Numerical Effects on Wave Propagation in Atmospheric Models

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Space weather motivation

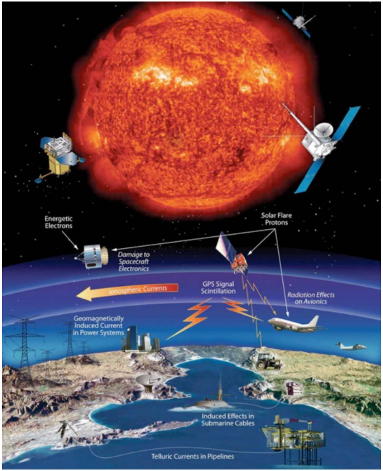

Figure 1 : Illustration of the potential impacts of space weather to technology in space and at high altitudes [The Met Office].

Ray tracing equations

The coupled system of equations we solve for $\boldsymbol{x} = (x, z)$ and $\boldsymbol{k} = (k, m)$ is

$$\frac{D_{\boldsymbol{c}_g}}{Dt}(\boldsymbol{x}) = \boldsymbol{c}_g(\boldsymbol{k}, T) \quad (1)$$
$$\frac{D_{\boldsymbol{c}_g}}{Dt}(\boldsymbol{k}) = -\boldsymbol{\nabla}\omega, \quad (2)$$

where $\boldsymbol{c}_g = \boldsymbol{\nabla}_{\boldsymbol{k}}\omega$ is the group velocity, ω is the frequency and

$$\frac{D_{\boldsymbol{c}_g}}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{c}_g \cdot \boldsymbol{\nabla}$$

represents the rate of change of a variable with time at a position moving with the group velocity $\boldsymbol{c}_g$.

Analytical wave propagation for different angles

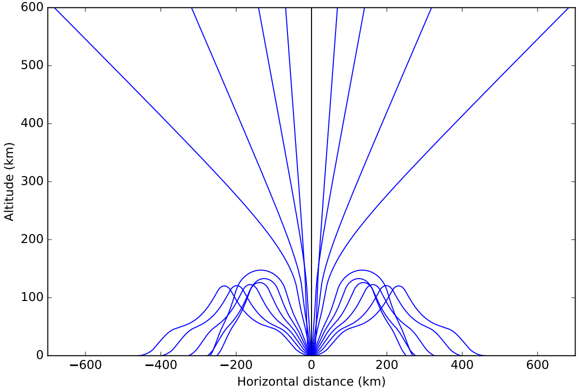

Figure 2 : Ray plot of acoustic waves with a total wavenumber of 1×10^{-4} initiated with a range of wavevector angles from 0 to π, using the analytical wave equation with the USSA temperature profile.

Effect of space discretisation on wave propagation

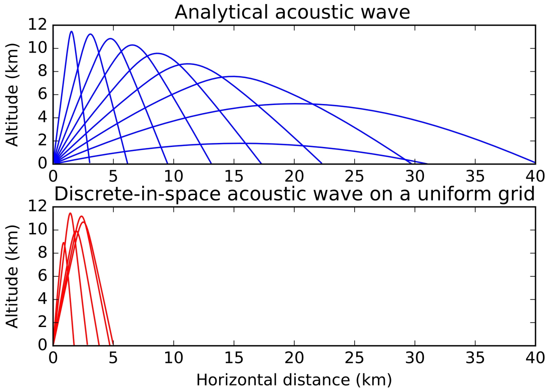

Figure 3 : A comparison of ray plots of acoustic waves with a total wavenumber of 4×10^{-5} initiated with a range of wavevector angles measured from the horizontal, from 0 to $\pi/2$ (above: the analytical wave equation, and below: the spatially-discrete wave equation with $\Delta x = 100\,\text{km}$, $\Delta z = 1\,\text{km}$).

The discrete-in-space group velocity

For waves to be resolved in the numerical grid, we require

$$0 < k < \pi/\Delta x \tag{3}$$

where k is the horizontal wavenumber and Δx is the horizontal resolution. *This limits the angles of wavevectors that can be used.*

The numerical version of the horizontal group velocity: $\partial\omega/\partial\hat{k}$, where $\hat{k}$ is the numerical horizontal wavenumber, is

$$\frac{\partial\omega}{\partial\hat{k}} = \frac{\partial\omega}{\partial k}\cos\left(\frac{\hat{k}\Delta x}{2}\right). \tag{4}$$

Note: $\cos(\hat{k}\Delta x/2) \sim 0$ at the limits of k in equation (3).
If $\partial\omega/\partial\hat{k} \approx 0$, then *the wave propagates straight upwards, even for large horizontal wavenumbers k.*

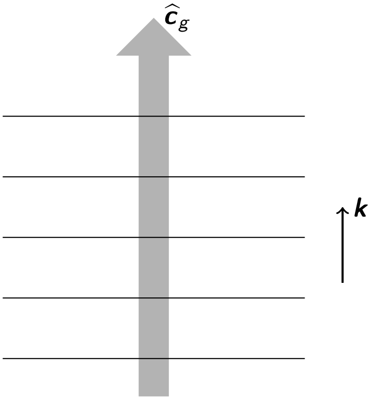

Figure 4 : $k = 0$, then the wavevector and group velocity go in the same direction.

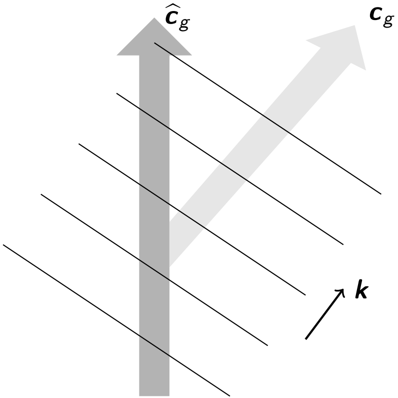

Figure 5 : $k = \pi/\Delta x$, so the wavevector angle changes. The analytical group velocity matches the wavevector direction, but the numerical group velocity still points up.

The discrete-in-space group velocity

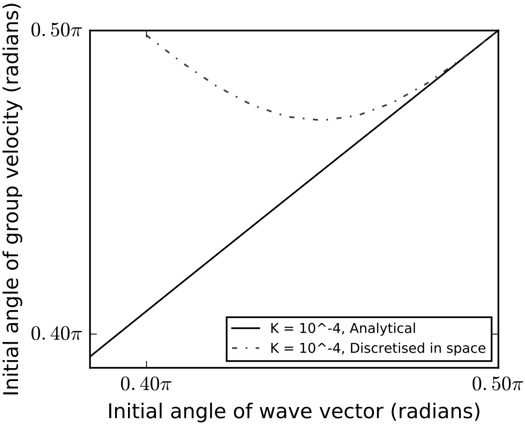

Figure 6 : Initial group velocity angles up to $\pi/2$ for a range of initial wavevector angles up to $\pi/2$ for acoustic waves with a total wavenumber of 1×10^{-4}.

Effect of time discretisation on wave propagation

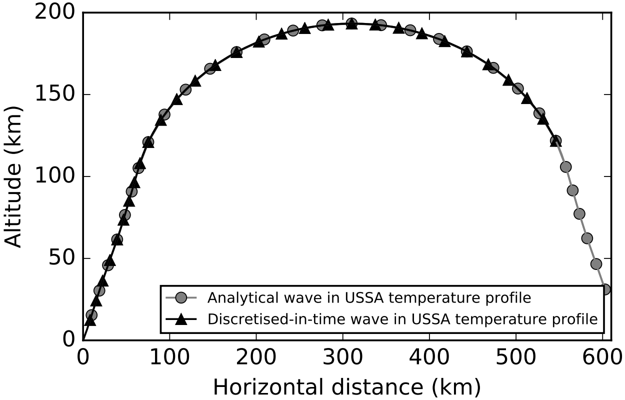

Figure 7 : Ray plot of acoustic waves with a total wavenumber of 1×10^{-3} initiated with a wavevector angle of 0.3π, run for 30 minutes, using either the analytical wave equation or time-discrete wave equation with a 3 second timestep.

Wave amplitude growth

The background density varies like

$$\rho_0 \propto \exp\left(-\frac{z}{H}\right). \tag{5}$$

Relative density perturbations ρ'/ρ_0 vary like:

$$\frac{\rho'}{\rho_0} = W\rho_0^{-1/2}, \tag{6}$$

where W is the wave energy.

W is described by the wave energy conservation law, which is given by

$$\frac{D_{\boldsymbol{c}_g}}{Dt}(\log W) + \boldsymbol{\nabla}\cdot\boldsymbol{c}_g + \frac{2}{\tau} = 0, \tag{7}$$

where τ represents timescales for decaying processes (either molecular viscosity or numerical damping).

Wave amplitude growth

Wave energy conservation law:

$$\frac{D_{\boldsymbol{c}_g}}{Dt}(\log W) + \boldsymbol{\nabla} \cdot \boldsymbol{c}_g + \frac{2}{\tau} = 0, \tag{8}$$

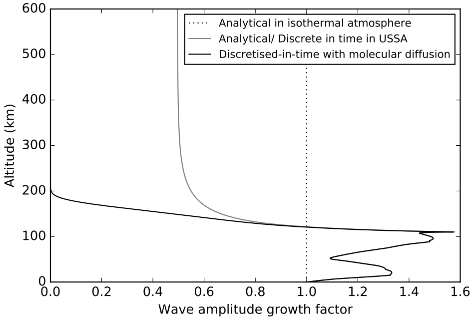

Figure 8 : The wave amplitude growth factor for a vertically propagating acoustic wave with a total wavenumber of 1×10^{-4} under different conditions. The discretised-in-time wave equation used 5 minute timesteps.

Conclusions

In summary:

- We have used ray tracing techniques to gain insight into how resolution affects wave propagation in numerical models compared with the analytical equation.
- We learnt that molecular viscosity and diffusion has a significant damping effect in the thermosphere.
- Further experiments have confirmed that including molecular viscosity and diffusion in atmospheric models can prevent excessive wave growth at high altitudes, and improves their stability.

D. J. Griffin and J. Thuburn (2017)
Numerical Effects on Wave Propagation in Atmospheric Models
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