no need for (re)normalization of variances
- requires only the action of deformation on the state vector (and adjoint)
- How can we *statistically* estimate the deformation from the error samples?

Deformed correlations from a sequence of operators

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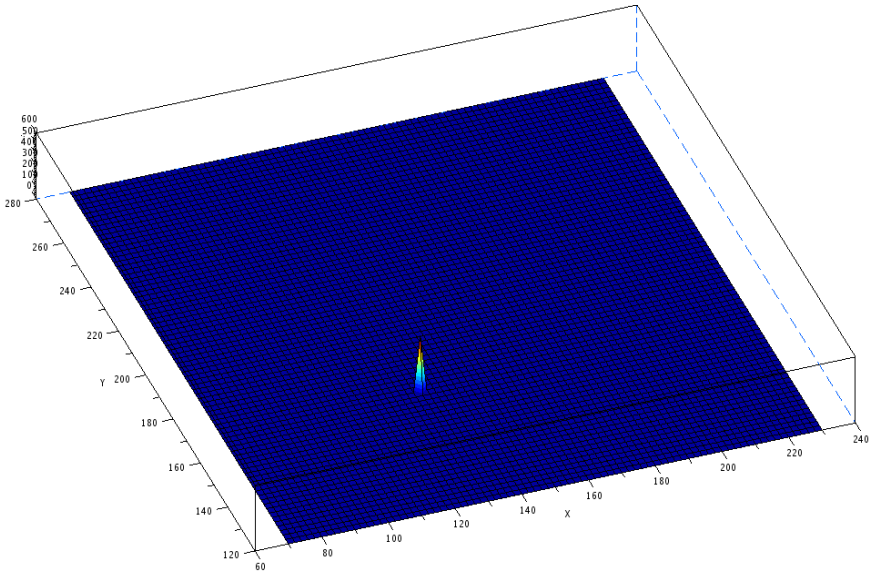
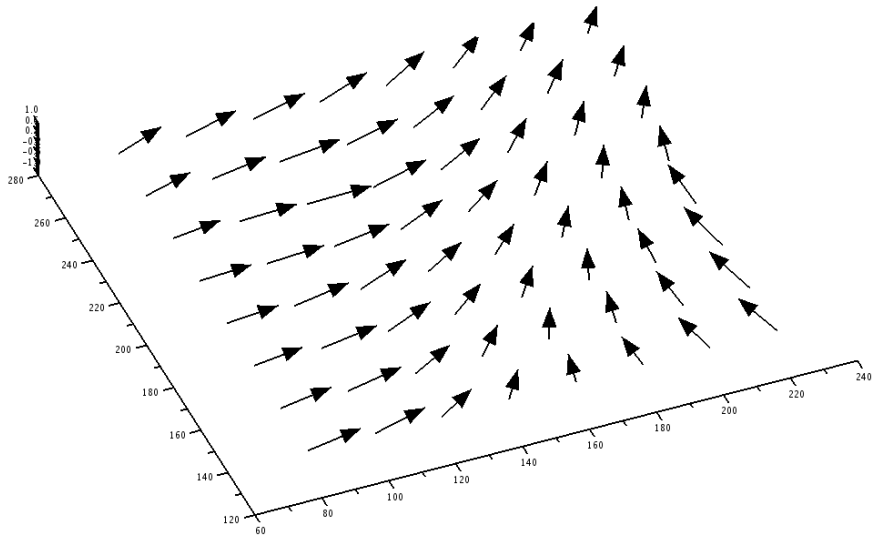
Adjoint of deformation





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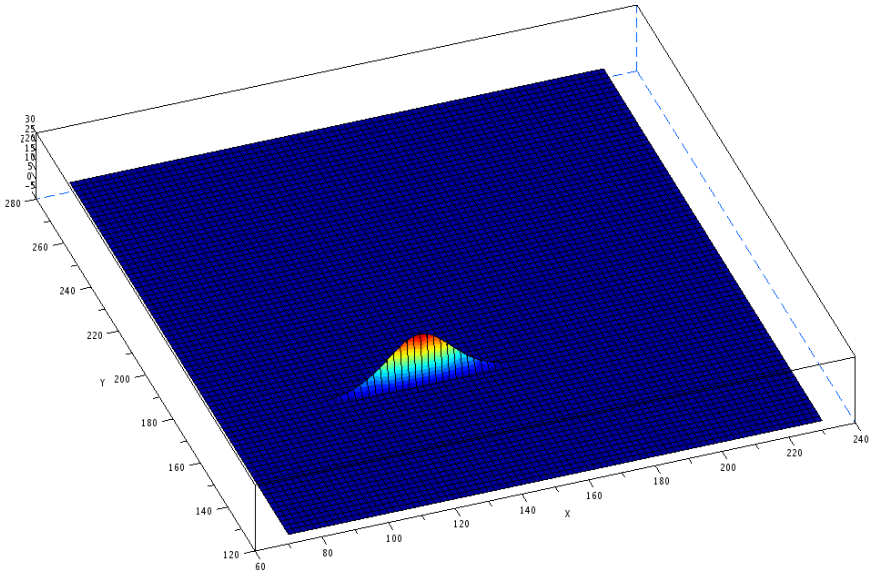
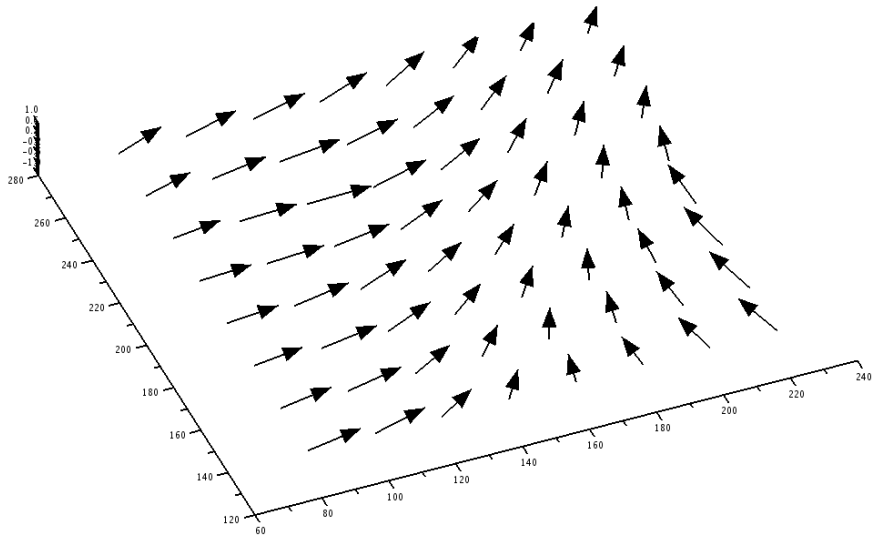
Recursive filter in x direction





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Deformed correlations from a sequence of operators

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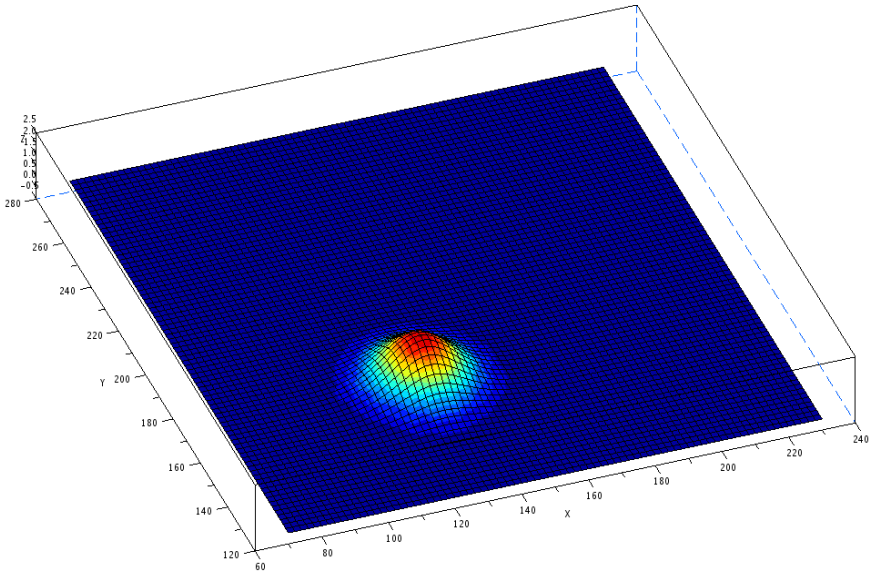
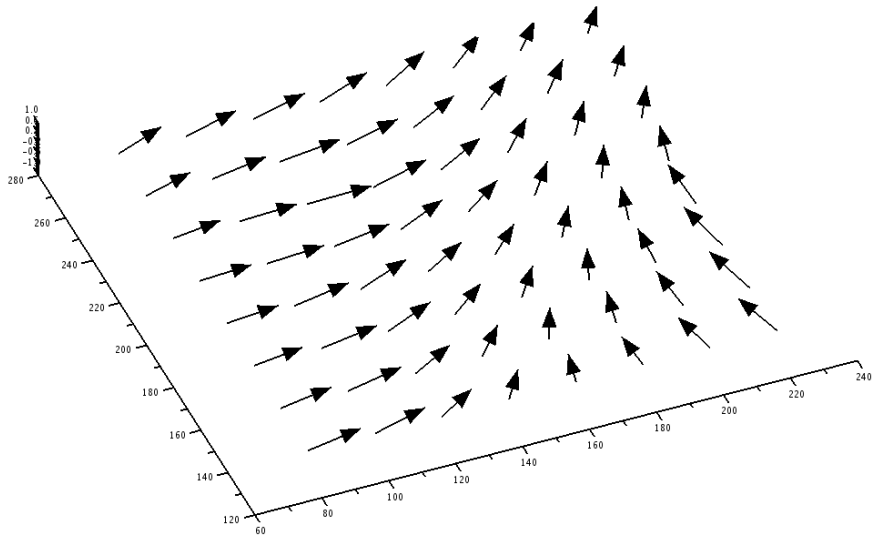
Recursive filter in y direction





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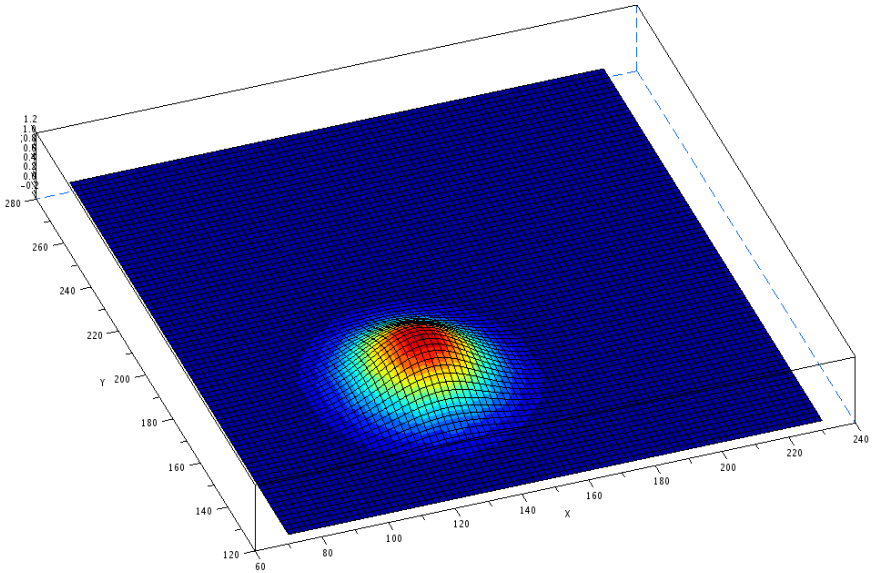
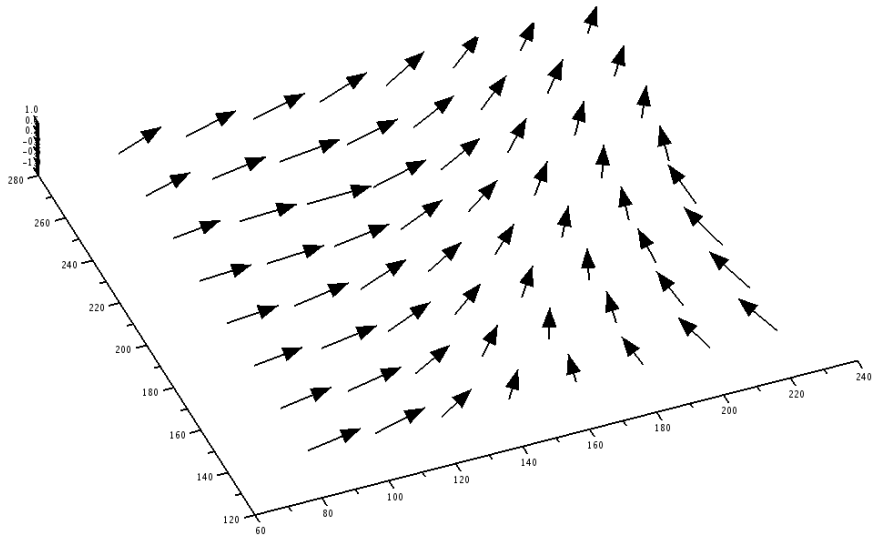
Recursive filter in x, y direction





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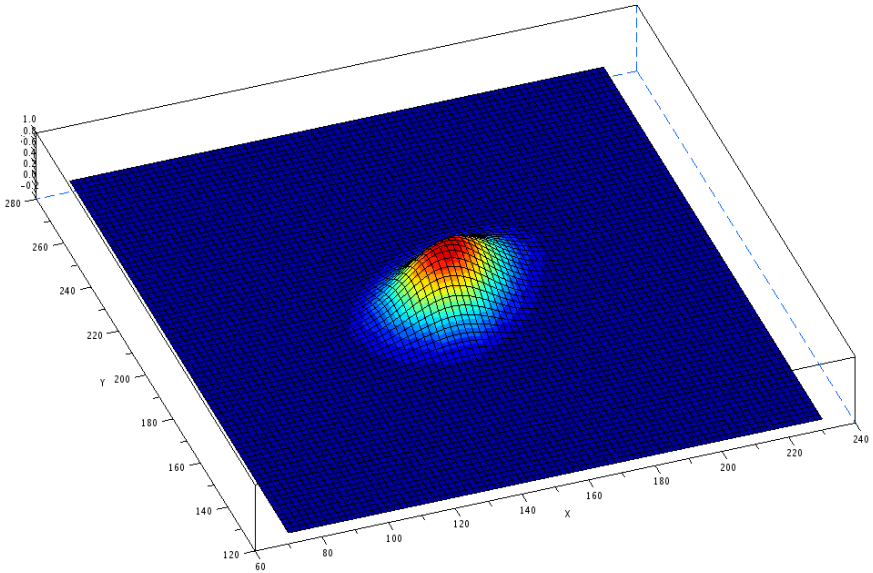
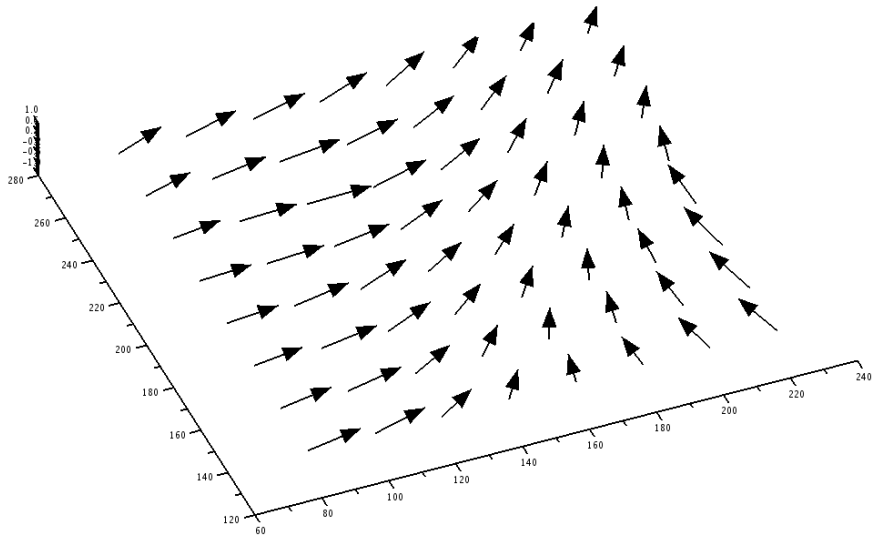
Deformation





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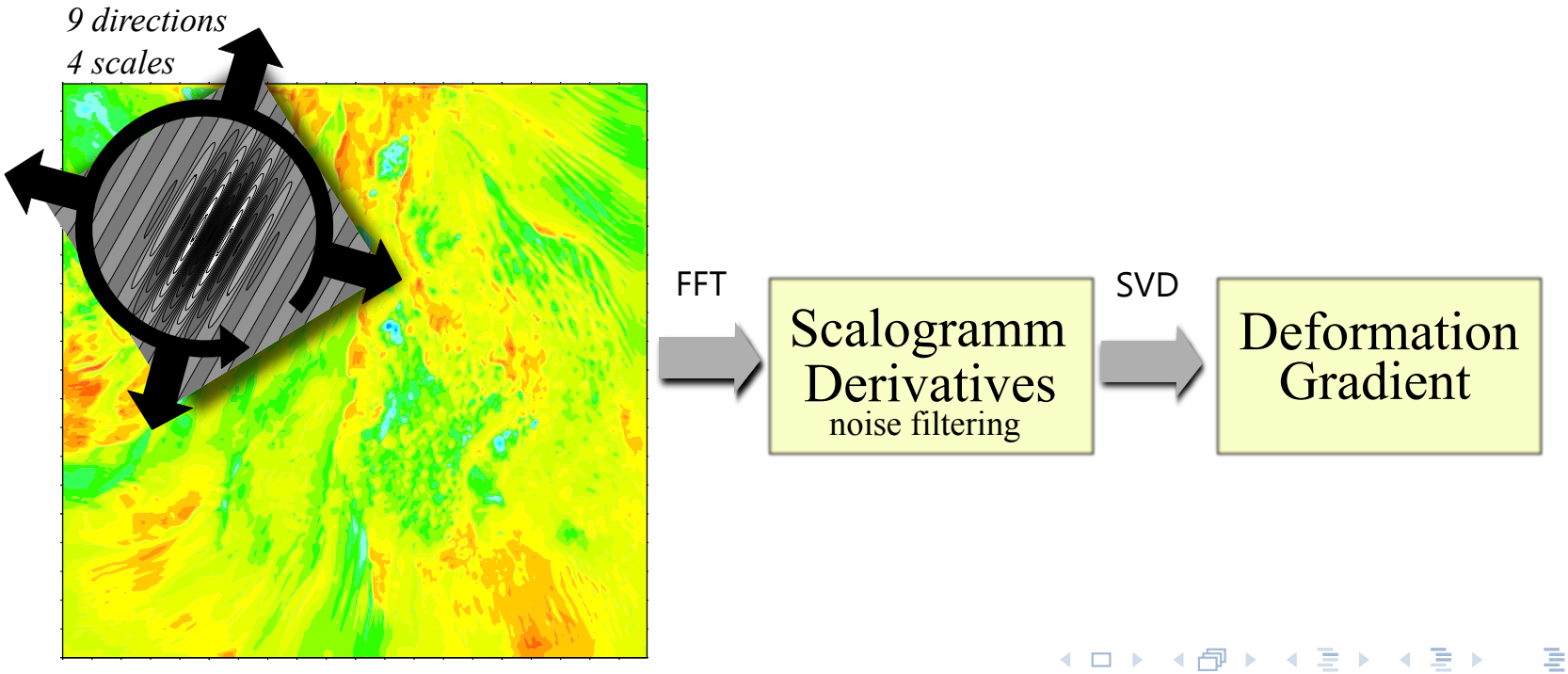
The bidimensional "Shape from Texture" problem in computer vision

Problem of measuring the deformation gradient from randoms fields has been tackled by [Clerc and Mallat 2002] using a wavelet-based approach. The deformation gradient is defined as:

$$G^{1,2} \equiv J_d(x)^{-1} \partial_{1,2} J_d(x), \text{ where } J_d(x) = \begin{pmatrix} \frac{\partial d_1(x)}{\partial x_1} & \frac{\partial d_1(x)}{\partial x_2} \\ \frac{\partial d_2(x)}{\partial x_1} & \frac{\partial d_2(x)}{\partial x_2} \end{pmatrix}$$

It is possible to estimate this deformation gradient from the 6-dimensionnal derivative of the scalogram $w = \mathbb{E}\{|| < DF, \psi_{\mathbf{u},S} > ||^2\}$, using a set of deforming matrices $S_1, \ldots, S_P$ matching different scales and orientations.



Integrating the deformation gradient: numerical solving

Once we have an estimate of the deformation gradient, we need to recover the deformation. A new result is that that the deformation can be recovered by solving a linear system of elliptic PDE:

$$-\Delta u_1 + [G^1_{11} + G^2_{12}, G^1_{21} + G^2_{22}] \cdot \nabla u_1 = -G^1_{11} - G^2_{12}$$
$$-\Delta u_2 + [G^1_{11} + G^2_{12}, G^1_{21} + G^2_{22}] \cdot \nabla u_2 = -G^1_{21} - G^2_{22}$$

where $u = d - I$ is the deviation of the deformation from identity mapping.

Discretisation of $-\Delta u + b \cdot \nabla u = f$ with finite differences yields:

$$-\frac{u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j}}{h^2} + b^{(1)}_{i,j} \frac{u_{i+1,j} - u_{i-1,j}}{2h} + b^{(2)}_{i,j} \frac{u_{i,j+1} - u_{i,j-1}}{2h} = f_{i,j}$$

This can be solved as a sparse linear system $Ax = y$ with sparse LU-factorization (UMFPACK). A depends on the chosen boundary conditions.

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symmetric Laplacian term

unsymmetric gradient term

This can be solved as a sparse linear system $Ax = y$ with sparse LU-factorization (UMFPACK). A depends on the chosen boundary conditions.

Applying the "Shape from Texture" approach to 2D correlation model

Dataset is a 30 member EnKF with WRF-model at 3 km resolution described by [Dowell and Stensrud 2008] and used in [Michel et al 2011] to construct a covariance matrix for hydrometeors.

We will work with error samples as pairwise differences from 6h forecasts from ensemble members, for specific humidity variable at the surface.

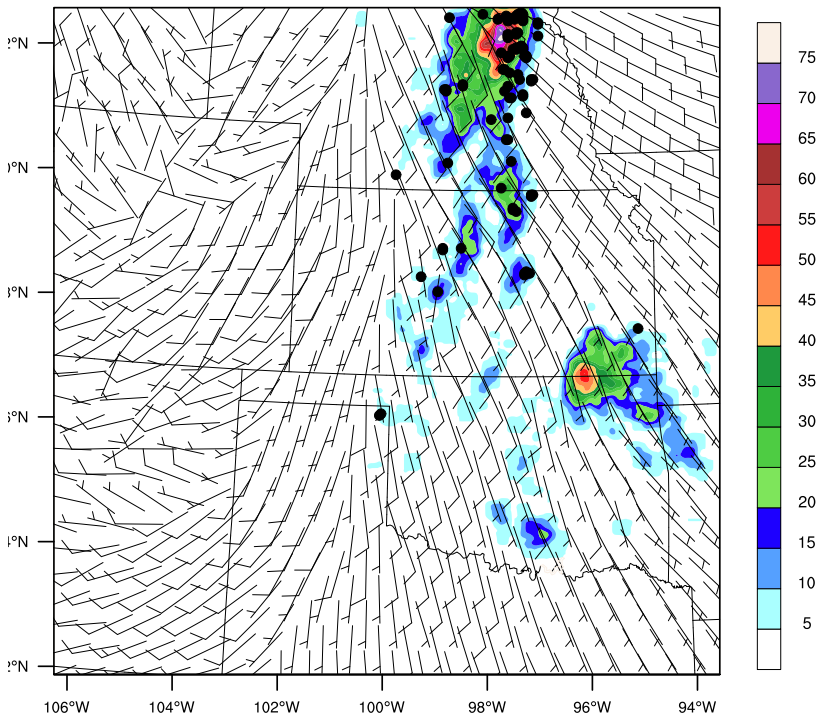
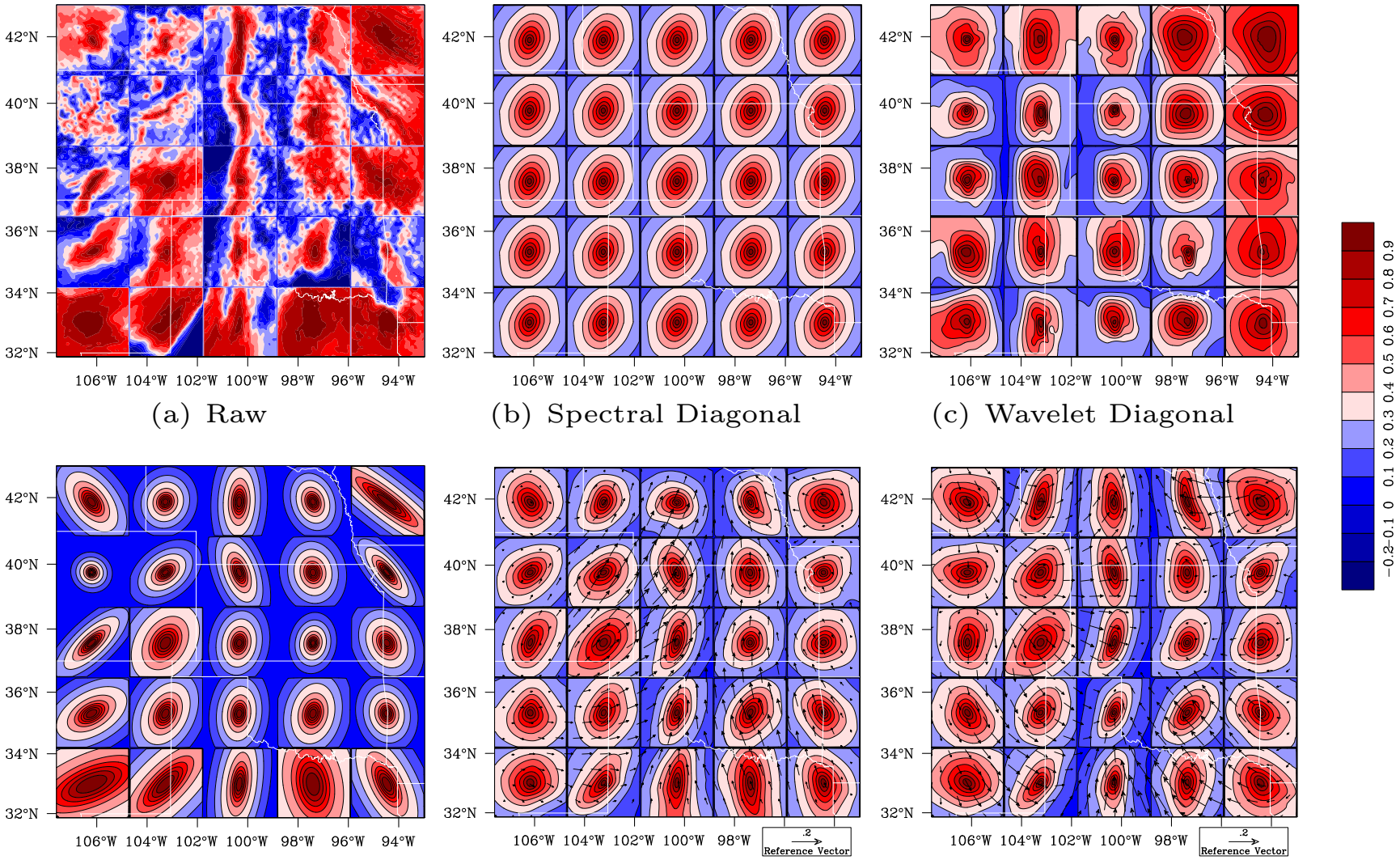


Figure: Mean wind, supercells, probability (%) of high reflectivity

Raw and **B**-implied correlations



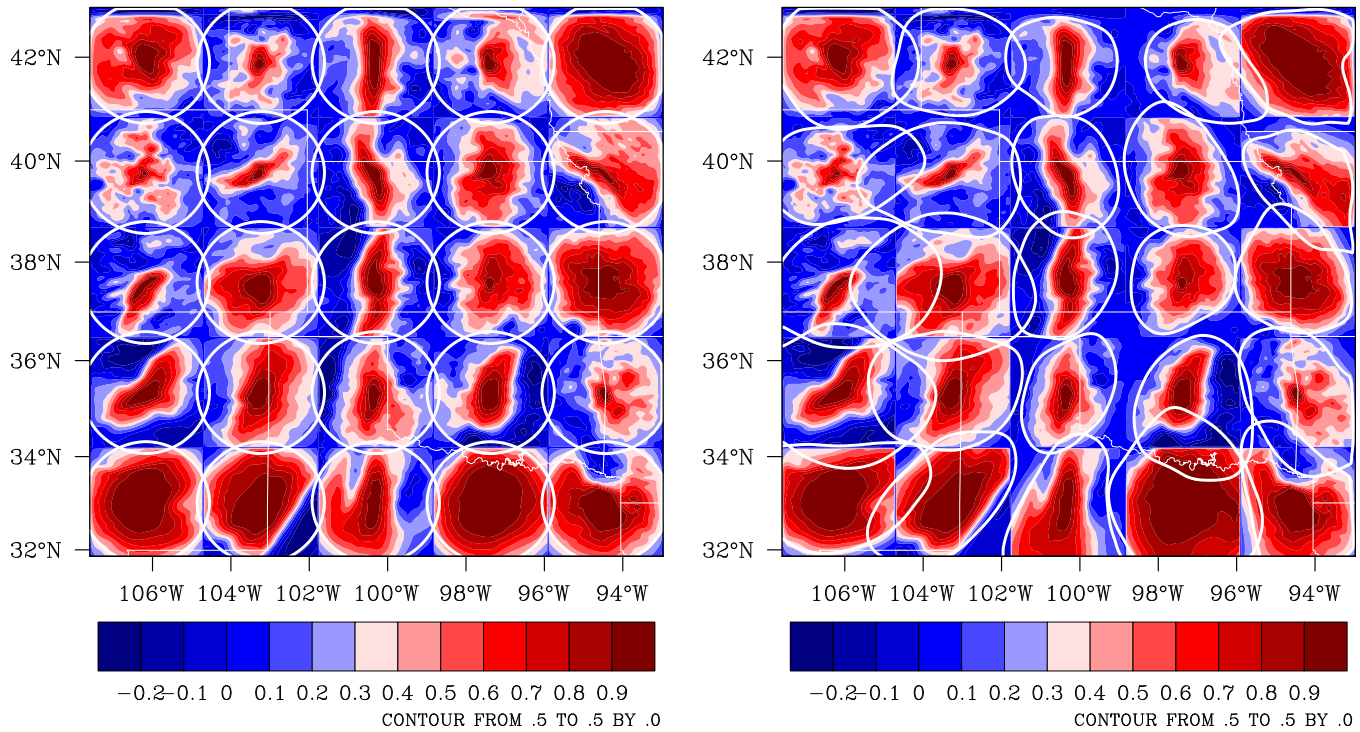
(a) Raw (b) Spectral Diagonal (c) Wavelet Diagonal

(d) RF-like, BPB Tensor (e) Deformed Dirichlet b.c. (f) Deformed Periodic b.c.

Figure: Spatial Correlations for specific humidity errors at the surface

Flow-dependent Schur-filtering in α-Hybrid DA

In a hybrid framework, ensemble covariances are improved by a Schur product with a correlation function with compact support. Some of the benefit of EnKF is lost in the use of isotropic functions - as the ones from [Gaspari and Cohn 1999].
Using deformed correlation functions for the Schur product may prove useful in retaining more covariance in the directions of the structure functions.



(a) With Standard Schur-filtering (b) With Deformed Schur filtering

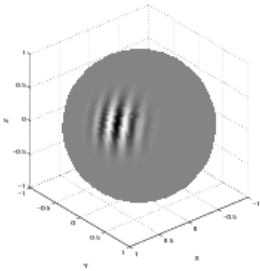
Conclusions

A new correlation model is being developped following the "shape from texture" approach by M. Clerc, in which structure functions are locally distorted by a deformation.

- The 2D deformation gradient may be numerically inverted with a finite difference approach;
- The algorithm is well suited to large scale data assimilation: low computationnal cost, square root form for the control variable transform, quasi-inverses available, accurate variances, no grid artifacts
- The method looks promising at extracting "flow-dependence" from an ensemble, and is able to represent some anisotropic structures

Perspectives

- Comparison with "poor's man" deformations such as wind or semi-geotrophic transform (R. Legrand)
- Approach in spherical geometry (post-doc financed by RTRA) as Gabor wavelets exist on the sphere [Antoine et al 2002]
- Treatment of boundaries for application in Oceanographic Data Assimilation
- Three-dimensional aspects



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