Influence of atmospheric waves on the formation and the maintenance of the subtropical jet during the Northern Hemisphere winter

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Motivation
1. We found a system which can describe forcing-response problem using “zonal-mean primitive equation”.

\[ \chi(\text{obs.}) = \chi(A) + \chi(B) + \ldots \]

2. We apply this tool to understand the “subtropical jet problem”:

What drives the jet?
How the jet is maintained?
What causes the variability of the jet?
....
Zonal-mean primitive equations

\[
\frac{\partial \bar{u}}{\partial t} = 2\Omega \bar{v} \sin \phi + \bar{F}_e + \bar{F}_n + \bar{X},
\]

\[
2\Omega \bar{u} \sin \phi + \frac{1}{a} \frac{\partial \bar{\Phi}}{\partial \phi} = \bar{J},
\]

\[
\frac{\partial \bar{\Phi}}{\partial p} = -\frac{RT}{p},
\]

\[
\frac{\partial \bar{T}}{\partial t} = \Gamma \bar{\omega} + \bar{Q}_e + \bar{Q}_n + \bar{S},
\]

\[
\frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (\bar{v} \cos \phi) + \frac{\partial \bar{\omega}}{\partial p} = 0,
\]

Wave-forcings

\[
\bar{F}_e = -\frac{1}{a \cos^2 \phi} \frac{\partial}{\partial \phi} \left[ u'v' \cos^2 \phi \right] - \frac{\partial}{\partial p} \left[ u' \omega \right],
\]

\[
\bar{Q}_e = -\frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \left[ T'v' \cos \phi \right] - \frac{\partial}{\partial p} \left[ T' \omega \right] + \frac{\kappa}{p} T' \omega,
\]

Non-linear terms

\[
\bar{F}_n \equiv -\frac{\bar{v}}{a} \frac{\partial \bar{u}}{\partial \phi} - \frac{\partial \bar{u}}{\partial p} + \frac{\bar{u} \bar{v}}{a} \tan \phi,
\]

\[
\bar{Q}_n \equiv -\frac{\bar{v}}{a} \frac{\partial \bar{T}}{\partial \phi} + \left( -\frac{\partial \bar{T}}{\partial p} + \frac{\kappa \bar{T}}{p} - \Gamma \right) \bar{\omega}.
\]

Stability

\[
\Gamma(p) = -\frac{\partial T_0}{\partial p} + \frac{\kappa T_0}{p}
\]
From primitive equations

\[ \frac{1}{\cos \phi} \frac{\partial}{\partial \phi} \left( \frac{\cos \phi}{\sin^2 \phi} \frac{\partial \bar{\omega}}{\partial \phi} \right) + \frac{4\Omega^2 a^2 p}{R\Gamma} \frac{\partial^2 \bar{\omega}}{\partial p^2} \]

\[ = \frac{2\Omega a p}{R\Gamma \cos \phi} \frac{\partial}{\partial \phi} \left[ \frac{\cos \phi}{\sin \phi} \frac{\partial}{\partial p} \left( \bar{F_e} + \bar{F_n} + \bar{X} - \frac{\dot{J}}{2\Omega \sin \phi} \right) \right] \]

\[ - \frac{1}{\Gamma \cos \phi} \frac{\partial}{\partial \phi} \left[ \frac{\cos \phi}{\sin^2 \phi} \frac{\partial}{\partial \phi} (\bar{Q_e} + \bar{Q_n} + \bar{S}) \right]. \]

\[ \frac{D\bar{\Phi}}{Dt} = 0 \quad \bar{\omega} = 0 \quad \bar{v} = 0 \text{ at } \phi = \pm \pi / 2 \]

\[ \bar{\omega} = A_1 (\bar{F_e} + \bar{F_n} + \bar{X}) + A_2 \dot{J} + A_3 (\bar{Q_e} + \bar{Q_n} + \bar{S}) \]
Meridional circulation or acceleration of zonal wind can be **linearly decomposed** into those produced by each forcing including NL terms!

\[
\mathbf{L} \ddot{\omega} = A_1 (\overline{F_e} + \overline{F_n} + \overline{X}) + A_2 \dot{J} + A_3 (\overline{Q_e} + \overline{Q_n} + \overline{S})
\]

\[
\ddot{v} = M \ddot{\omega}
\]

\[
\overline{F_n} = B \ddot{\omega}, \quad \overline{Q_n} = C \ddot{\omega}.
\]

\[
B = \left( \frac{u}{a} \tan \phi - \frac{1}{a} \frac{\partial u}{\partial \phi} \right) \mathbf{M} - \frac{\partial u}{\partial p} \mathbf{I},
\]

\[
C = -\frac{1}{a} \frac{\partial T}{\partial \phi} \mathbf{M} + \left( -\frac{\partial T}{\partial p} + \frac{\kappa T}{p} - \Gamma \right) \mathbf{I},
\]

\[
\ddot{\omega} = \left( \mathbf{L} - A_1 B - A_3 C \right)^{-1} \left[ A_1 (\overline{F_e} + \overline{X}) + A_2 \dot{J} + A_3 (\overline{Q_e} + \overline{S}) \right]
\]

\[
\frac{\partial u}{\partial t} = \left\{ (2\Omega \sin \phi \mathbf{M} + B) \left( \mathbf{L} - A_1 B - A_3 C \right)^{-1} A_1 + \mathbf{I} \right\} (\overline{F_e} + \overline{X})
\]

\[
+ \left\{ (2\Omega \sin \phi \mathbf{M} + B) \left( \mathbf{L} - A_1 B - A_3 C \right)^{-1} \right\} \left[ A_2 \dot{J} + A_3 (\overline{Q_e} + \overline{S}) \right].
\]
Data: ERA-Interim 1989/90～2008/9 (20-winter)
Resolution of diagnosis model
  Vertical 1001 (1hPa-thickness)
  Horizontal 121 (equal spacing in sine-latitude)

Output: daily

Wave: Departure from zonal-mean field

Stationary Waves 31-day running ave. comp.
LFT waves 10-day ＜T＜ Stationary
Synoptic Waves 2-day ＜T＜ 6-day
Medium-scale Waves 1.75-day ＜T (Excl. Tidal waves)
January Climate (Accel., Meridonal Circ., Wave Amp.)

CI: du/dt(Upper): 0.5 m/s/day  
 du/dt(Lower): 0.2 m/s/day

Chi(Upper): 1x10^{10} kg/s  
 Z (Lower): 20 m

Sh: Heat: 1.2/0.6/0.3 K/day  
 FRICT: 3/1/0.5 m/s/day

Arrow : EP-flux
Zonal wind and acceleration on the Jet (1991/2)

Blue = Acceleration
Red = Acceleration by eddies

Correlation of DJF for 20-winter (1800 days) is 0.86

Variation is mostly produced by the interference between synoptic and LFT+stationary waves
Month-to-month regression against the zonal wind at the jet core (using DJF for 20-winter)

CI: U: 1m/s
du/dt: 0.1 m/s/day
Heating: 0.1 K/day
Friction: 0.5 m/s/day
Arrow: EP-flux
Sh: Sig. 95%
(Student’s t test)
Acceleration on the jet core (Lagged regression)

Forcing lead Forcing lag

Total acceleration

Synoptic waves

Stationary waves

LFT waves

Friction

Diabatic Heating

m/s/day

$\frac{dU}{dt}$ (30N, 200hPa)
Conclusion

1. Based on zonal-mean primitive equation on the sphere, a new method of linear decomposition that can isolate forcing and its response (including NL effects) for zonal wind acceleration etc. is developed.

2. The method is applied to the problems of the formation and variabilities of the subtropical jet in the NH winter.

3. Climatological balance of the subtropical jet for the equatorward flank is established between the acceleration produced by diabatic heating by tropical heating and subtropical cooling and deceleration by stationary waves propagating from mid-latitude, whereas poleward flank is between acceleration by stationary waves and deceleration by frictional forcing.

4. For the day-to-day variabilities of the jet, most of them are produced by activity of waves. If it is examined more in detail, it is mostly produced by the interference of synoptic waves and waves with frequencies lower than 10 days.

5. For the month-to-month variability of the jet, it is causally created as follows:
   (1) Synoptic waves in mid-latitude tend to propagate equatorward with development and they tend to accelerate the jet. (2) With development of the jet, diabatic heating associated with these waves and surface friction against the jet tend to create the meridional circulation whose Coriolis acceleration decelerate the jet. By these deceleration, the jet is peaked. (3) Although stationary waves always accelerate the jet, its acceleration is almost proportional to the jet speed, and almost no contribution on the causality of the jet.
Thank you!

Reference: Kuroda (2016) JGR-Atmosphere
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