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Topographic coupling at core-mantle boundary in rotation and orientation changes of planets

V. Dehant (1,3), M. Puica (2), M. Folgueira (2), A. Trinh (1,3) and T. Van Hoolst (1)
(1) Royal Observatory of Belgium, Brussels, Belgium, (2) Universidad Complutense de Madrid, Spain, (3) Catholic University of Louvain-la-Neuve, Belgium (v.dehant@oma.be)

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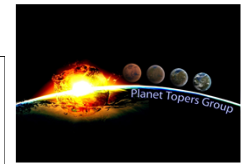
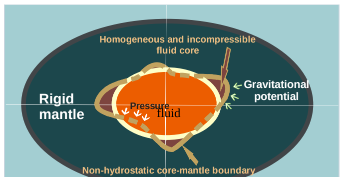


General Approach

Topography of the bumpy core-mantle boundary (CMB): expressed in spherical harmonics
Fluid motion → pressure torque on CMB
Ellipsoidal boundary → Poincaré fluid motion for nutations & global rotation for length-of-day variations in response to a tidal potential
Bumpy boundary → incremental flow at the CMB with respect to these global motions
→ incremental velocity field coefficients in terms of the topography coefficients
→ incremental torque
Wu and Wahr (1997): *numerical* approach*;* this work: *analytical* approach → understand the reasons of enhancement
We have found that there are particular velocity field coefficients that are enhanced due to resonance effects with inertial waves that arise from the incremental flow at the boundary, depending on the geometry of the boundary.

We have computed the solutions for the incremental velocity field at the core-mantle boundary (CMB) induced by the external gravitational potential and by the CMB topography for tidal effects on length-of-day (LOD) variations and on nutations.



We start from:

Linearized Navier-Stokes equation:

$$\frac{\partial \vec{V}}{\partial t} = -\frac{1}{\rho_f}\nabla P + \vec{b} - \vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2\vec{\omega}\times\vec{V} - \frac{\partial \vec{\omega}}{\partial t}\times\vec{r}$$

Velocity field; fluid density; Initial and incremental pressures; body force related to initial, external, and incremental potentials; uniform rotation; global rotation

Equation of hydrostatic equilibrium

$$\frac{1}{\rho_f}\nabla P_0 + \vec{\Omega}\times(\vec{\Omega}\times\vec{r}) - \nabla\Phi_0 = 0$$

Incremental rotation

$$\frac{\partial \vec{V}}{\partial t} + 2\vec{\Omega}\times\vec{V} + \frac{1}{\rho_f}\nabla p - \nabla\phi_m + \Omega\frac{\partial \vec{m}}{\partial t}\times\vec{r} = 0$$

With boundary conditions: **with** $\vec{V} = \vec{v} + \vec{u}$ **and** $\vec{q} = \frac{\vec{u}}{\Omega L}$

At CMB $\hat{n}\,.\vec{V} = 0$ and incompressibility $\nabla\,.\vec{V} = 0$

We solve for the case without topography (except flattening) to get $\vec{v}$

$$\begin{cases} \frac{\partial \vec{v}}{\partial t} + 2\vec{\Omega}\times\vec{v} + +\Omega\frac{\partial \vec{m}}{\partial t}\times\vec{r} - \nabla\phi_m = \nabla\chi \\ \nabla\cdot\vec{v} = 0 \end{cases}$$

χ for nutations is $\chi = \Omega^2(m_1^f xz + m_2^f yz)$
χ For LOD is χ =0

and consider that the incremental velocity field $\vec{q}$ obeys:

$$\begin{cases} i\sigma_m\vec{q} + 2\vec{\hat{z}}\times\vec{q} + \nabla\Phi = 0 \\ \nabla\cdot\vec{q} = 0 \\ \vec{n}\cdot\vec{q} + \Omega^{-1}L^{-1}\vec{n}\cdot\vec{v} = 0 \end{cases}$$

where $\Phi = \frac{\phi}{\Omega^2 L^2}$ and $\phi = \frac{p}{\rho_f} + \chi$.

χ (or ϕ) chosen in order for this incremental $\vec{q}$ to be small.

The solution for $\vec{q}$ is:

$$\vec{q} = \frac{-i\,\sigma_m}{4-\sigma_m^2}\left[\nabla\Phi - \frac{2}{i\,\sigma_m}\vec{\hat{z}}\times\nabla\Phi - \frac{4}{\sigma_m^2}(\vec{\hat{z}}\cdot\nabla\Phi)\,\vec{\hat{z}}\right]$$

$$\nabla^2\Phi - \frac{4}{\sigma_m^2}\frac{\partial^2\Phi}{\partial Z^2} = 0 \qquad \Phi = \sum_{l=1} a_l^k P_{lk}\left(\frac{\sigma_m}{2}\right) Y_l^k(\theta,\lambda)$$ at the boundary.

The associated torque at the CMB has the following expression:

$\Gamma_{topo}^{\phi} = -\iint_{CMB}\vec{r}\times\hat{n}\;\rho_f\,\phi\,\mathrm{dS}$ where $\hat{n}$ is the normal to CMB.

For **LOD case**, the boundary condition provides the equation relating the a_l^k coefficients with the global velocity induced by the external potential without CMB topography and the topography coefficients:

$$\sum_{l,k} Y_l^k\left[kP_{lk}\left(\frac{\sigma_m}{2}\right) - \left(1-\frac{\sigma_m^2}{4}\right)P'_{lk}\left(\frac{\sigma_m}{2}\right)\right]a_l^k + 2\left(1-\frac{\sigma_m^2}{4}\right)\sum_{n=1} m_3\;\epsilon_n^m Y_n^m = 0$$

where Y_l^k are spherical harmonics, P_{lk} are the Legendre polynomials, ε_l^k are coefficients of the spherical harmonics expansion of the topography, and σ_m is the forcing frequency of the tidal potential in cycle/day.

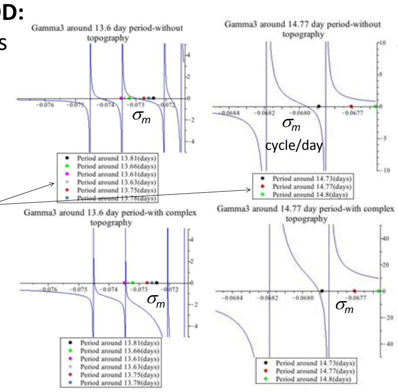
Above topographies → effects of a bumpy CMB on the LOD
for a generic topographic behaviour ("without topography")
or the real one (only known for Earth).
→ sensitive to resonances induced by inertial waves
? Inertial waves ← solving for the zeros of the coefficient of a_l^k of relation between the incremental velocity field and CMB topography
Torque on the CMB → amplitude resonance near inertial waves

Results for LOD:

Case 13.6 days and 14.7 days.

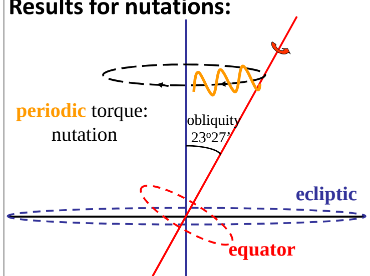
For a scaled topography (any planet) with Earth tidal freq.

For the Earth



Several tidal frequencies (in color in the graphs) close to some of the resonances and thus very much enhanced.

Results for nutations:



Changes in the orientation of a planet in space (see left graphic) due to the tidal (luni-solar) attraction; amplified by normal modes such as FCN (Free Core Nutation) for an ellipsoidal planet. For a bumpy CMB, there are additional resonances when the inertial wave frequencies are close to nutation frequencies.

We have recomputed analytically these Resonances (see table).

Resonance frequencies	(degree,order) of topography
1.00 cycle/day	(15,2) (15,6) (16,5) (20,15)
0.99 cycle/day	(12,6) (18,12)
1.01 cycle/day	(13,8) (14,6) (15,6) (15,11) (18,2) (18,6) (19,5) (19,9) (19,10)

Results for librations:

Torque → similar resonances near 1cycle/day (for icy satellites) and near 1.5 cycle/day (for Mercury).

Resonance frequencies	(degree,order) of topography
1.50 cycle/day	(3,2) (15,10) (18,0) (18,6)
1.49 cycle/day	(13,2) (13,4) (20,12)
1.51 cycle/day	(12,6) (17,8) (19,15) (20,6)

Conclusions:

Inertial waves may play an important role in LOD variations or librations or nutations as the involved forcing frequencies may be close to one of the inertial waves, therewith enhancing the response to the tidal or external potential.
The presence of an inner core is not consider in this study and may change the resonance periods, while the conclusion remains.