Surface-layer turbulence and Monin-Obukhov similarity theory revisited

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Air-land/water interaction

Vertical turbulent fluxes linking the atmosphere with land/water surface are calculated on the basis of

- Monin-Obukhov Similarity Theory MOST (1954)
  Flux-profile relationships in the surface layer

- Roughness lengths for momentum and scalars
  Drag and heat/mass transfer at the very surface

However, MOST fails in both very stable and unstable stratification
Virtual parameters and hypothetical concepts

Flux-profile relations expressing turbulent fluxes of momentum $\tau = u_*^2 = -K_M \frac{\partial U}{\partial z}$, temperature $F_\theta = -K_H \frac{\partial \Theta}{\partial z}$, etc. through wind-speed $U$, potential temperature $\Theta$, etc. at a given height $z$ are based on turbulent viscosity $K_M$, conductivity $K_H$, diffusivity $K_D$

$$K_{M,H,D} = (u_* z) f_{M,H,D}(z/L)$$

where $L = u_*^3 / (-g / T_0) F_\theta$ is Obukhov length-scale, and $f_{M,H,D}$ are universal functions whose shapes are prescribed by MOST

E.g. MOST-based flux-profile relation: $U(z) = \frac{\tau^{1/2}}{k} \left( \ln \frac{z}{z_0} + C_u \frac{z}{L} \right)$

allows determining $\tau$, from measured $U$; known roughness length for momentum $z_0$; and $F_\theta$ determined from its own relation.

Historically, much efforts focused on MOST, while roughness lengths of natural surfaces remained poorly understood
MOST in stable stratification

Ignores self-control of temperature flux, $F_{\theta}$, and prescribes similar turbulent viscosity and conductivity: $Pr_T = K_M / K_H = \text{constant}$, which entails erroneous turbulence cut off at $Ri > Ri_c = 0.25$

Turbulent Prandtl number vs. Richardson number

black line shows modern theory (EDB, Z et al., 2007-2017)

MOST shown by red line prescribes collapse of turbulence at $Ri > 0.25$

Data **Atmospheric:** Kondo et al., 1978, Bertin et al., 1997; **Laboratory:** Rehmann, Koseff, 2004, Ohya, 2001, Strang, Fernando, 2001; **DNS:** Stretch et al., 2001; **LES:** Esau, 2009
Self-control of turbulence  
via counter-gradient heat flux missed in MOST

\[ F_\theta = C_1 t_T \beta \left\langle \theta^2 \right\rangle - C_2 t_T E_z \frac{\partial \Theta}{\partial z} \]

The key feedback:

→ Larger temperature gradient \( \partial \Theta / \partial z \) causes
→ larger negative heat flux \( F_\theta \)
→ hence larger turbul. potential energy \( E_p = \frac{g \left\langle \theta^2 \right\rangle}{2T_0 \left( \partial \Theta / \partial z \right)} = -C_3 t_T \frac{g}{T_0} F_\theta \)
→ hence, stronger potential temperature fluctuations \( \left\langle \theta^2 \right\rangle \) and
→ stronger counter-gradient (positive) heat flux \( C_0 \beta \left\langle \theta^2 \right\rangle \)
→ which compensates for firming up the negative heat flux \( F_z \) and prevents collapse of turbulence in super-critical startification
The temperature flux vs. temperature gradient

Data and theory show non-gradient heat transfer at $Ri > 0.25$

Almost neutral ($0 < z/L < 0.5$) MOST OK

Moderate ($0.5 < z/L < 10$) MOST acceptable

Very stable ($z/L > 10$) MOS fails

$$\frac{g F_\theta}{T_0 E_K S}$$

wind shear

$$S = \frac{\partial U}{\partial z}$$

B/V frequency

$$N = \left( \frac{g}{T_0} \frac{\partial \Theta}{\partial z} \right)^{1/2}$$

Ri number

$$Ri = \left( \frac{N}{S} \right)^2$$

$$F_\theta \sim N^2, ~ K_H \sim E^{1/2}_K z$$

$$F_\theta \sim N, ~ K_H \sim E_K / N$$

$$F_\theta \text{ independent of } N, ~ K_H \sim E_K S / N^2$$

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“Long-lived” stably-stratified planetary boundary layer (LL PBL) in Bergen visualized by water haze (winter 2012, courtesy T. Wolf). Strong static stability above LL PBLs make them very shallow and affects the surface layer via additional length-scale, $L_N = \tau^{1/2}/N$, where $N$ is the buoyancy-frequency in the free atmosphere. MOST is inapplicable.
Conventionally Neutral (CN) PBL

= PBL with zero heat flux at the surface, affected by stable stratification in the free atmosphere. Figures show CN PBL evolving against stable stratification, which causes the downward heat flux at the upper boundary, affecting the entire PBL.

Traditional theories/models overwarm PBL and overestimate its height; MOST yields wrong mean profiles in surface layer.
Conventionally neutral PBL: velocity gradient

Initially homogeneous PBL evolves against stable stratification in the free atmosphere causing negative (downward) heat flux at the upper boundary. The surface heat flux is kept equal to zero.

MOST: wrong mean profiles, overestimated friction velocity…
MOST in unstable stratification

- Neglects principal difference between the shear-generated *mechanical* turbulence and buoyancy-generated *anarchy* turbulence characterised by *inverse energy transfer from smaller to larger plumes towards self-organised structures*.

- Wrongly formulates *horizontal* velocity fluctuations, strongly overestimates horizontal fluxes and diffusion.

- Considers large-scale self-organised convective structures as large turbulent eddies, which causes *underestimation of the surface fluxes*, especially strong in calm-weather convection.

Now these failures are improved but *eclectically*: keeping MOST untouched and without understanding of real physical processes.
Self-organisation in turbulent convection

**Cells in viscous convection** (Benard 1900, Rayleigh 1916)
**Cells / rolls** in **turbulent convection** atmosphere/LES/DNS/lab

No analogy to rolls in viscous convection
Rolls are excited by turbulent large-scale instability triggered by non-gradient horizontal heart flux (Elperin et al, 2002, 2005)

Self-organisation is missed in the conventional turbulence paradigm and in old theories such as
- Convective heat/mass transfer law
- MOST
Self-organised structures in the atmosphere

Williams & Hacker (1992) airborne measurements: Arrows show self-organised velocity field; solid lines, deviations of potential temperature $\theta$ from its averaged value $<\theta>$. Lines $\theta - <\theta> = 0$ mark the updraught.

The near-surface convective winds (up to a few m/s) generate mechanical turbulence, strongly enhancing the heat transfer. MOST overlooks this mechanism.
Strengthening heat/mass transfer in free convection: non-classical mechanism

Large-scale self-organised structures

Convective winds towards the plume base

Near-surface wind shears generate mechanical turbulence (*overlooked in traditional theories*)

**Strongly enhanced heat/mass transfers**
Heat-transfer in calm-weather convection

Failure of classical heat-transfer law:

\[ F_\theta = 0.14 \left( \frac{g \kappa^2}{T_0 \nu} \right)^{1/3} \Delta \Theta^{4/3} \]

Blue points: observations over natural surfaces from calm sea to urban landscape

Red points: large-eddy simulation


is much stronger than in classical theories and depends on CBL height, \( h \), and wind-roughness length, \( z_0u \)

New law, \( F_\theta = f(h / z_{0u}) \left( \frac{gh}{T_0} \right)^{1/2} \Delta \Theta^{3/2} \) involves factor \( h^{1/2} \) and function \( f(h/z_{0u}) \) shown in the Figure but missed in the classical heat-transfer law,
What is wrong in MOST?

MOST is based on **TURBULENCE PARADIGM** rooted in Kolmogorov (1941, 1942), who considered only the shear-generated *mechanical* turbulence in neutrally stratified flows. His followers plainly extended his vision to stratified flows:

- Turbulent flow = superposition of organised mean flow and chaotic turbulence → **NOT TRUE** in unstable stratification, which causes additional type of motion: self-organised convective structures

- Turbulence = multi-scale chaos with the forward cascade of kinetic energy towards smaller scales – aiming at viscous dissipation → **NOT TRUE** in unstable stratification, which causes the buoyancy-generated *anarchy* turbulence with inverse energy transfer towards self-organised structures, co-existing with *mechanical* turbulence

- Turbulent fluxes = mean gradients multiplied by turbulent viscosity, conductivity or diffusivity (analogy with molecular-diffusion) → good approximation only for neural and weakly stratified flows; **NOT TRUE** for horizontal fluxes and also for vertical fluxes in strong convection and strongly stable stratification
Revised paradigm of stratified turbulence:

*chaos out of order* (Kolmogorov)

co-exists with order out of chaos* (Prigogine)