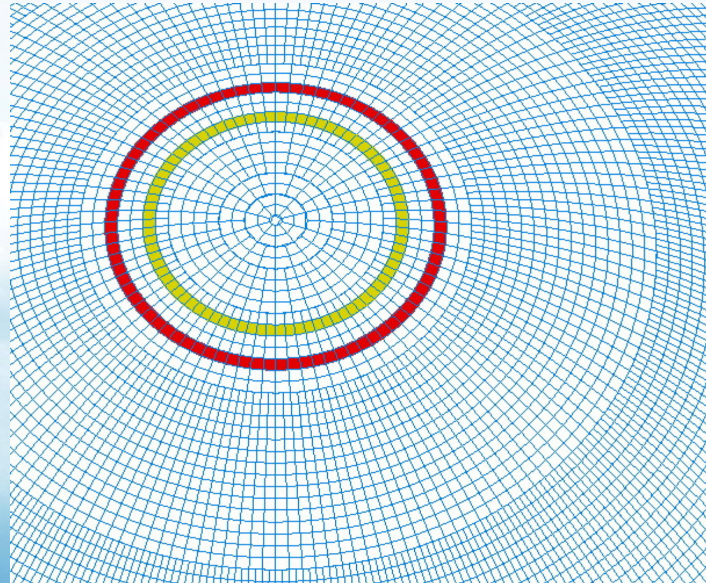
An efficient multi-resolution grid for global models and coupled systems

– Spherical Multiple-Cell (SMC) grid



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5 September 2018

Three points to be explained

1. Why a Spherical Multiple-Cell (SMC) grid is introduced.
2. What advantages the SMC grid has over other grids.
3. Where the SMC grid is used so far and possibly in the future.

Advection equation on spherical grid

- The advection equation on spherical grid with standard longitude λ and latitude φ (-π/2, π/2) is given by

$$\frac{\partial \psi}{\partial t} + \nabla \bullet (\mathbf{v}\psi) = \frac{\partial \psi}{\partial t} + \frac{\partial(\psi u)}{r\cos\varphi\partial\lambda} + \frac{\partial}{r\cos\varphi\partial\varphi}(\upsilon\psi\cos\varphi) = 0$$

Using the standard dx (along longitude, east positive) and dy (along meridian, northward positive) geophysical notation, it becomes

$$\frac{\partial(\psi\cos\varphi)}{\partial t} + \frac{\partial(u\psi\cos\varphi)}{\partial x} + \frac{\partial(\upsilon\psi\cos\varphi)}{\partial y} = 0$$

It is equivalent to a Cartesian grid advection equation except for the singularity at the Poles.

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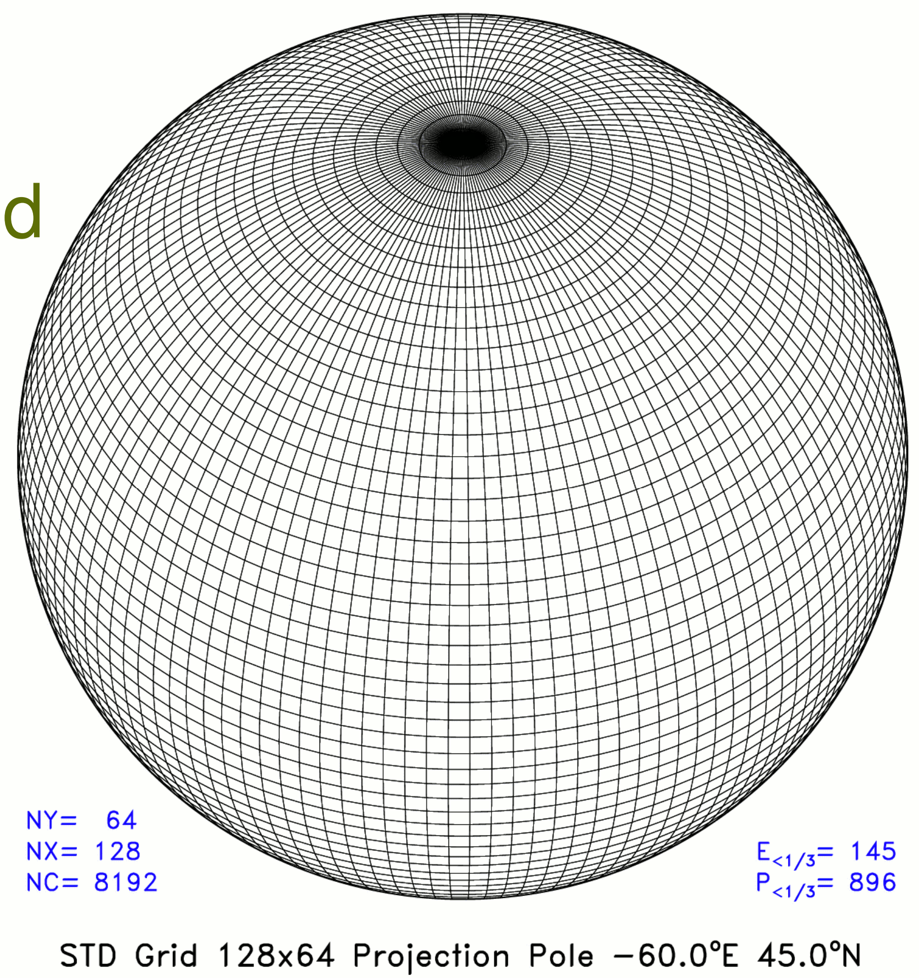
Polar problems in lat-lon grid

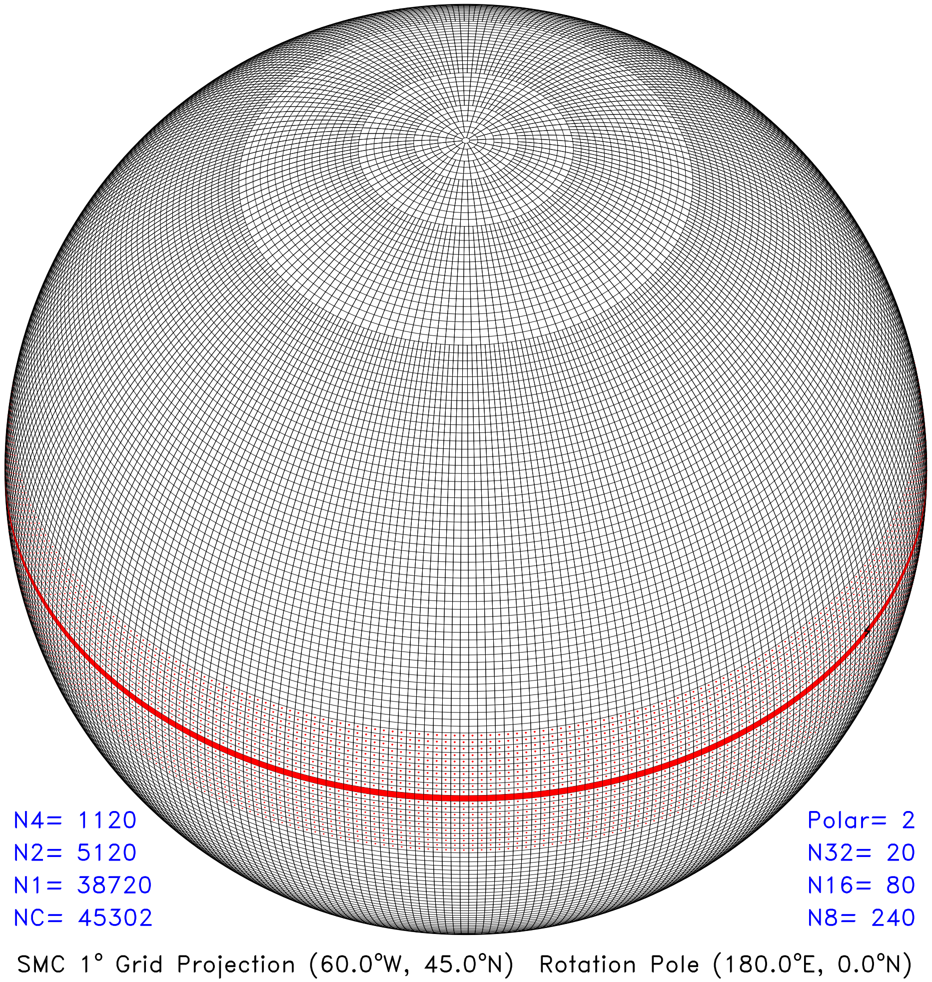
1. Severe CFL restriction on Eulerian advection time step at high latitudes.
2. The Pole is a singular point and flow has to go around it, not crossing it.
3. Scalar assumption of vector components becomes invalid near the Poles.



Spherical Multiple-Cell grid

- Merged cells at high latitudes to relax CFL limit on time step, like a reduced grid.
- Introduce round polar cells with integral equation to avoid polar blocking and singularity.
- Use fixed reference direction for vector components in polar regions.

- Reference: Li, J.G. 2011: *Mon. Wea. Rev.*, **139**, 1536-1555.



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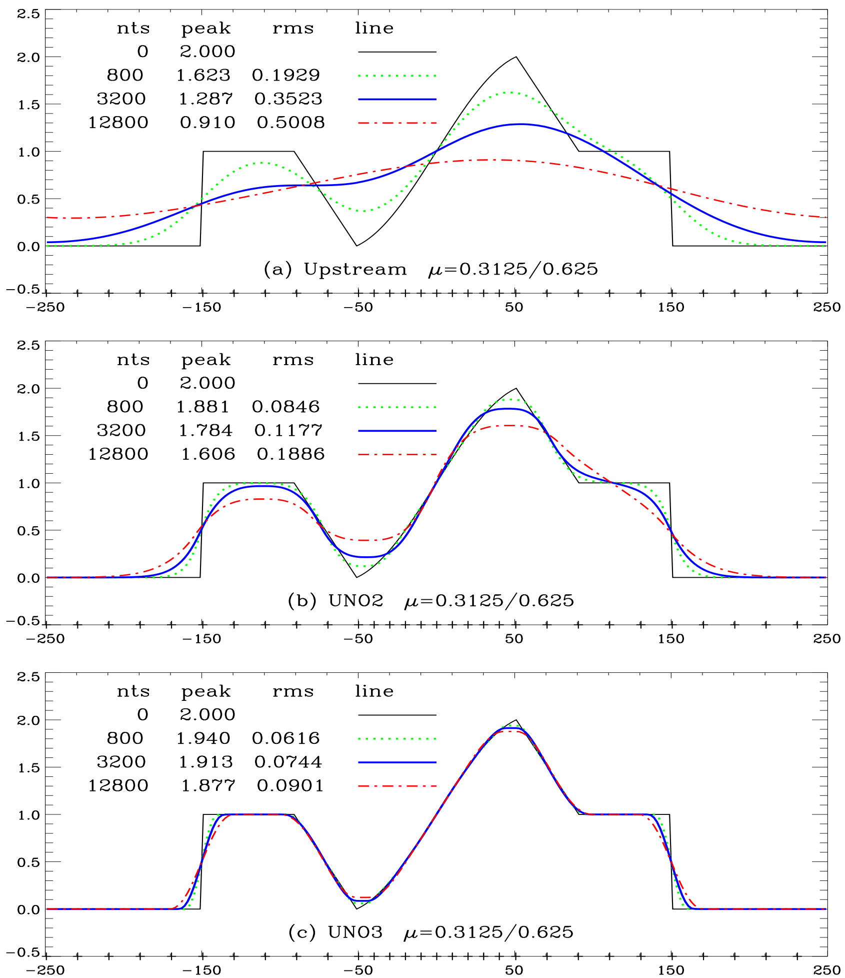
Upstream Non-Oscillatory advection schemes

- Choice of 1st, 2nd and 3rd order UNO advection schemes are available on SMC grid.
- Recommend the 2nd order UNO2 scheme for atmospheric models, fast and accurate enough.

Reference: Li, J.G. 2008: *Mon. Wea. Rev.*, **136**, 4709-4729.



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Upstream Non-Oscillatory 2nd Order (UNO2) Advection Scheme

$$\psi_j^{n+1} = \psi_j^n + \left(u_{j-1/2}\psi_{j-1/2}^{MF} - u_{j+1/2}\psi_{j+1/2}^{MF}\right)\Delta t / \Delta x_j$$

$$\psi_{j+1/2}^{MF} = \psi_C^n + \left(x_{MF} - x_C\right) G_C$$

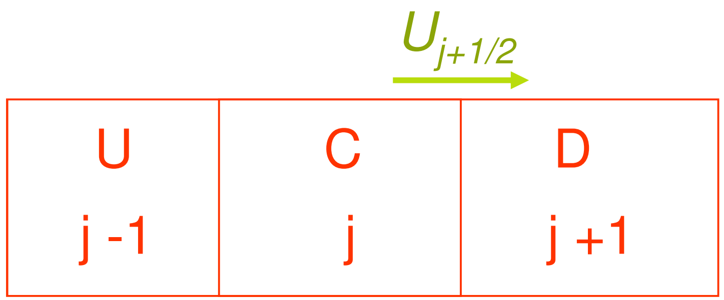
$$x_{MF} - x_C = 0.5\, sign\left(u_{j+1/2}\right)\left(\Delta x_C - \left|u_{j+1/2}\right|\Delta t\right)$$

$$G_C = Sign\left(G_{DC}\right)\min\left(\left|G_{DC}\right|, \left|G_{CU}\right|\right) \qquad G_{AB} \equiv \left(\psi_A - \psi_B\right) / \left(x_A - x_B\right)$$

Details see:

Li, J.G. (2008) *Mon. Wea. Rev.*, **136**, 4709-4729.

Upstream, Central and Downstream cells relative to velocity *u*.



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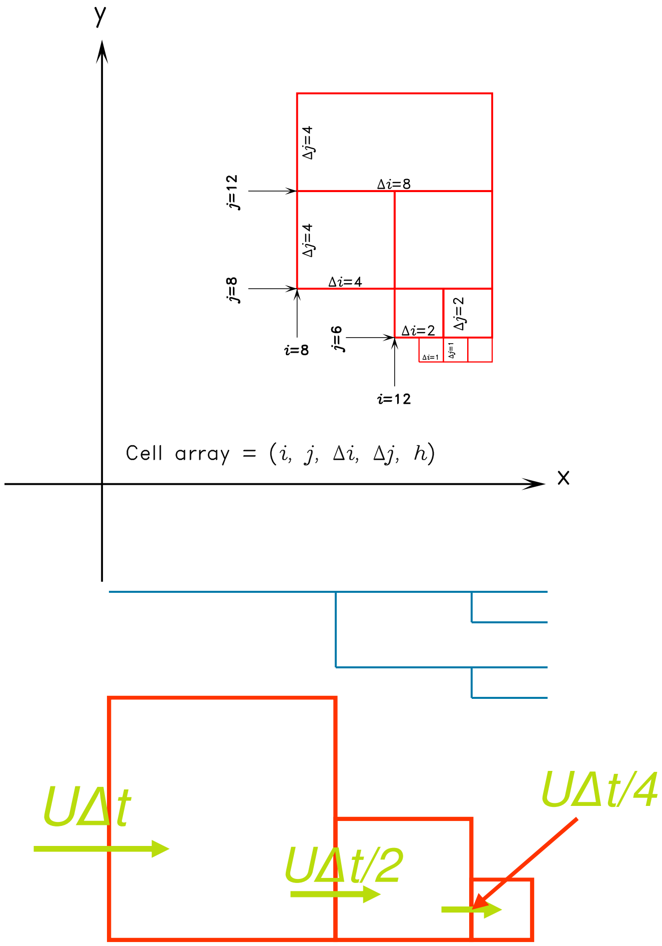
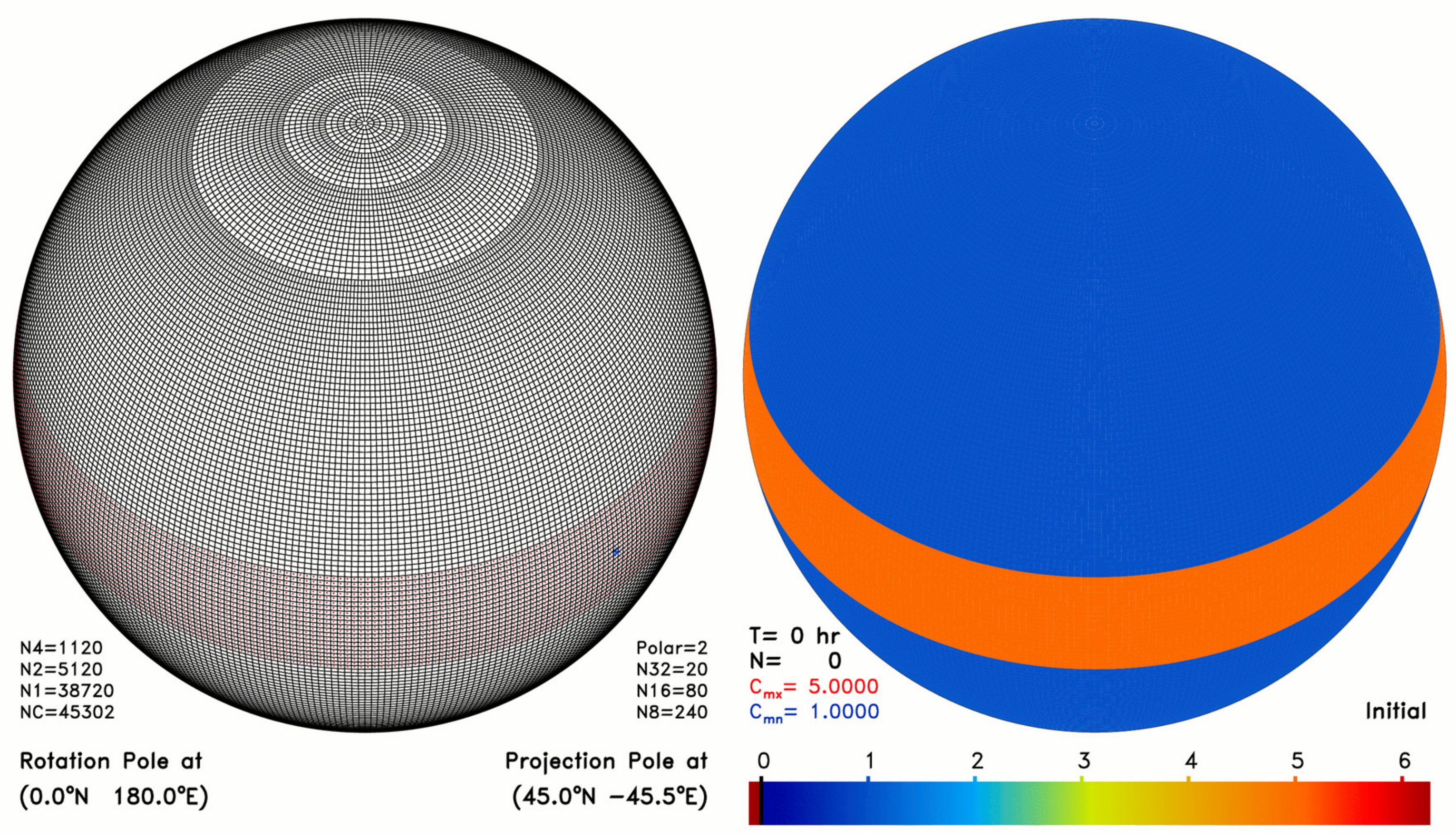
Unstructured grid with rectangular cells and pointer-oriented loops

- Cells are defined by location and size indexes and multi-resolution by refinement.
- Transport fluxes are calculated with face-array or pointer-oriented loops.
- Time-steps are automatically adjusted for different-sized cells for efficiency.
- One-dimensional array loop convenient for parallelization.

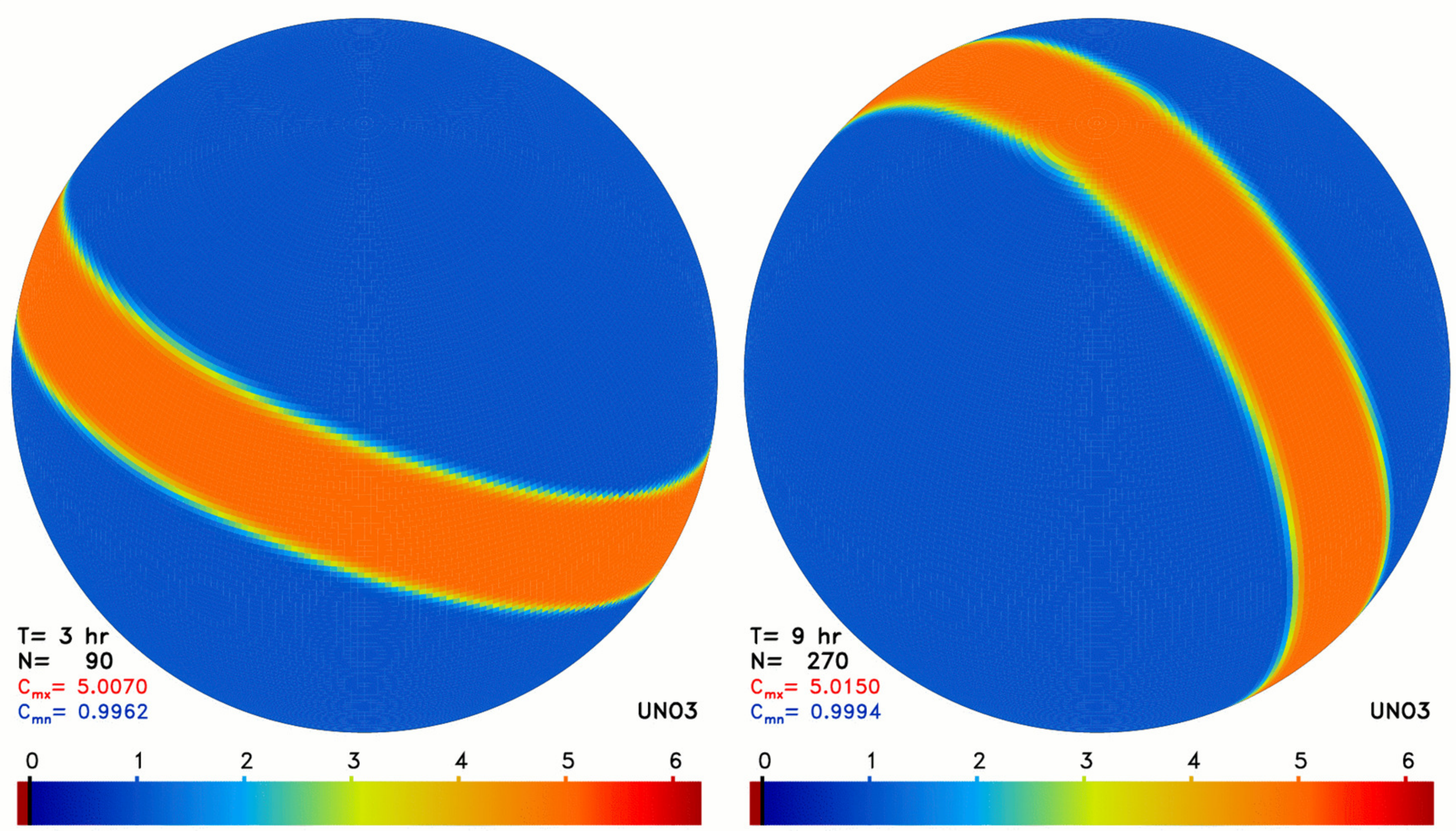
Reference: Li, J.G. 2012: *J. Comput. Phys.* **231**, 8262-8277.



Solid rotation on SMC grid with UNO3



Solid rotation on SMC grid with UNO3



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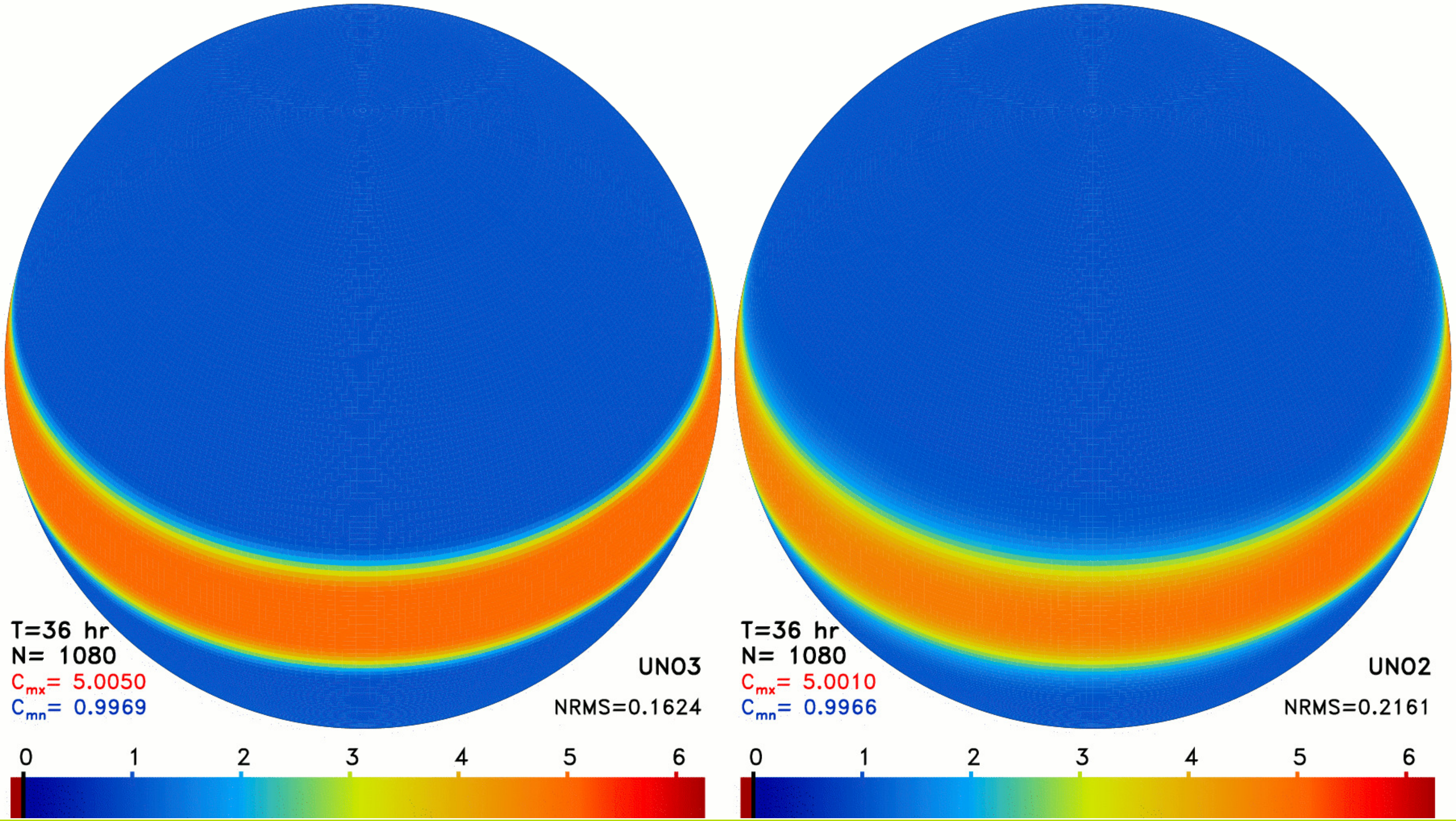
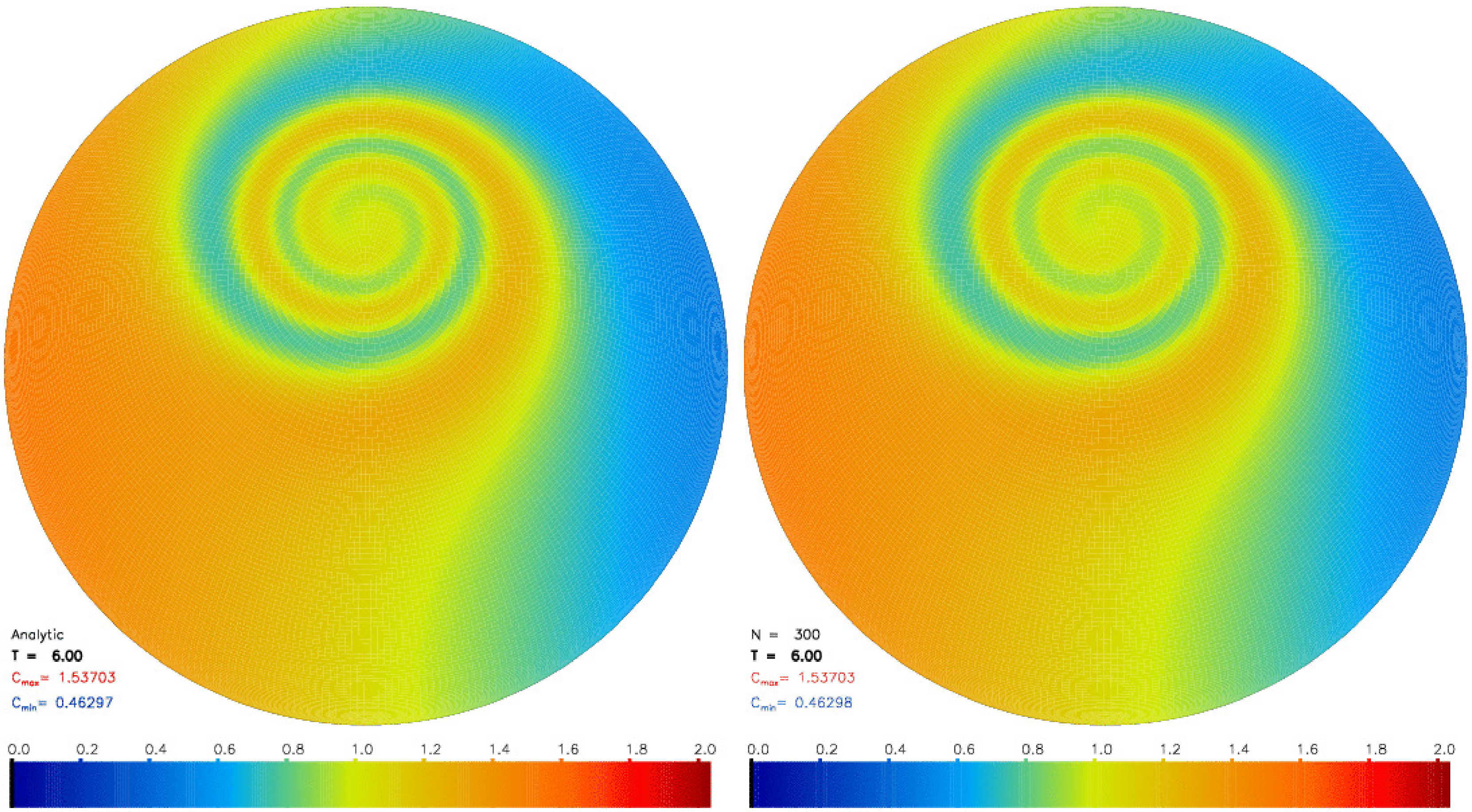
SSR on SMC: Comparing UNO3 and UNO2



Deformation flow on SMC grid with UNO3



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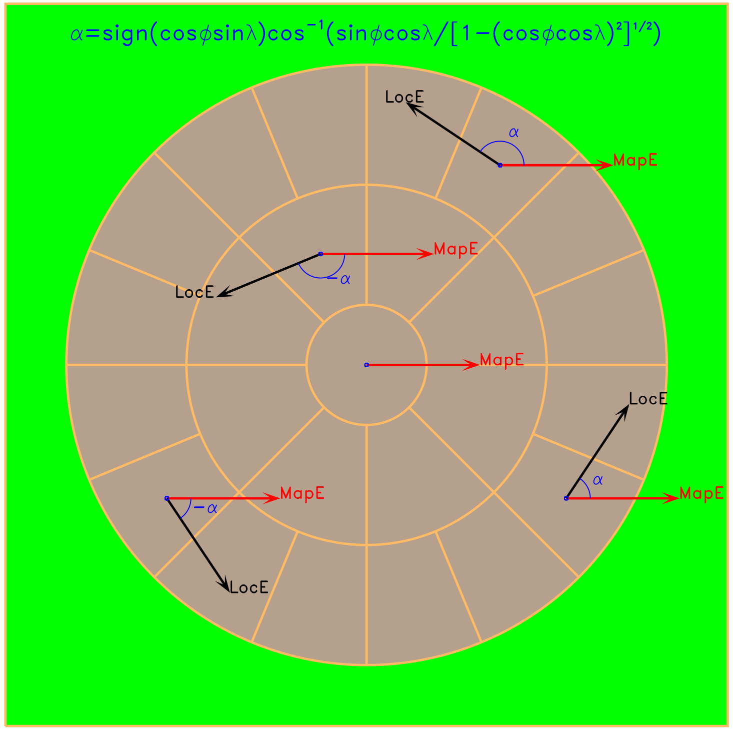
Map-east reference direction – Vector polar problem

- SMC grid uses merged cells at high latitudes to relax CFL limit on time step like a reduced grid.
- Local east changes rapidly from cell to cell in polar regions, rendering scalar assumption of vector component invalid.
- Define vector components with fixed reference direction --- the map-east, instead of the rapidly changing local east in polar regions.
- Reference: Li, J.G. 2016: *Ocean Dynamics*, **66**, 989-1004.



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Shallow water equations on a SMC grid

t the time,
r radius of the earth,
φ latitude,
λ longitude,
κ diffusion speed,
h water column height,
h_+ upstream water height,
$\mathbf{v}$ horizontal velocity,
$\mathbf{i},\mathbf{j},\mathbf{k}$ unit vectors,
b bottom topography,
g gravity constant,
$f=2\omega\sin\varphi$ Coriolis parameter,
ω earth angular speed,
γ damping frequency.

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{v}) = \nabla \cdot \left(\kappa h_+ \nabla (h+b)\right), \quad \nabla \equiv \mathbf{i}\frac{\partial}{r\cos\varphi\partial\lambda} + \mathbf{j}\frac{\partial}{r\partial\varphi}$$

$$\frac{\partial \mathbf{v}}{\partial t} + \eta\,\mathbf{k}\times\mathbf{v} + g\nabla(h+b+K) + \gamma\mathbf{v} = 0$$

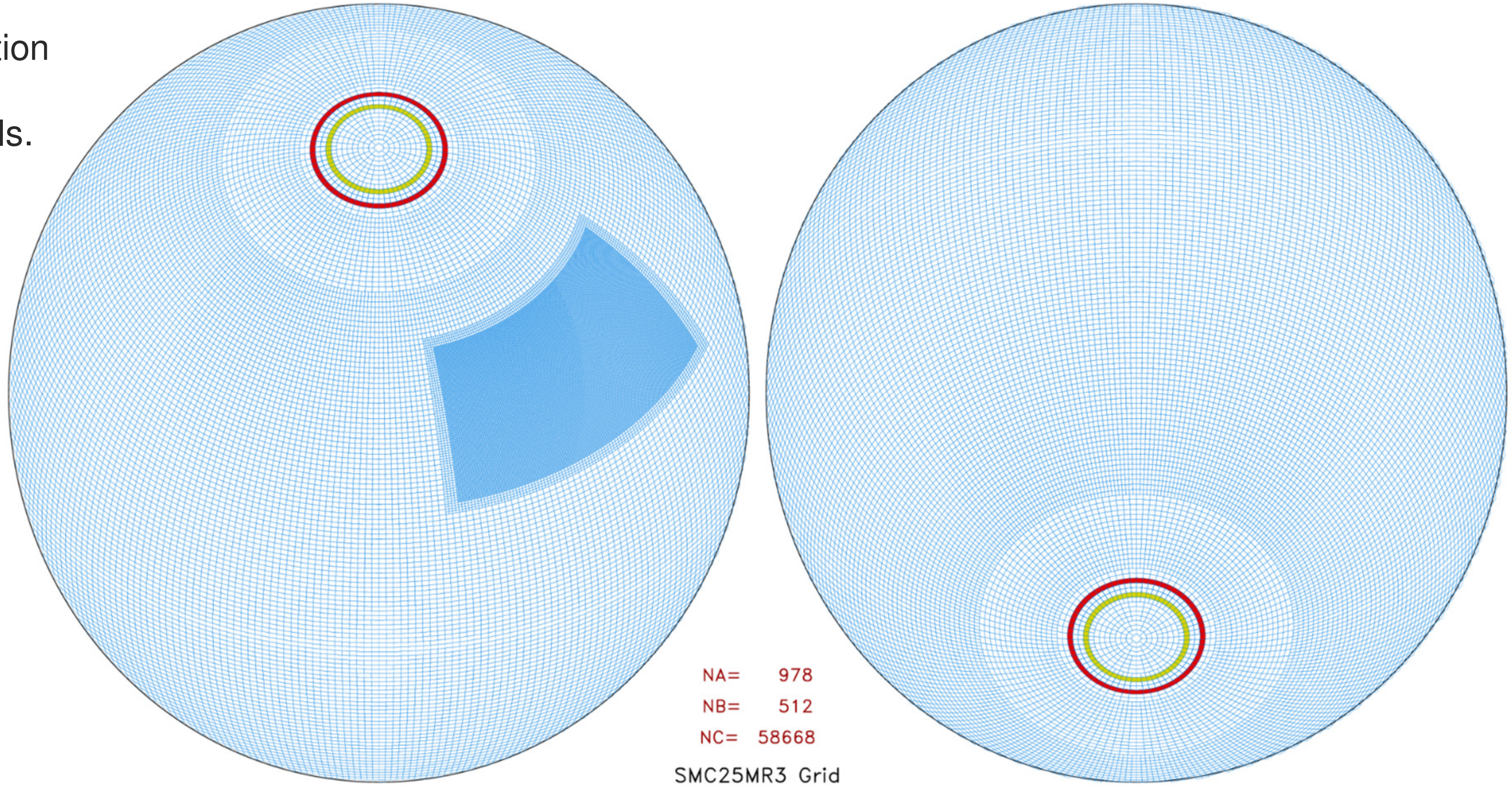
where $K = |\mathbf{v}|^2/2$ is the kinetic energy and η is the absolute vorticity, defined by $\eta = \mathbf{k}\cdot\nabla\times\mathbf{v} + f$

Li, J.G. 2018: *Quarterly J. Royal Meteor. Soc.* **144**, *1-12.*

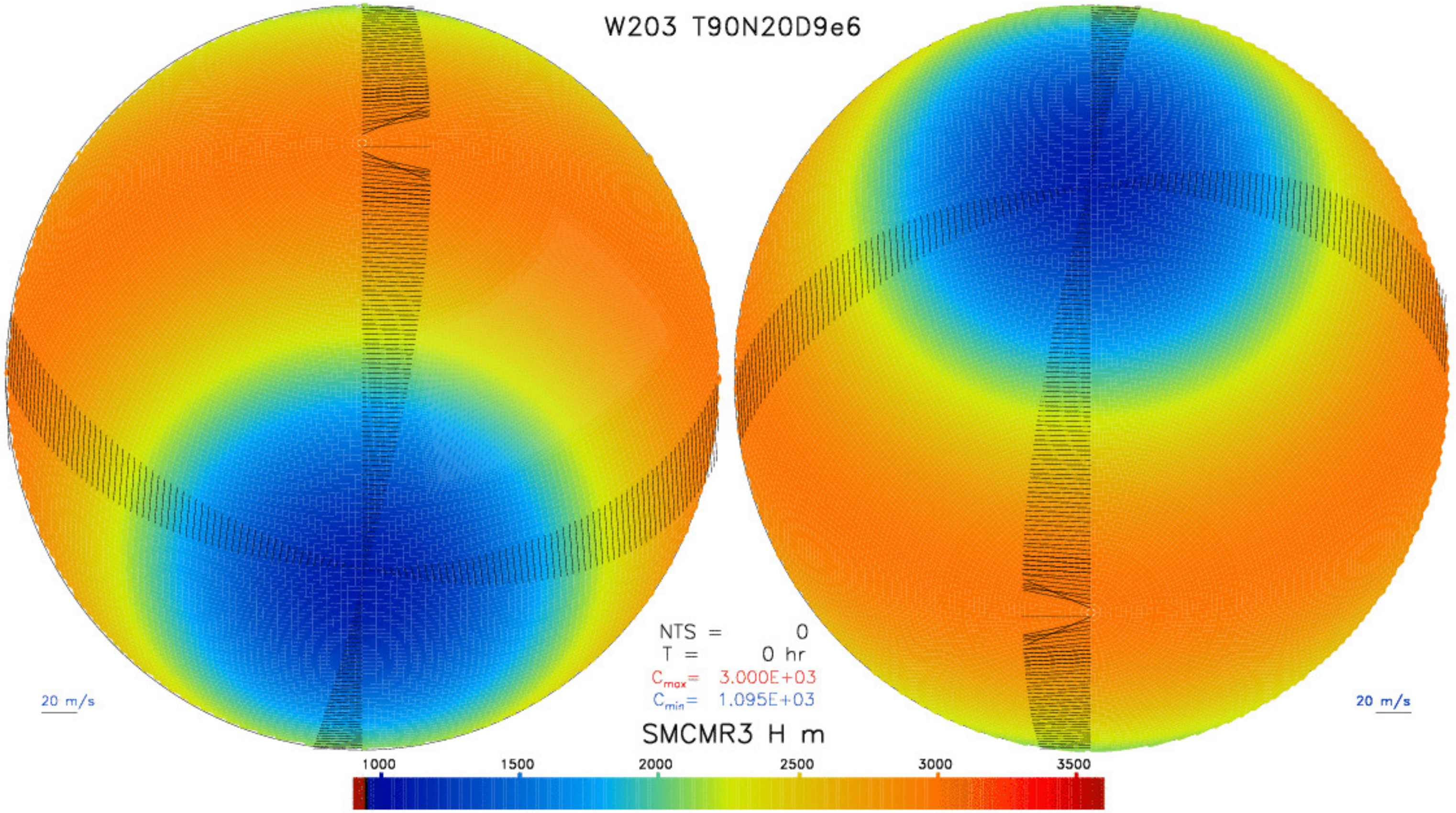
SMC1° grid with refined area up to ¼ °

Link up 2-direction systems with overlapping cells.

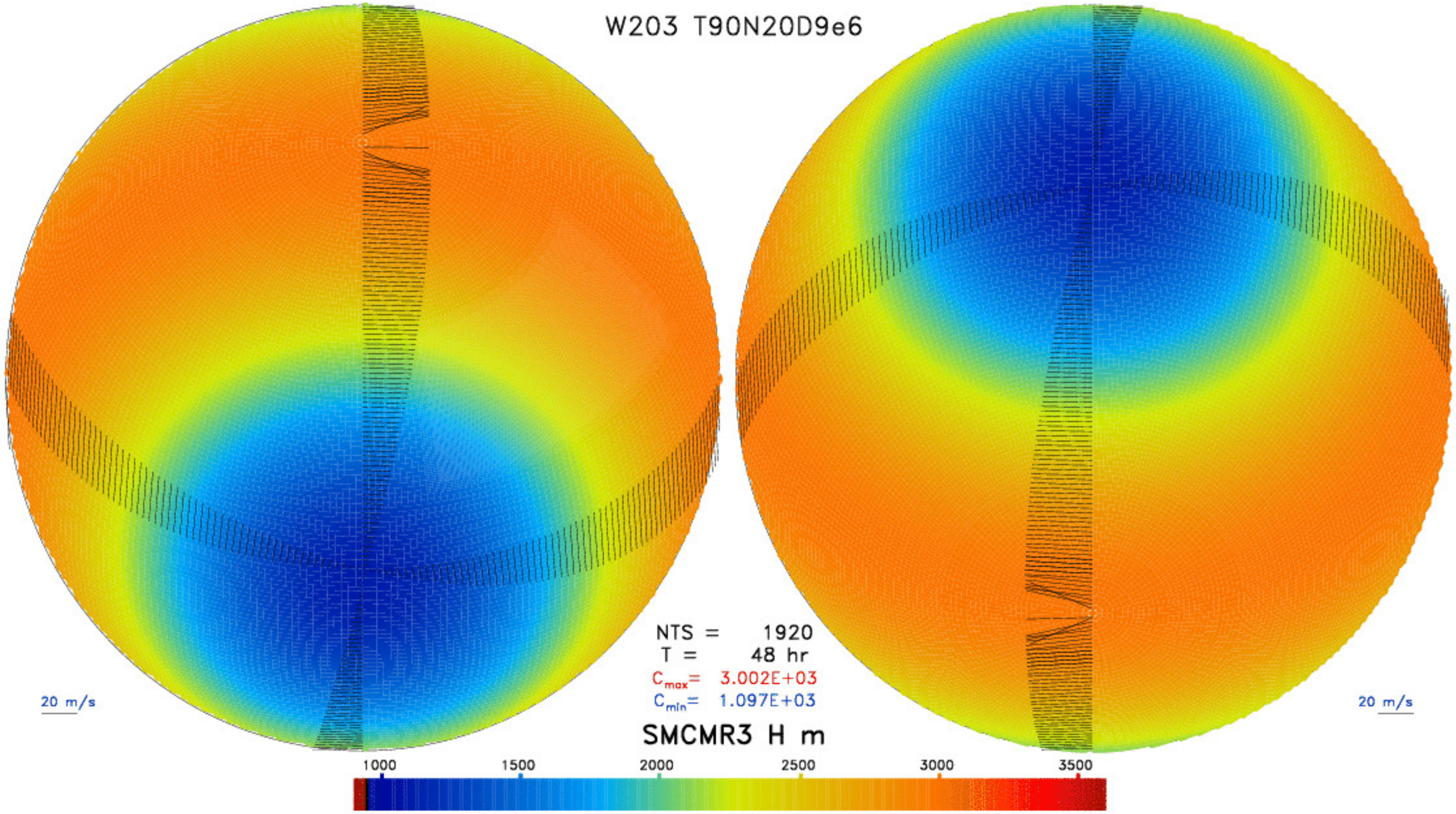
Multi-resolution as mesh refinement as SMC grid uses unstructured techniques.



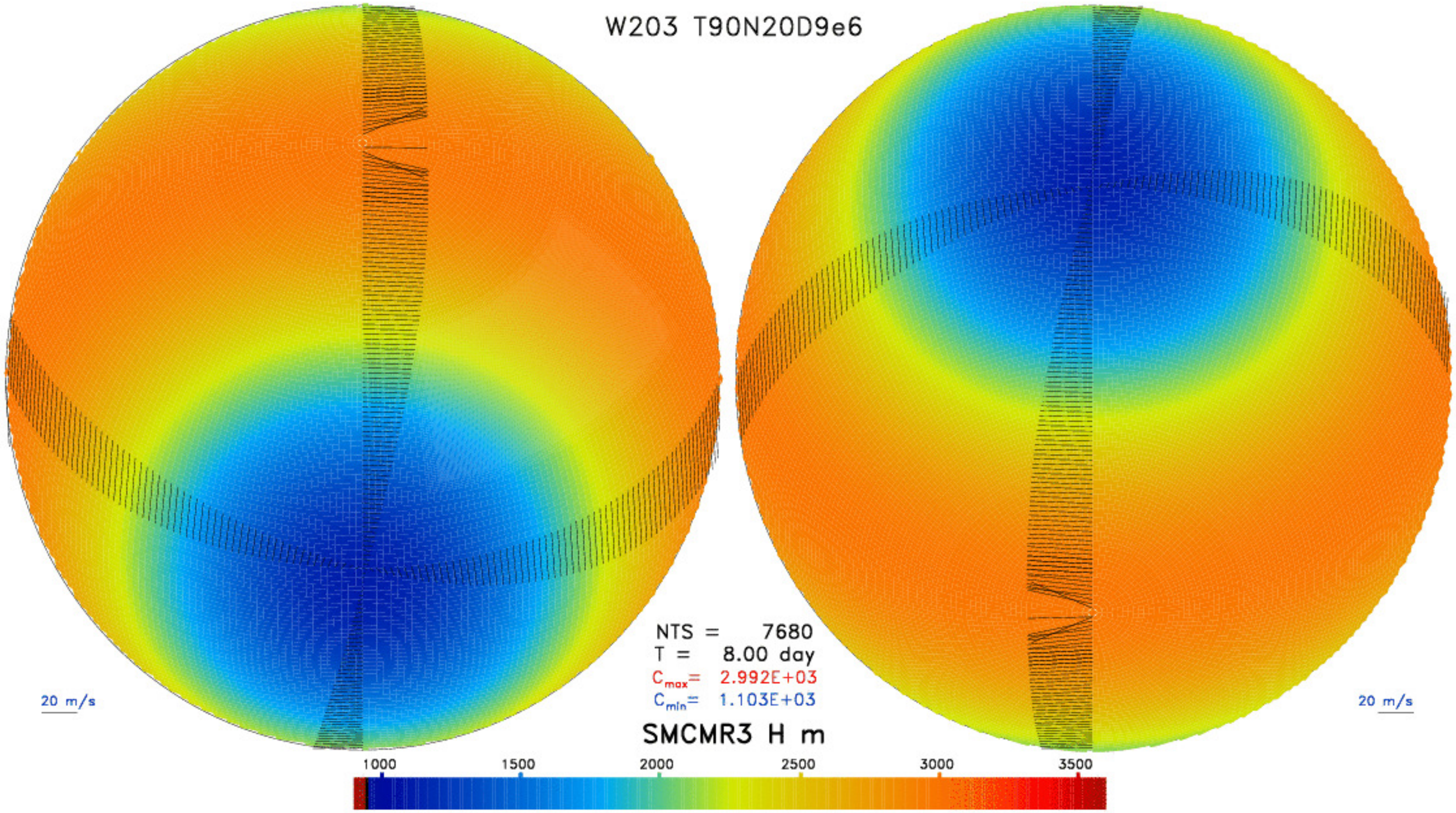
SWE Test W2, α=π/2-0.05, Initial



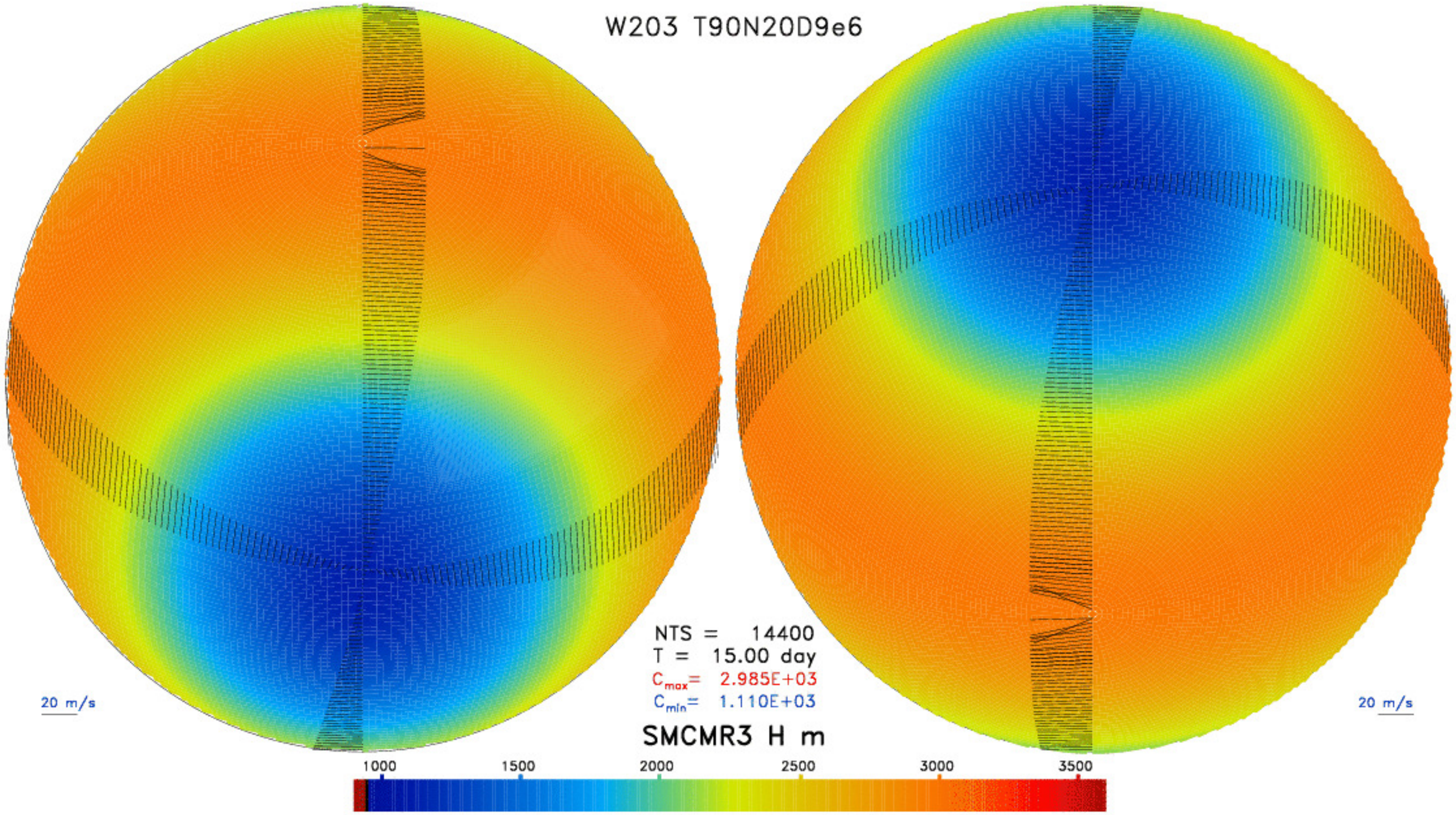
SWE Test W2, α=π/2-0.05, t = 2 days



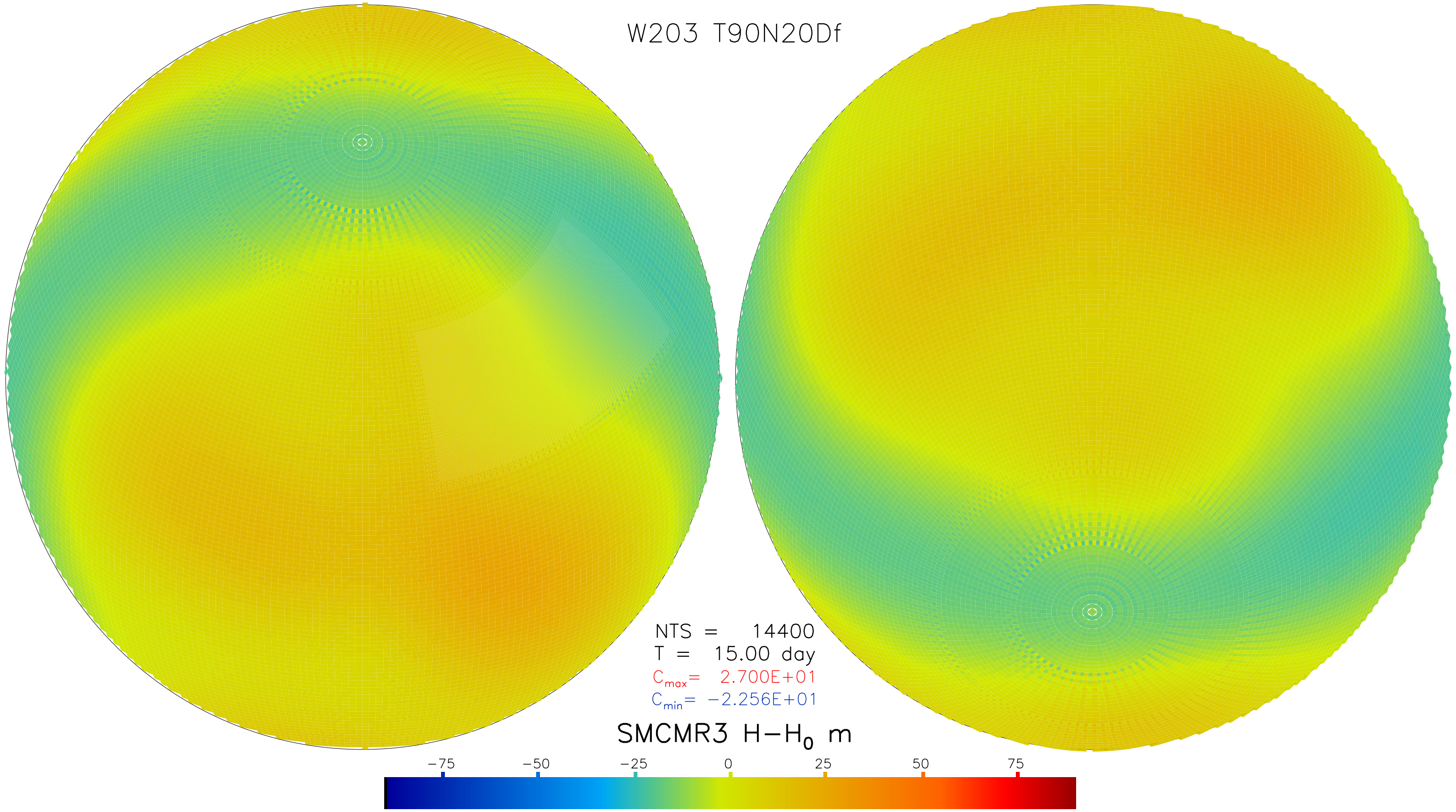
SWE Test W2, α=π/2-0.05, t = 8 days



SWE Test W2, α=π/2-0.05, t = 15 days



SWE Test W2, H(t=15d) - H(0)



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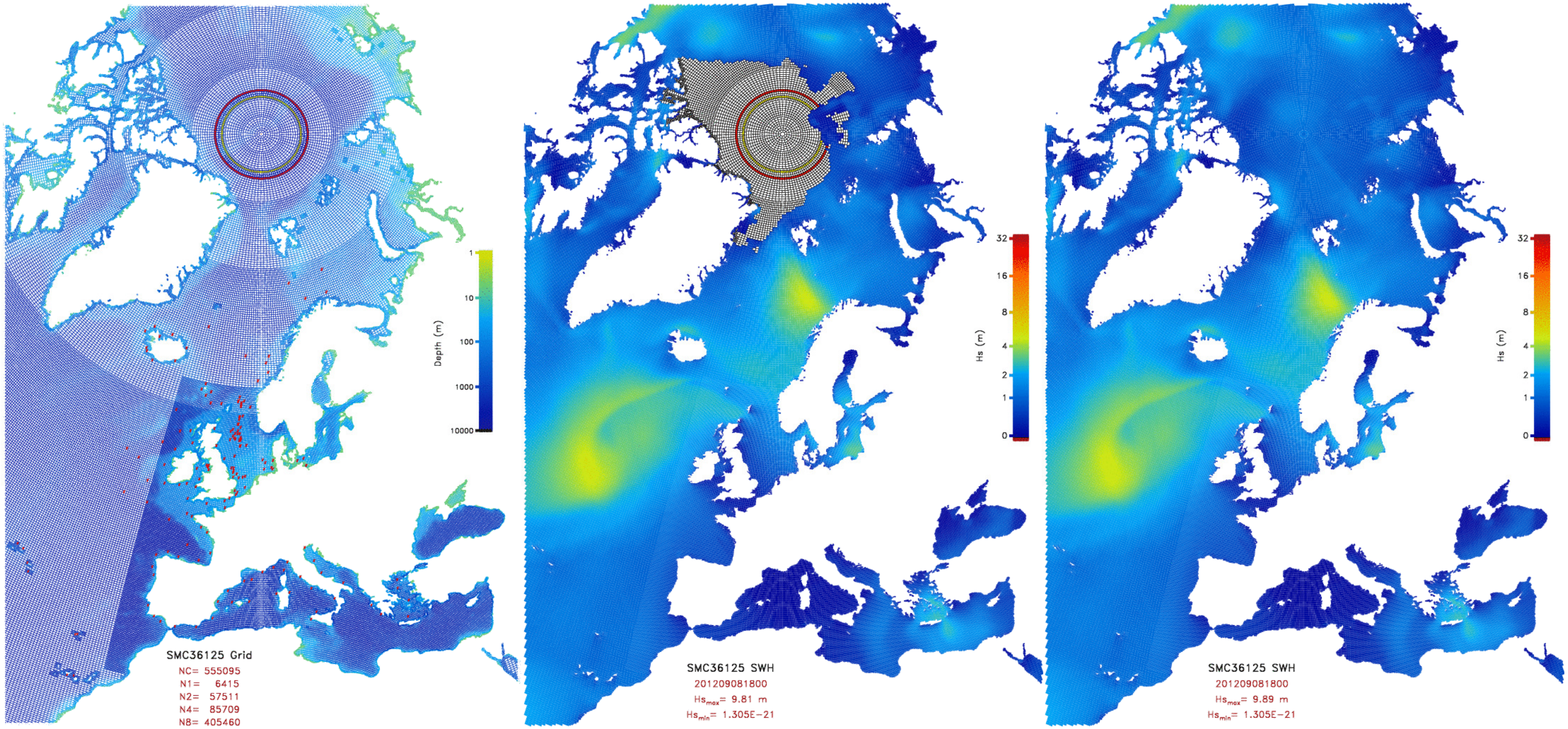
Applications of SMC grids so far

- Implemented in WAVEWATCH III® ocean surface wave model and applied in Met Office global (SMC3-6-12-25km) and regional (UK1.5-3km) wave forecasting models and coupled climate models (wave model only SMC50km).
- Environment Canada and Ocean University of China used for Arctic wave climate studies (Global SMC100km + Arctic SMC12-25 km).
- University of Melbourne for simulation of hurricane wind high waves in Gulf of Mexico (SMC2-4-8-16km).
- Collaboration with European partners for wave modelling in the Mediterranean Sea (SMC36125) and with other users in New Zealand, Japan, China & Russia.
- London Thames Valley air pollution model (horizonal1-2-4 km and flat levels).

Unified global & regional wave forecasting model

Reference: LI, J.G. & A. Saulter 2014: *Ocean Dynamics*, **64**, 1657-1670.



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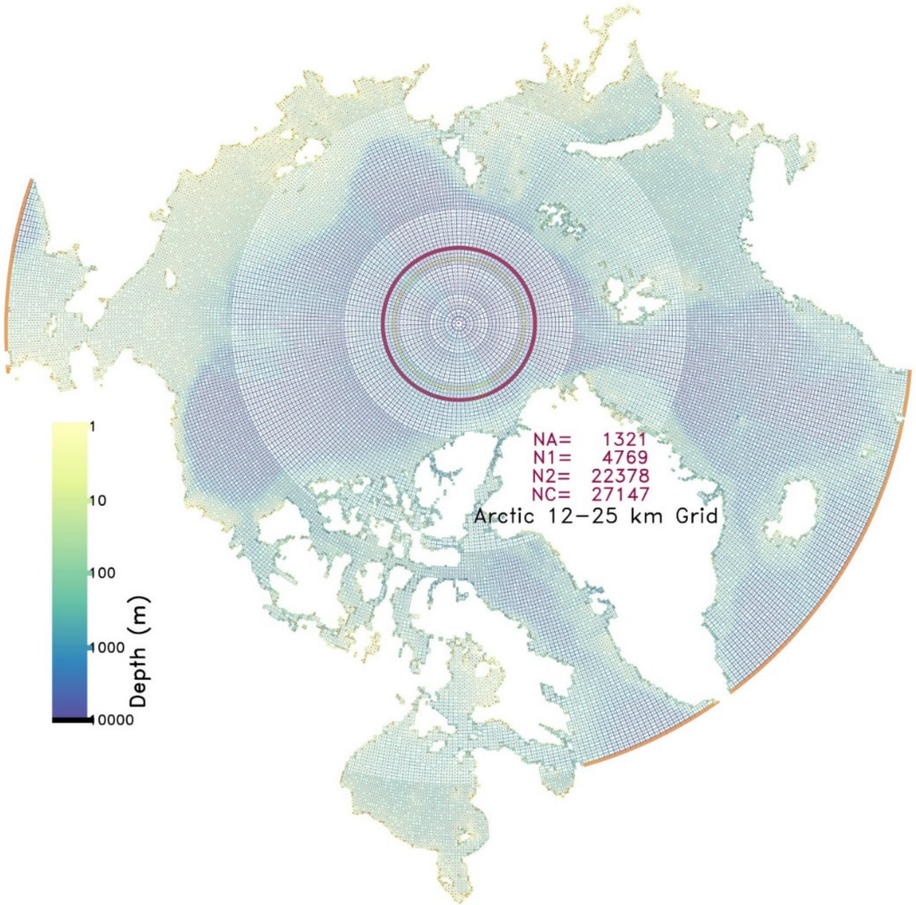
Environ. Canada Arctic model

- Courtesy of **Dr Mercè Casas-Prat** Environment Canada

References:

- Li, J.G. 2016: *Ocean Dynamics*, **66**, 989-1004.
- Casas-Prat et al 2018: *Ocean Modelling*, **123**, 66-85.



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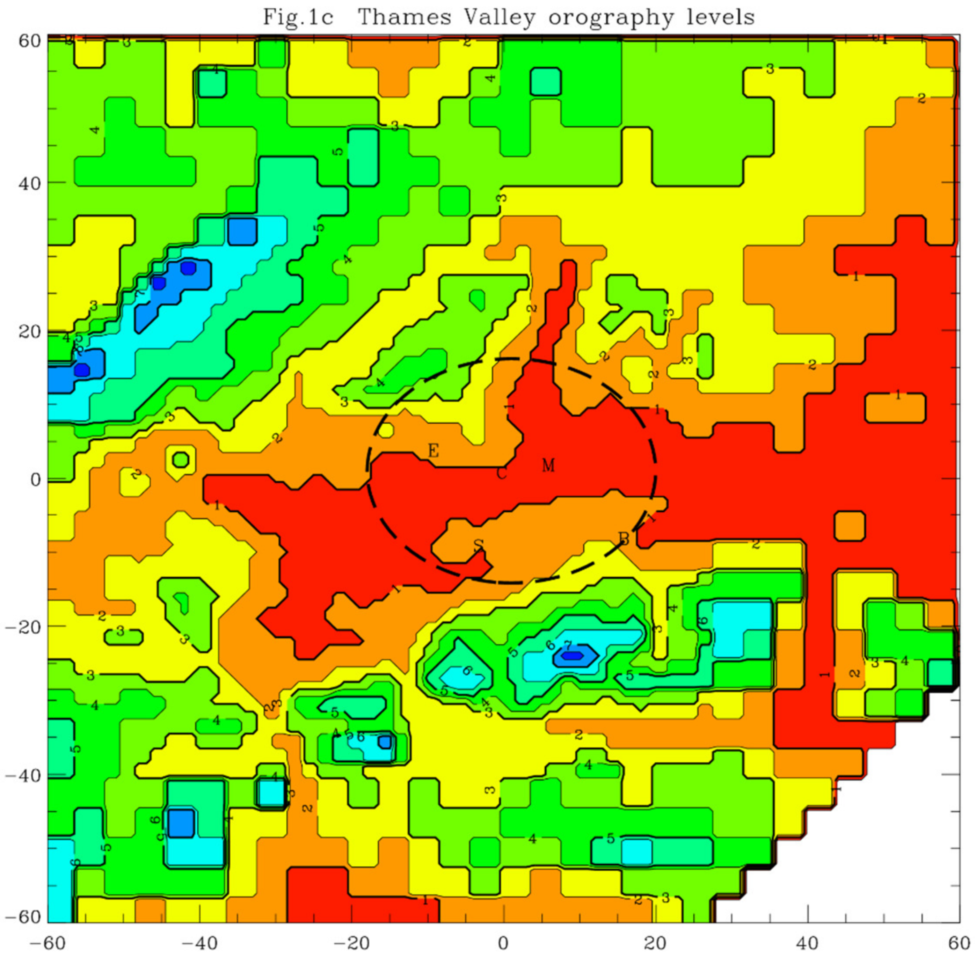
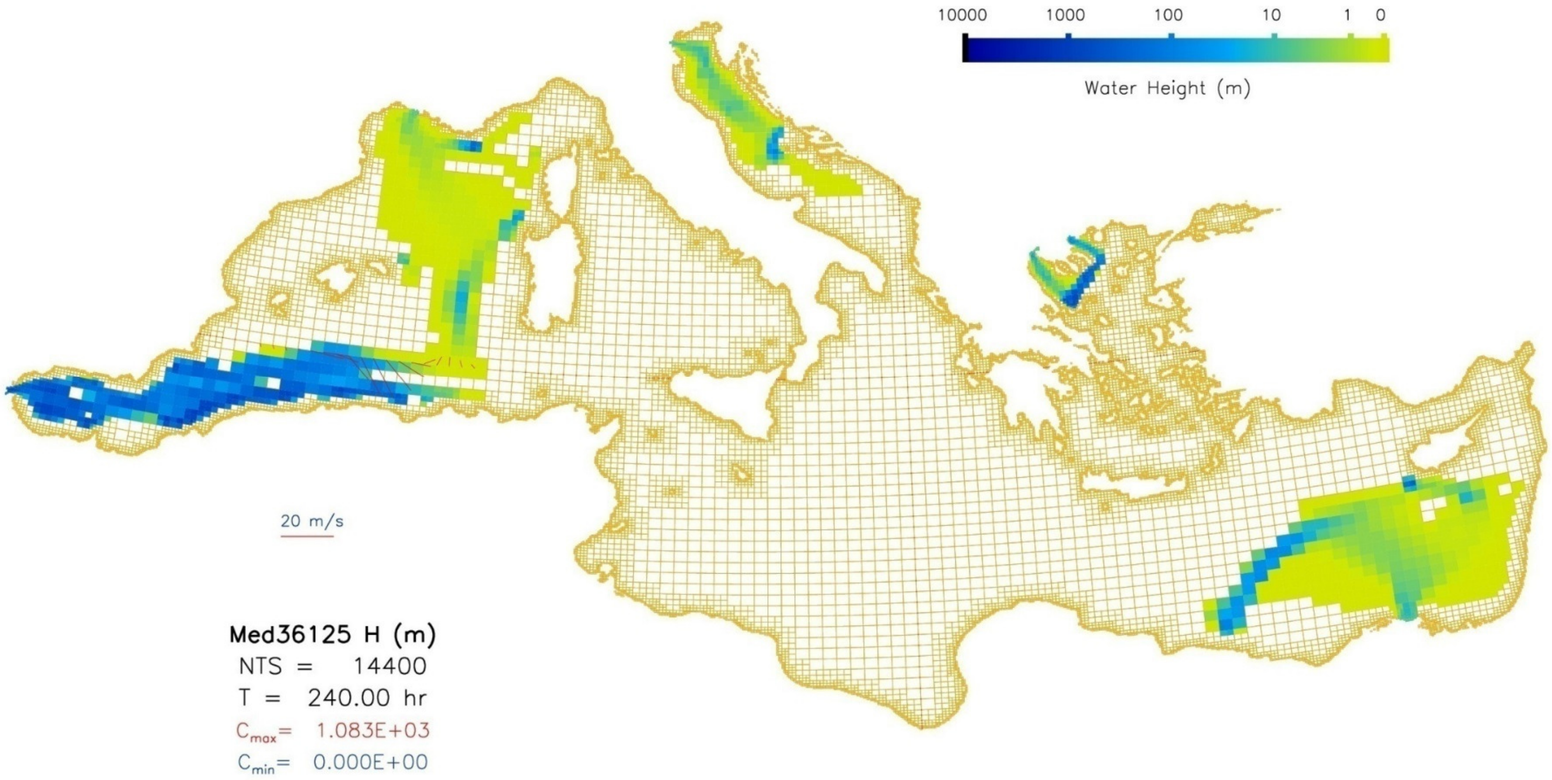
Multiple-Cell grid for Thames Valley

London Thames Valley multiple-cell 3-D grid for London air pollution study. The contour numbers indicate vertical levels.

Li, J.G. 2003: Boundary-Layer Meteorology, **107**, 289-322.



Filling the Mediterranean Sea SMC36125



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Possible applications in the future

- Global transportation in chemical/biological models even if dynamical models are on different grids. Simply switching to the UNO2 scheme may save you a lot of time.
- Earth system with a unified horizontal grid for all components, ocean, wave, atmosphere, chemistry, land/soil and biological sphere etc.
- Regional multi-resolution model for air pollution, coastal surge, environmental studies.
- Anything else you could image, which involves transportation.

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Summary

- SMC grid is a unstructured grid but retains the lat-lon grid rectangular cells so simple finite-difference schemes could be used. It relaxes the CFL limit at high latitudes by merging cells like a reduced grid and allows multi-resolutions like mesh refinement. It extends the scalar assumption to the polar regions by defining vector components with fixed map-east direction.
- SMC grid has been implemented in the WAVEWATCH III® wave model and used in UK Met Office operational wave forecasting models and coupled systems. It is also applied in other wave modelling projects through international collaborations.
- It has the potential for global transportation in chemical and biological models and could be adapted for dynamical models and even as a unified grid for different components in earth systems.

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Thanks! Questions?