Attribution of trends in daily rainfall statistics to the global mean temperature

Global statistics mean $\mu$

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Precipitation

Two aspects: if it rains ($f_w$), and if, how much it rains ($\mu$).

Mean $\bar{x}$ mixes dry days with wet days, and different conditions and processes.

Long time scales and large spatial scales
Wet-day mean $\mu$
($X \geq x_0$)

Mean $\bar{x}$
($f_w \times \mu$)

Wet-day frequency $f_w$
$f_w \in [0, 1]$
The calculations...

- 7109 rain gauge records - singular vector decomposition (SVD) of $X = \mu / \mu_{\text{ref}}$
  - $X = U \Lambda V^T$.

- Generalised linear regression (GLM)
  - $\tilde{v}_i = \beta_{0,i} + \beta_{t,i} t + \beta_{T,i} T' + \beta_{N,i} N' + \xi$.

- *De-trended* temperature ($T'$) and NINO3.4 ($N'$)

- Reconstruct $X$ ($k$ is $t$, $T$, or $N$):
  - $\alpha_k = \sum_i u_i \lambda_i \beta_{i,k}$
Regression, p-values, binomial distribution, and combinations...

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Probability: 3 sign. (10%) results for $t$, 3 sign. (10%) results for $T'$, and both are the same combination of 3 of 11 is:

$$P = \binom{11}{3} \cdot (1.0-0.1)^8 \cdot 0.1^3 \cdot 1/\binom{11}{3} = 0.006 \quad [165 \text{ combinations of 3-out-of-11}]$$
Regression results for temperature

\[ \alpha_T \frac{dT}{dt} \text{ [% per °C]} \]

\[ \mu(x, t) = \alpha_0(x) + \alpha_t(x)t + \alpha_T(x)T'(t) + \alpha_N(x)N'(t) \]

Reconstructed from 11 PCA components
'Global' aggregate
Trends in wet-day mean and quantiles

Trends in $\mu$ and $q_{95}$

$\eta_t$ [% per decade]

$-\ln(1-p)\alpha_t$ [% per decade]

1945 – 1995 # pts= 7109 # pentads= 11 correlation= 0.637
Also spatially valid

Evaluation of q95-prediction

$r = 0.9$ (0.9 - 0.91)

$p$-value = 0 %

Benestad et al, (2012), 
Higher $\mu \rightarrow$ more convective type?

Several factors - local influences?

Large spatial scales (PCs1-3) and time scales > 5 years.
Implications for the future?
Are the connections for real?
Are they valid in the future?
Thank you for your attention!
Pentads + rain gauge records

Anomaly correlations

\[ \hat{\mu} = f(T) \text{ [%]} \]

7109 rain gauge records
Link between trends and $T'$

Trends in $\mu$ and link to $T'$

1945 – 1995 # pts= 7109 # pentads= 11 correlation= 0.707
Abstract

The question about trends in extreme precipitation and their causes has been elusive because of climate models' limited precision and the fact that extremes are both rare and occur at irregular intervals. Here a newly discovered empirical relationship between the wet-day mean and quantiles in 24-hr precipitation amounts was used to show that trends in the wet-day 95th percentiles $q_{95}$ - worldwide - have been influenced by the global mean temperature, consistent with an accelerated hydrological cycle caused by a global warming. It can be shown that most of the marginal distribution for 24-hr precipitation records, sampled from more than 30,000 rain gauges, can be specified from only four parameter: wet-day mean $\mu$, wet-day frequency $f_w$, distance from the coast $d$ and altitude $z$. Most of the temporal variations and trends in the upper quantiles, however, can be captured through $\mu$, which provides a more robust estimate of the upper tail of the distribution than small samples of extremes suffering from substantial statistical fluctuation. A multiple regression analysis was used as a basis for the attribution analysis by matching temporal variability in precipitation statistics with the global mean temperature and some of the dominant modes of internal variability. The regression was applied to both $\mu$ and $f_w$ respectively, using general linear models and a logit link for the latter.
There has been a number of reports suggesting that there is an increase in extreme precipitation amounts. It may be possible to attribute this with the changes in the global mean temperature based on statistical analysis.

Expect that the precipitation statistics respond to changes in the global mean temperature on long time scales and on large spatial scales.

24-hr precipitation from rain gauges, but the statistics is made for samples representing the large scales.
Precipitation

Two aspects: if it rains ($f_w$), and if, how much it rains ($\mu$).

Mean $\bar{x}$ mixes dry days with wet days, and different conditions and processes.

Long time scales and large spatial scales

Precipitation is a product of atmospheric conditions, which differ from dry and wet days and between type of precipitation events. Rather than looking at the traditional mean, I want to focus on the wet-day mean. The mean amount when it rains.
The geographical distribution of the wet-day mean shows some differences to the mean, which is the product between the wet-day mean and the wet-day frequency.
The calculations...

- 7109 rain gauge records - singular vector decomposition (SVD) of \( X = \mu \mu_{\text{ref}}^{-1} \)
  \[ X = U \Lambda V^T. \]
- Generalised linear regression (GLM)
  \[ \tilde{y}_i = \beta_{0,i} + \beta_{t,i} t + \beta_{T,i} T' + \beta_{N,i} N' + \xi. \]
- De-trended temperature (\( T' \)) and NINO3.4 (\( N' \))
- Reconstruct \( X \) (\( k \) is \( t \), \( T \), or \( N \)):
  \[ \alpha_k = \sum_i u_i \lambda_i \beta_{i,k}. \]

In order to compute quasi-global statistics for the precipitation, each rain gauge was subject to 5-year aggregation, and then these were divided by the climatology and represented as columns of matrix \( X \). Then a PCA was used to find the singular vectors, and a GLM was used to link these with trends and the de-trended global mean temperature. Spatial patterns were then reconstructed by letting the regression coefficients replace the principal components in the PCA.
Regression, p-values, binomial distribution, and combinations...

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[165 combinations of 3-out-of-11]

If we examine the ANOVA for each principal component, we see that the regression results were 10% significant only for the 3 leading PCs. We can estimate the probability for this to happen by chance, taking the binomial distribution to describe the probability of 3-of-11 cases being significant for both the trend and the temperature, and then the probability that these 3 cases match for trend and temperature.
The spatial pattern of the link to the global mean temperature. Noisy picture, and large site-to-site differences. Nevertheless, a tendency of more positive values.
We can use the regression model to predict the quasi-global mean precipitation if we use the original temperature and set the trend and the ENSO terms to zero.

The insert map shows the location of the sites used here.

Good reproduction of the long-term trend over the calibration period. The data availability outside this period was not so great (lower panel), and the divergence outside the calibration period may be due to different sampling.
We can also compare the trends derived from the wet-day mean and the trends in the 95-th percentile, given that the precipitation is close to being exponential.

There are some points that diverge – red symbols. The insert map shows their location. Bad data?
Also spatially valid

Higher $\mu \rightarrow$ more convective type?

Several factors - local influences?

Large spatial scales (PCs1-3) and time scales $> 5$ years.

What is the reason for higher $\mu$ and more intense extreme precipitation? One explanation could be a shift to more convective rainfall, where the precipitation is concentrated over shorter time and smaller area. Convection is affected by temperature.
If there is a dependency between the global mean temperature and the large-scale precipitation statistics, then we can use the projected global mean temperature to make a projection for the wet-day precipitation.

For a complete picture, it is necessary to know the wet-day frequency too.
Thank you for your attention!
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Anomaly correlations

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