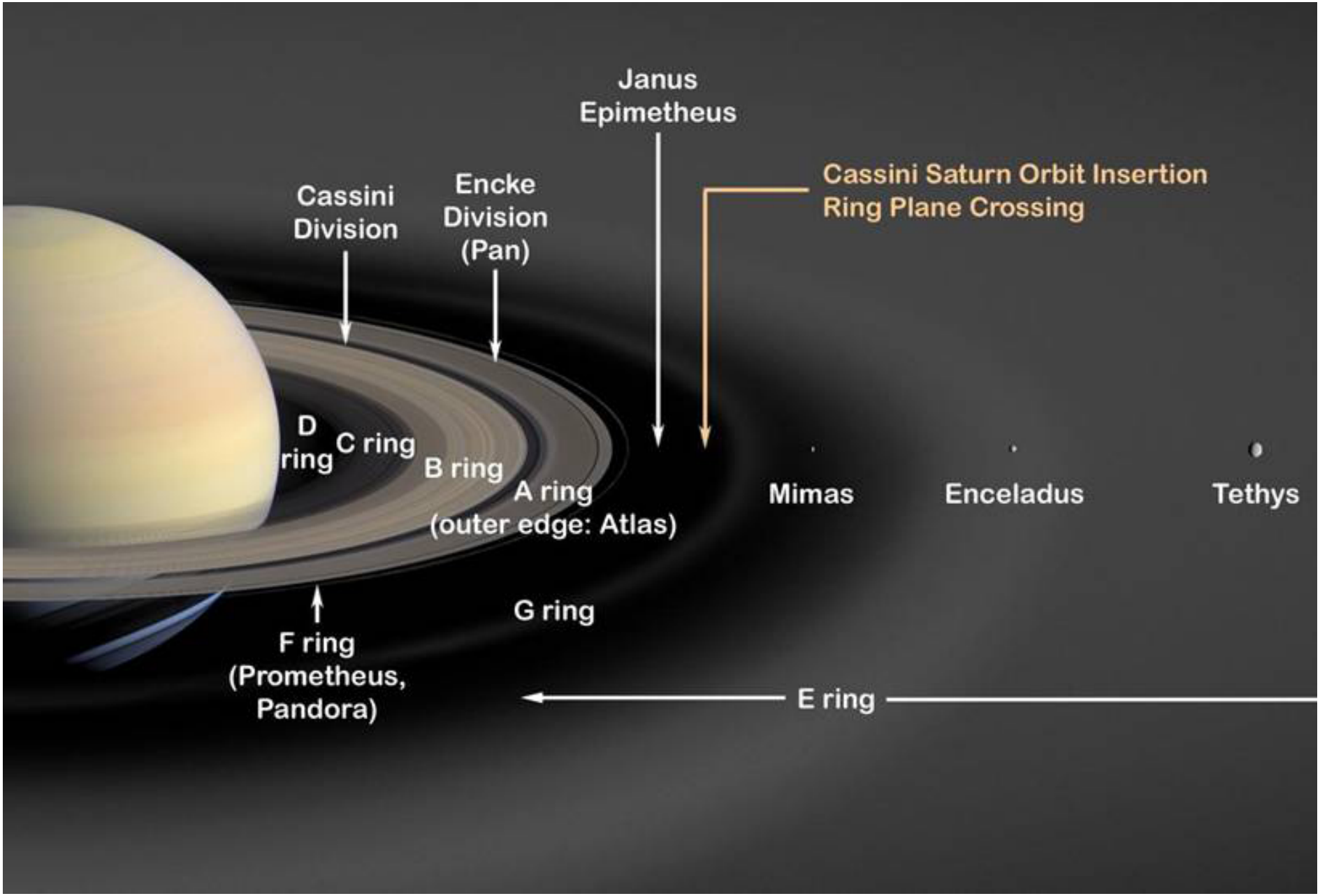
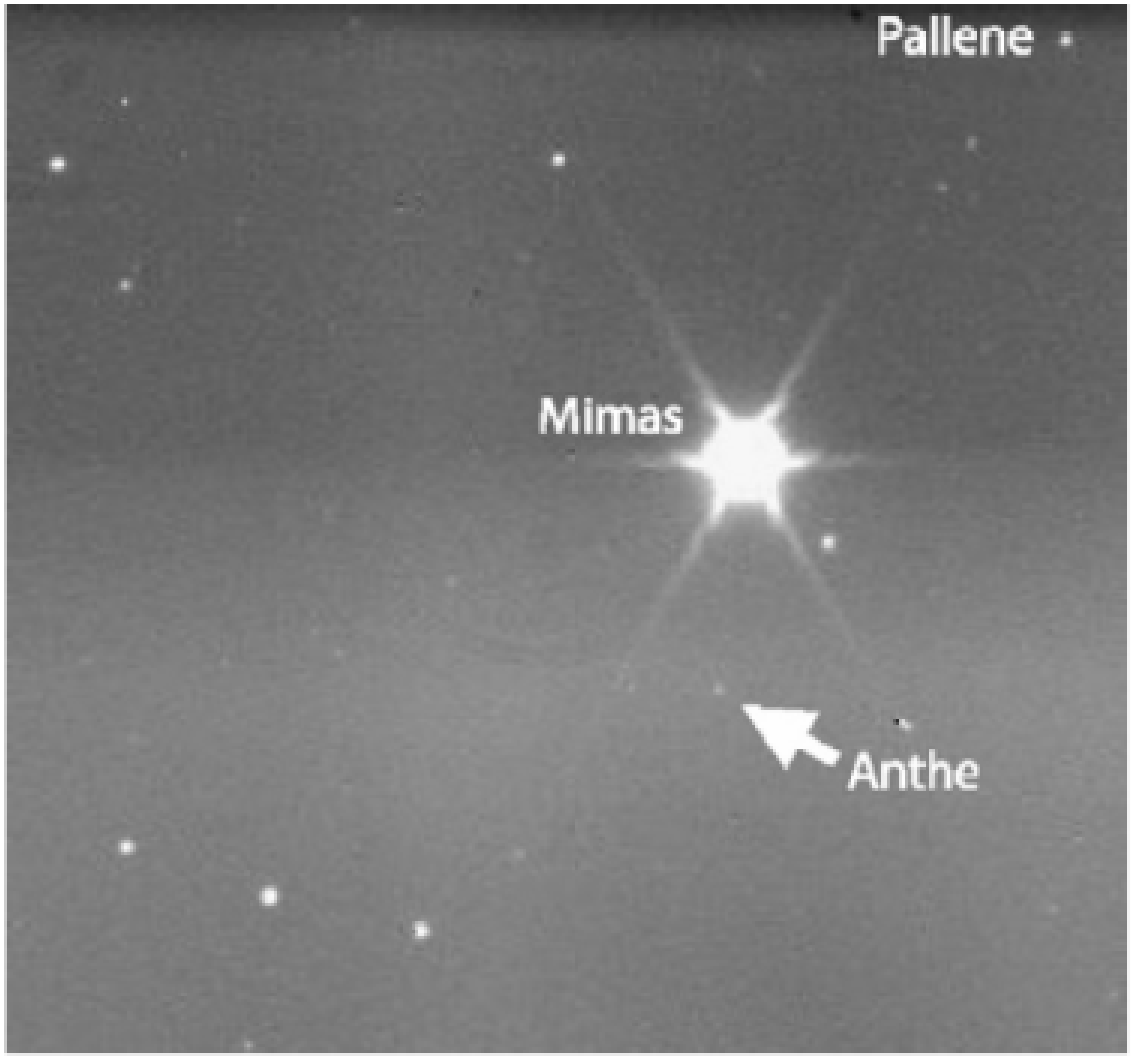
Dynamics of Saturn's small moons in coupled first order planar resonances

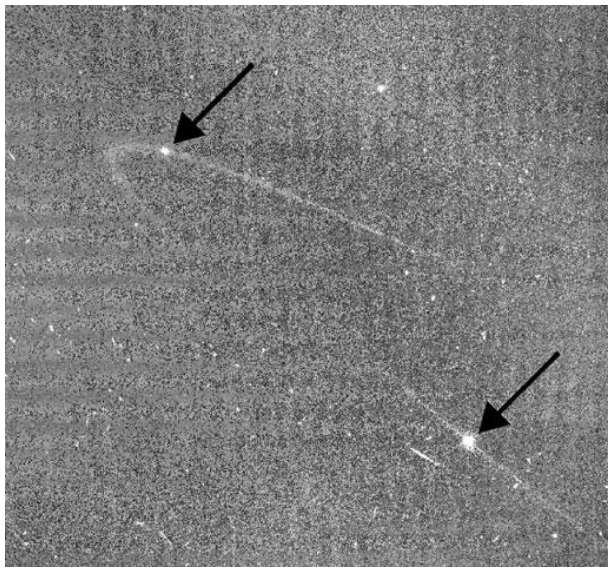
Maryame El Moutamid
Bruno Sicardy & Stéfan Renner

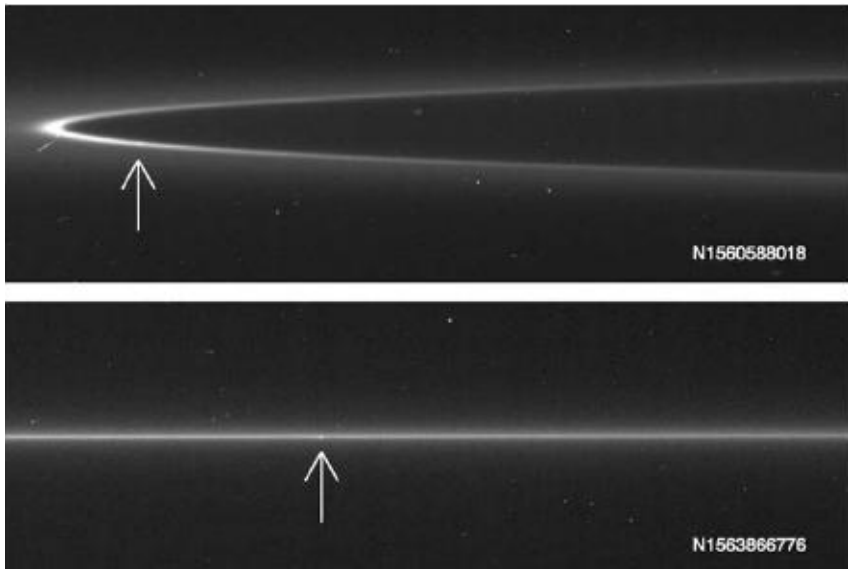

Janus
Epimetheus
Cassini Saturn Orbit Insertion
Ring Plane Crossing
Cassini
Division
Encke
Division
(Pan)
D
ring
C ring
B ring
A ring
(outer edge: Atlas)
Mimas
Enceladus
Tethys
G ring
F ring
(Prometheus,
Pandora)
E ring

Very small moons



New satellites : Anthe, Methone and Aegaeon





(Cooper et al., 2008 ; Hedman et al., 2009, 2010 ; Porco et al., 2005)

Very small (0.5 km to 2 km)
Vicinity of the Mimas orbit (outside and inside)

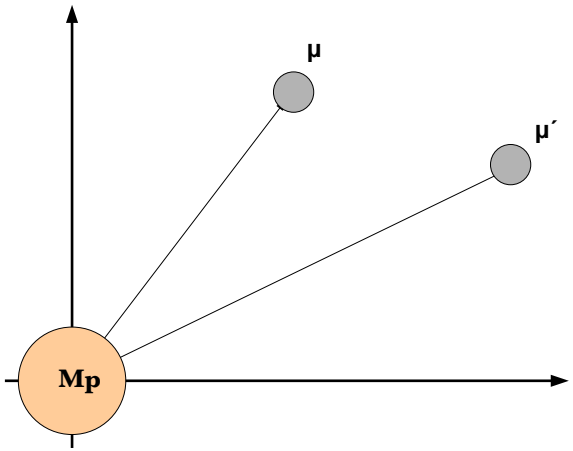
The aims of the work

A better understanding :
- of the dynamics of this population of news satellites
- of the scenario of capture into mean motion resonances

Dynamical structure of the system

We consider only :

- The resonant terms
- The secular terms causing the precessions of the orbit



When $\mu \to 0 \Rightarrow$ The symmetry is broken $\Rightarrow$ different kinds of resonances :

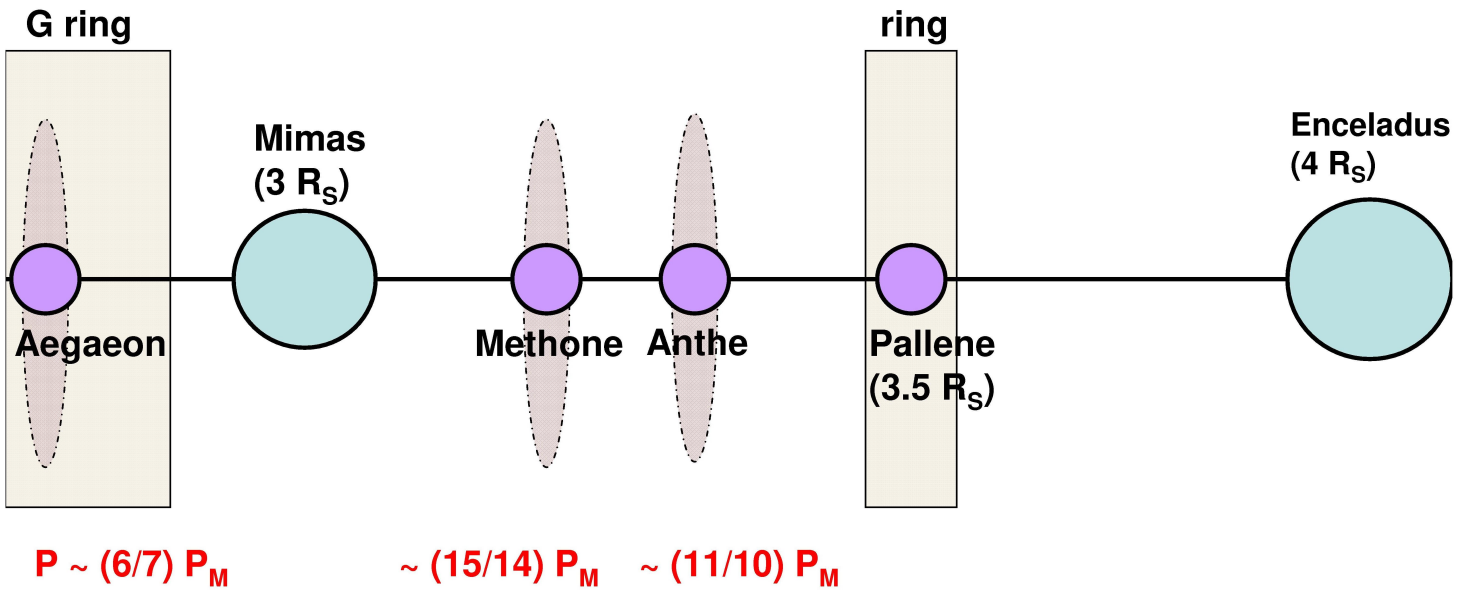
- Lindblad Resonance
- Corotation Resonance

D'Alembert rules :

$$\phi_c = (m+1)\lambda' - m\lambda - \varpi'$$
$$\phi_L = (m+1)\lambda' - m\lambda - \varpi$$

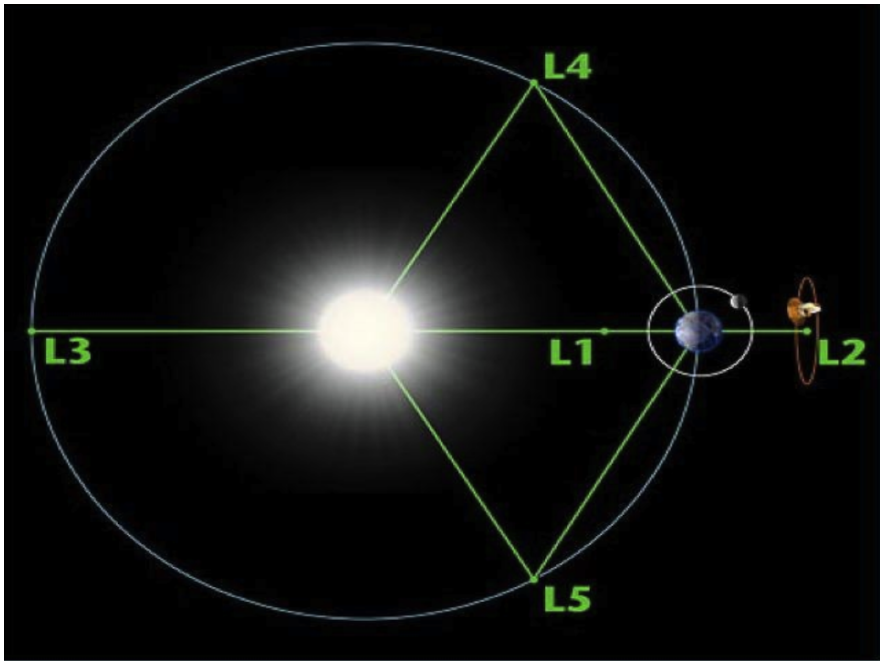
Corotation Resonance

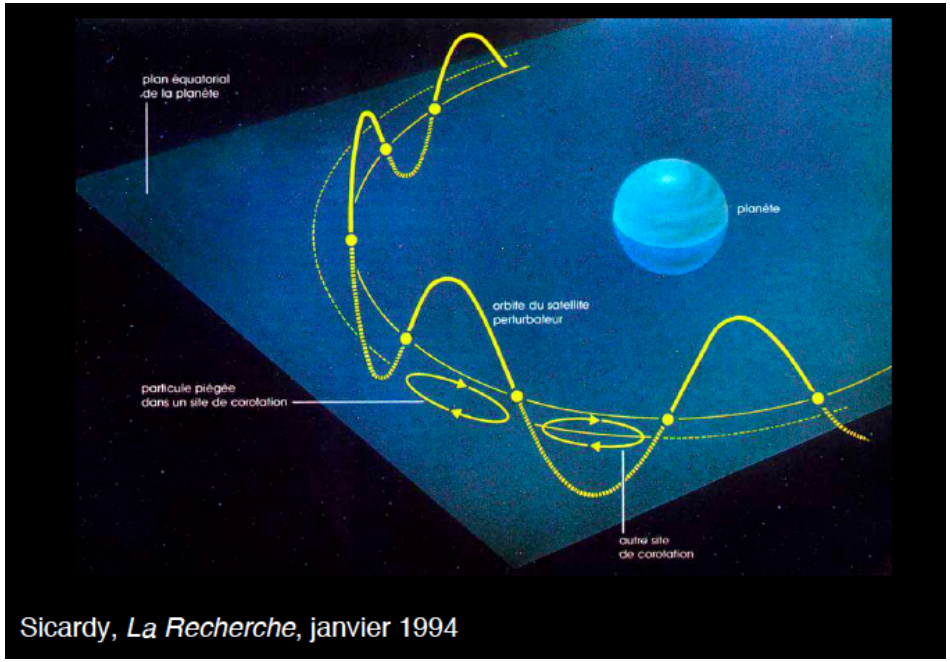
- Aegaeon (7/6) : $\phi_c = 7\lambda_{Mimas} - 6\lambda_{Aegaeon} - \varpi_{Mimas}$
- Methone (14/15) : $\phi_c = 15\lambda_{Methone} - 14\lambda_{Mimas} - \varpi_{Mimas}$
- Anthe (10/11) : $\phi_c = 11\lambda_{Anthe} - 10\lambda_{Mimas} - \varpi_{Mimas}$



Corotation resonances

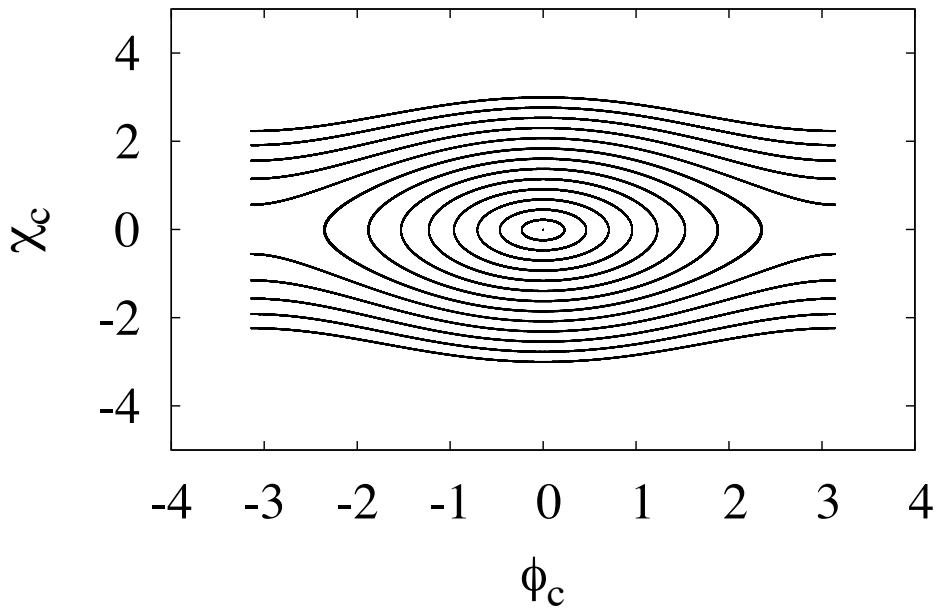
Mean motion resonance : $\frac{n_1}{n_2} = \frac{m+q}{m}$





Corotation resonances : Pendular motion

$$\begin{cases} \dot{\chi_c} = -\epsilon_c \sin(\phi_c) \\ \\ \dot{\phi_c} = \chi_c \end{cases} \quad \text{where : } \begin{cases} \chi_c = \frac{3}{2} m \frac{a - a_c}{a} \\ \\ \epsilon_c = 3m^2 \frac{M_s}{M_p} \frac{a}{a_c} Ee_s \end{cases}$$



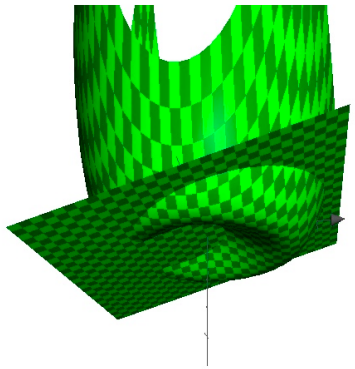
Lindblad Resonance

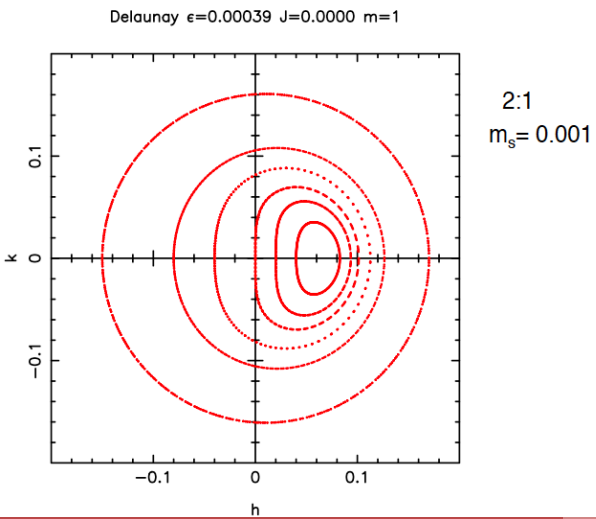
$$H_{And} = J_L^2 - (J_c + D)J_L - \varepsilon_L h$$

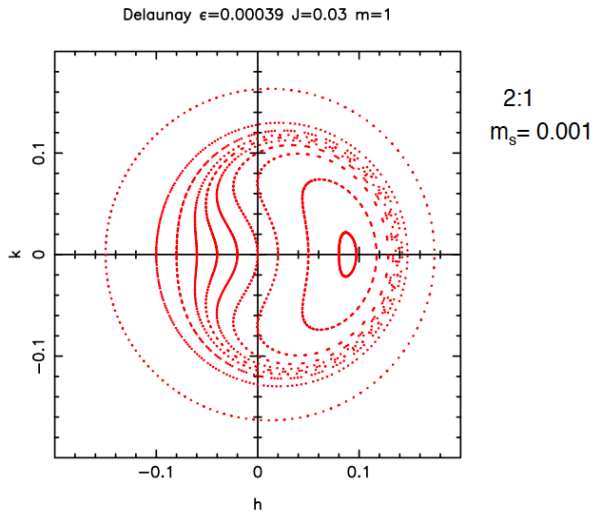
$$\begin{cases} \dot{h} = +\frac{\partial H}{\partial k} = -(J_C - J_L + D)k \\ \dot{k} = -\frac{\partial H}{\partial h} = +(J_C - J_L + D)h + \varepsilon_L \end{cases}$$

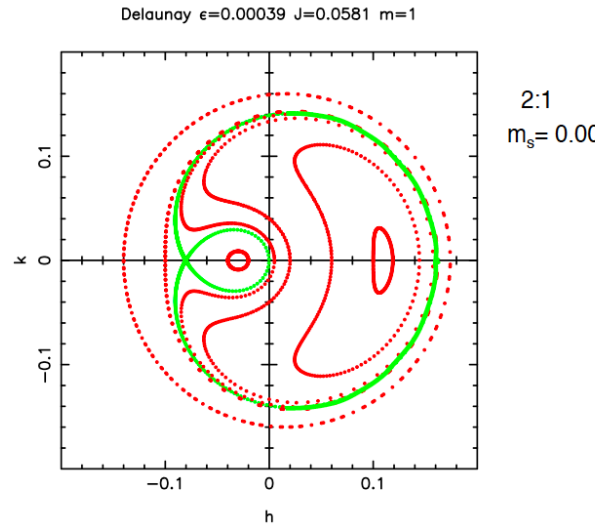
e=(h,k) : the vector eccentricity

$$\begin{cases} h \propto e\cos(\phi_L) \\ k \propto e\sin(\phi_L) \end{cases} \qquad J_L \propto \frac{e^2}{2}$$









Dynamical study

Equations of motions $\Rightarrow$ The CoraLin (CORotation And LINdblad) Model

$$\begin{array}{lll} \phi_c = (m+1)\lambda' - m\lambda - \varpi' & \text{near} & \dot{\phi}_c = 0 \\ \phi_L = (m+1)\lambda' - m\lambda - \varpi & \text{near} & \dot{\phi}_L = 0 \end{array}$$

The coupling leads to a chaotic motion

Asymptotic cases

General solutions based on adiabatic invariance arguments

Hamiltonian formalism : The Coralin Model

$$\left\{\begin{array}{l} \dot{J_c} = -\frac{\partial H}{\partial \phi_c} = -\sin(\phi_c) \\ \dot{\phi_c} = +\frac{\partial H}{\partial J_c} = J_C - [J_L] \end{array}\right\} \textit{Corotation}$$

$$\left\{\begin{array}{l} \dot{h} = +\frac{\partial H}{\partial k} = -([J_C] - J_L + D)k \\ \dot{k} = -\frac{\partial H}{\partial h} = +([J_C] - J_L + D)h + \varepsilon_L \end{array}\right\} \textit{Lindbald}$$

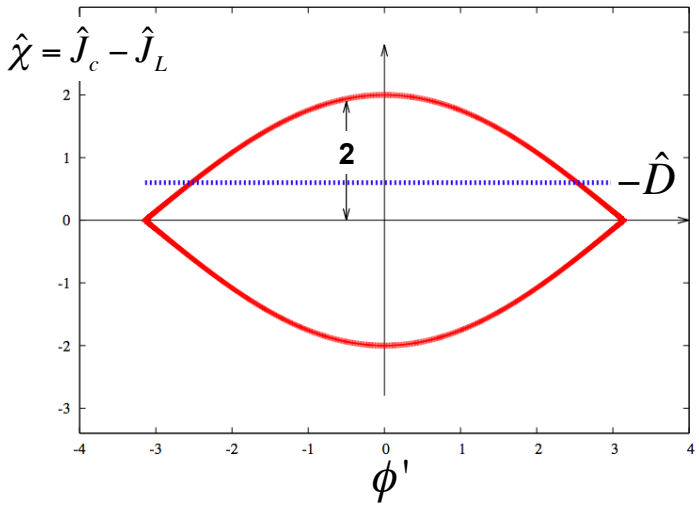
where the Hamiltonian H is given by :

$$H(J_c, \phi_C, J_L, \phi_L) = \frac{1}{2}(J_c - J_L)^2 - DJ_L - \cos(\phi_C) - \varepsilon_L\sqrt{3}|m|e\cos(\phi_L)$$

where : e=(h,k) : the vector eccentricity $\left\{\begin{array}{l} h \propto e\cos(\phi_L) \\ k \propto e\sin(\phi_L) \end{array}\right.$ $J_L \propto \frac{e^2}{2}$

El Moutamid, M., Sicardy, B., Renner, S. (2012) CeMDA

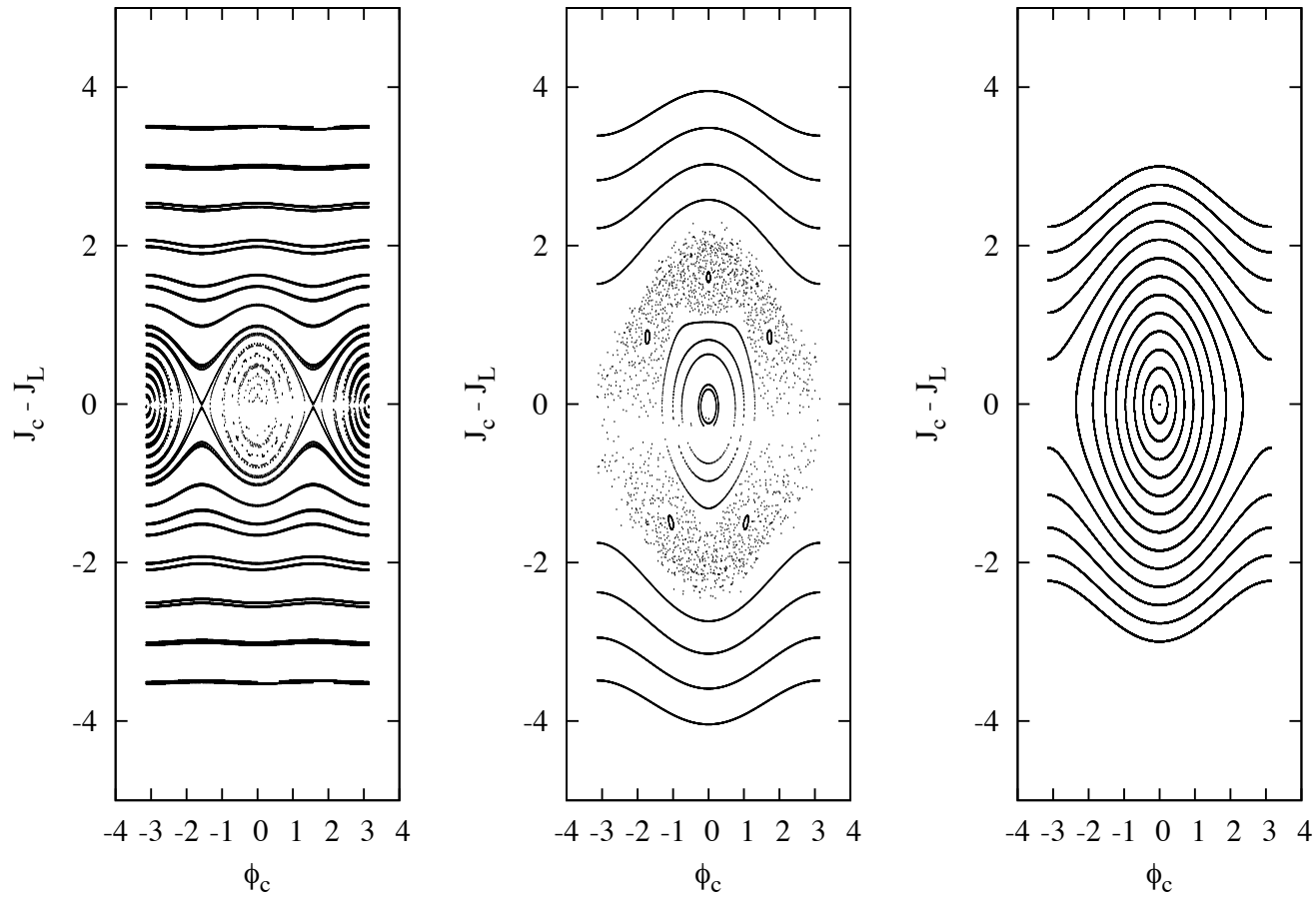
To simplify



This study depends essentially of D which gives the distance between the two resonances, and the forcing value ε_L for a given value of the width of CER.

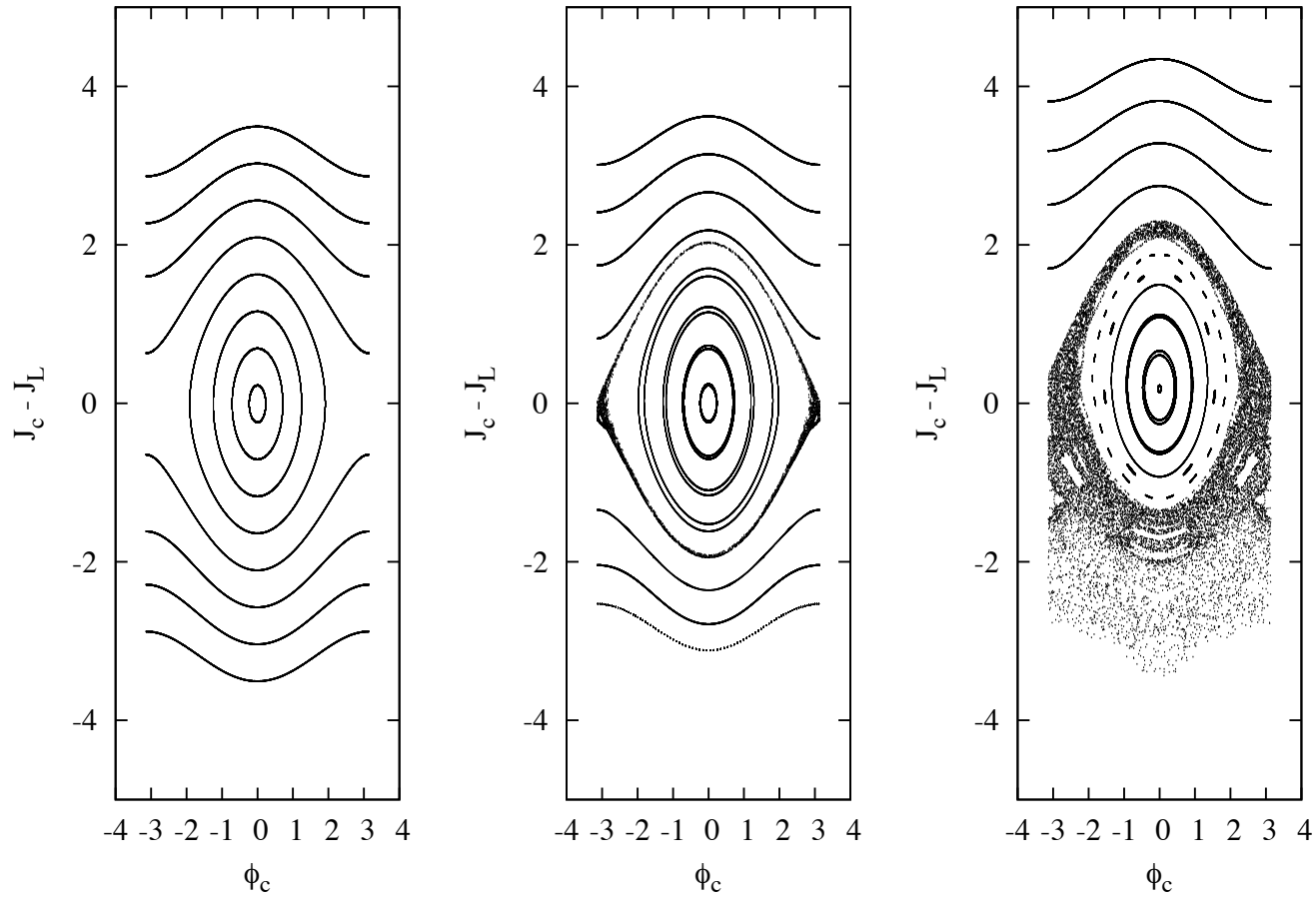
	D	ε_L
Anthe	0.256	-0.133
Aegaeon	-1.34	0.132
Methone	0.122	-0.115

$D = 0$ $D = 0.27$ $D = 20$



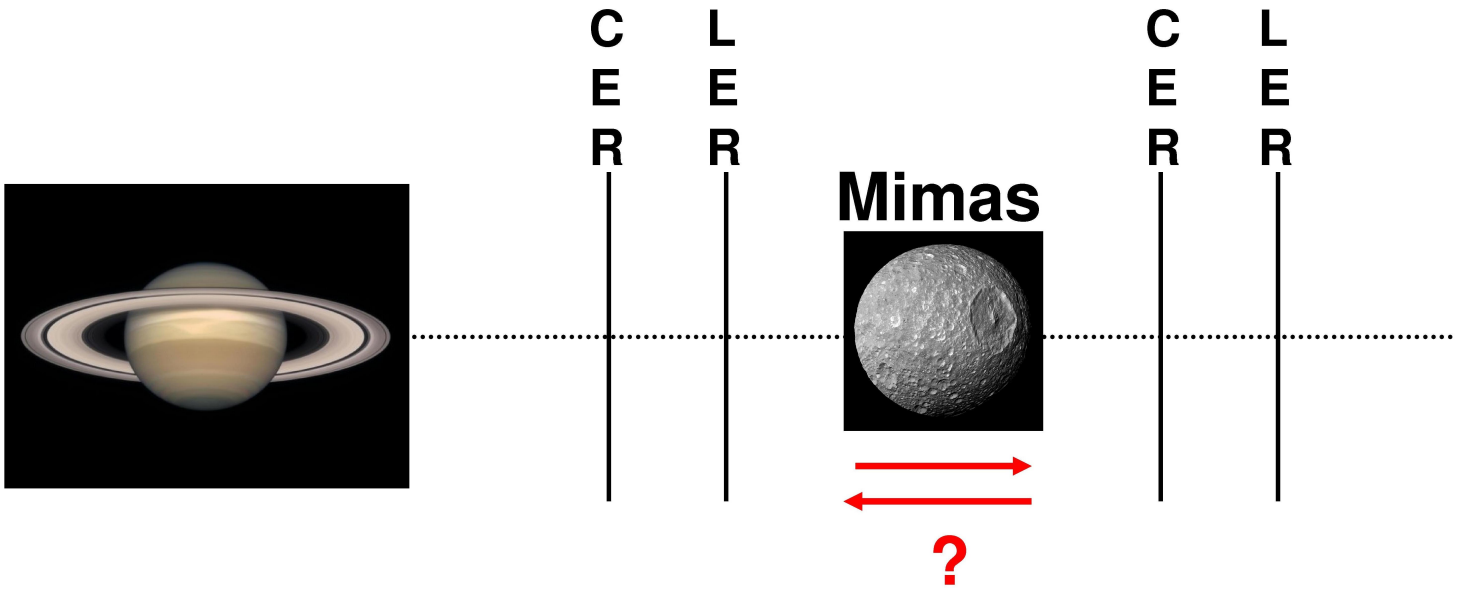
El Moutamid, M., Sicardy, B., Renner, S. (2012) CeMDA

$\varepsilon_L = 0$ $\varepsilon_L = -0.123$ $\varepsilon_L = -3$



El Moutamid, M., Sicardy, B., Renner, S. (2012) CeMDA

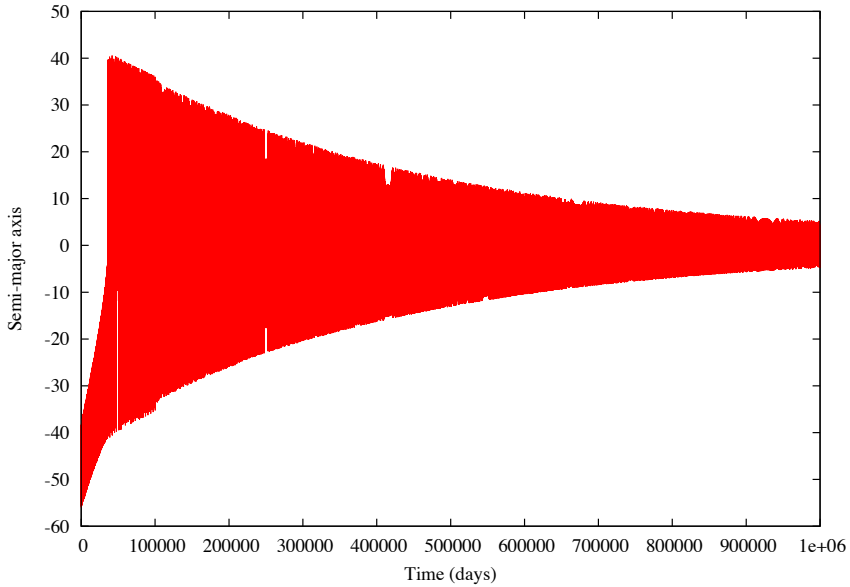
Probability of capturing into the Corotation Eccentric Resonance (CER)

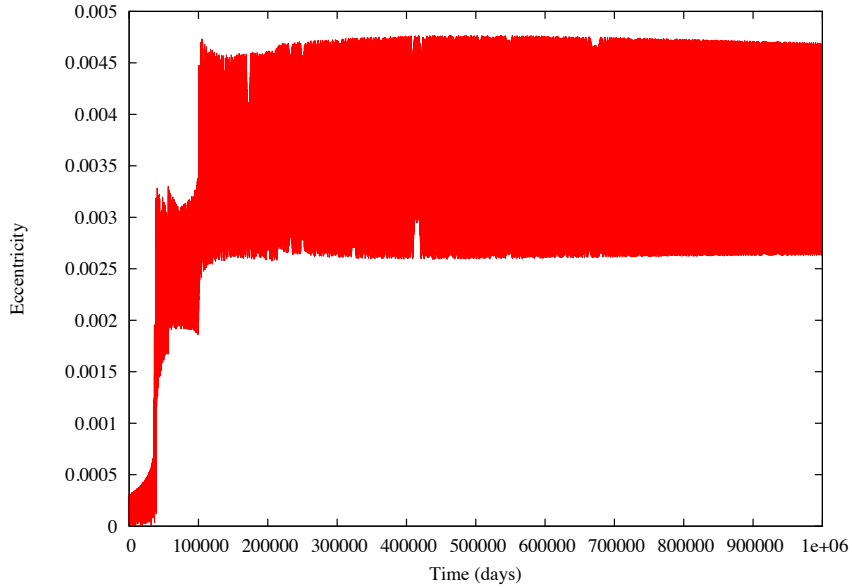


- add migration in the CoraLin model to estimate the capture probabilities into CER perturbed by LER

Capture into CER

$T_{av} = \alpha + \beta\chi$



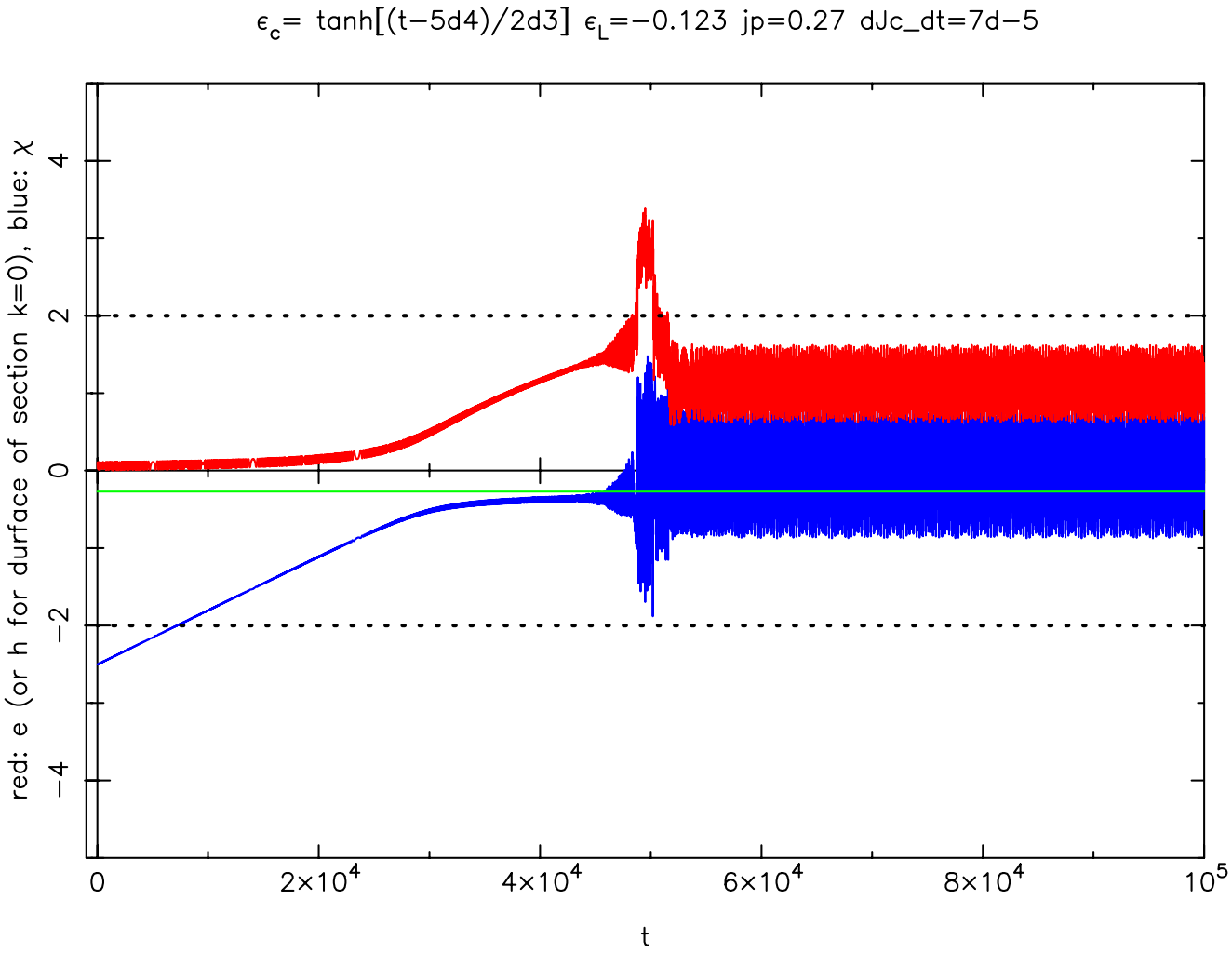


a = 197655.4 km

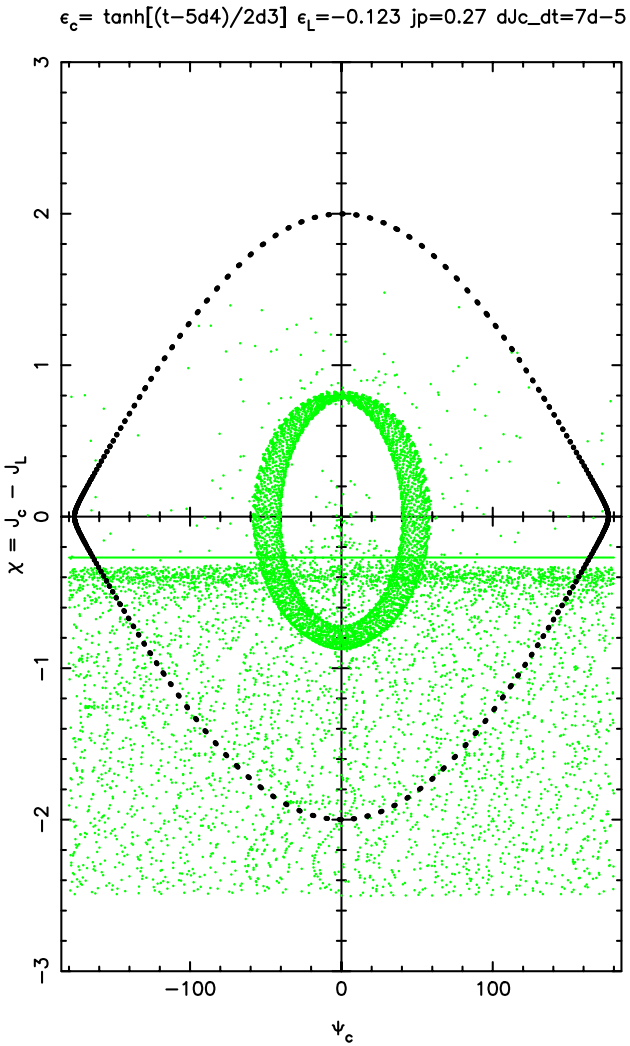
e = 0.0037

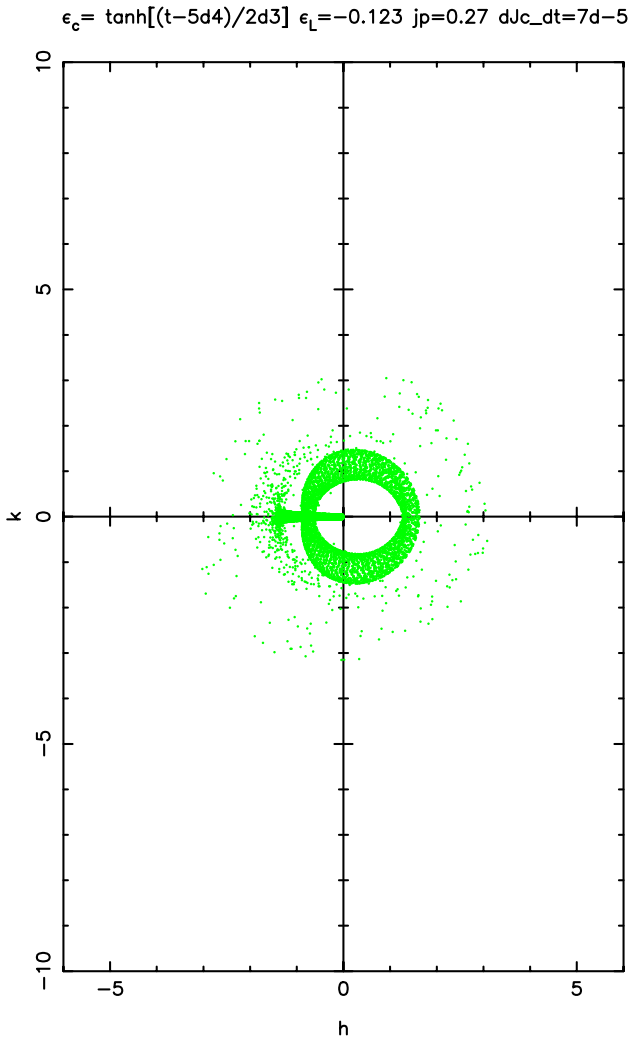
the same values than (Cooper et al., 2008)

Case Anthe


ϵ_c= tanh[(t−5d4)/2d3] ϵ_L=−0.123 jp=0.27 dJc_dt=7d−5
red: e (or h for durface of section k=0), blue: χ
4
2
0
−2
−4
0
2×10^4
4×10^4
6×10^4
8×10^4
10^5
t

Case Anthe


ϵc= tanh[(t−5d4)/2d3] ϵL=−0.123 jp=0.27 dJc_dt=7d−5
χ = Jc − JL
Ψc
3
2
1
0
−1
−2
−3
−100
0
100


ϵc= tanh[(t−5d4)/2d3] ϵL=−0.123 jp=0.27 dJc_dt=7d−5
k
h
10
5
0
−5
−10
−5
0
5

Conclusions and prospects

- CoraLin : simple model to investigate the dynamics of a satellite on CER/LER
- Capture probabilities into CER + applications to our satellites
- Anthe crosses a chaotic region favors the capture
- The capture is sure in the second scenario
- For Aegaeon : need a migration in the other direction.