Outreach Testing of Ancient Astronomy

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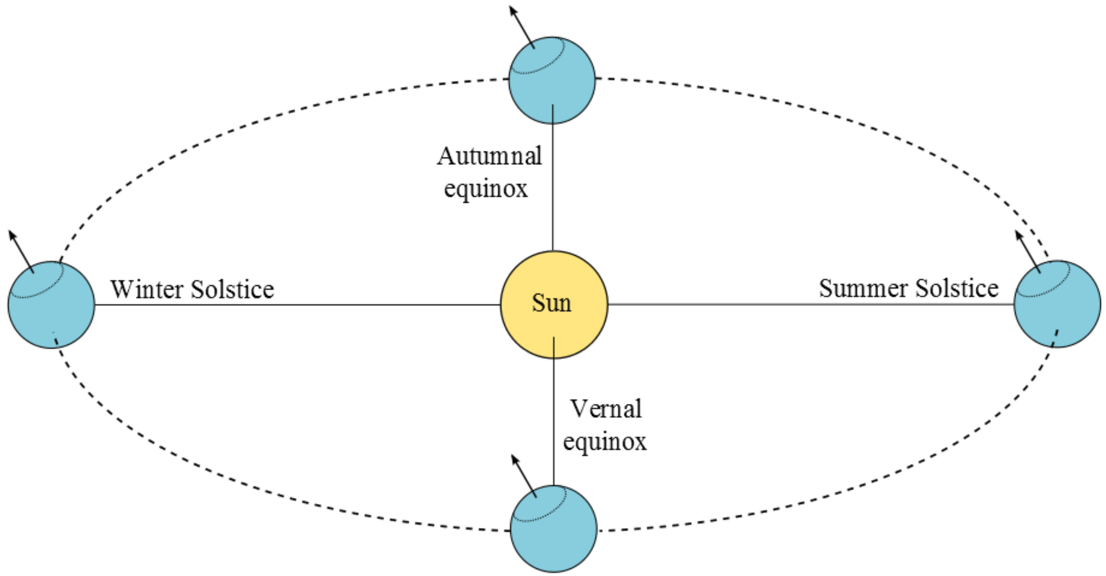
This work is an *outreach* approach to certain ubiquitous problem in secondary-school education:

How to face back the decreasing interest in natural sciences shown by too many students under 'pressure' of convenient resources in digital devices/applications.

The approach rests on 2 features.

First, *empowering* of teen-age students to understand some regular natural events around as very few educated people they meet could do.

Secondly, *understanding* that rests on personal capability to test and verify experimental results from the oldest science, Astronomy, with simplest instruments as used from antiquity down to the Renaissance,
a capability just allowed for the solar and lunar motions,… those most relevant, however.



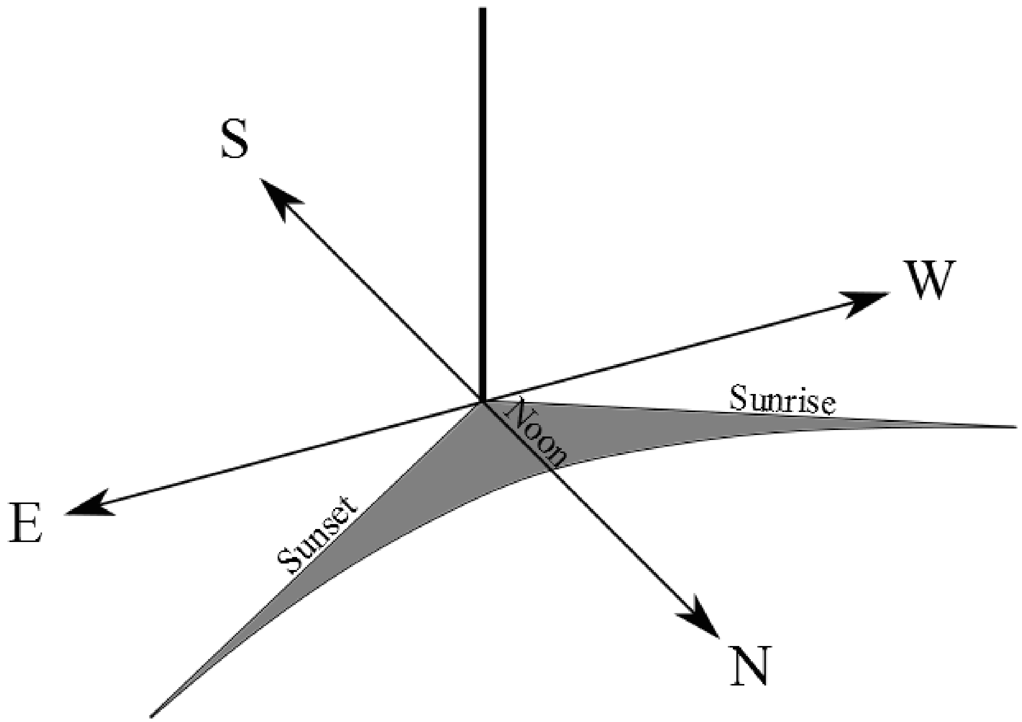
Astronomy basically involved observing and registering values of angles (along with times), be either

1) on the ground, or
2) in space from the ground.

1) On-the-ground measurements are particularly simple:

The *gnomon*, a simple stick planted vertically on the ground, allows understanding solar motion, as observed from Earth, as opposed to the heliocentric picture (above).
It allows determining the latitude at the point of observation on Earth, and the angle between Earth equator and *ecliptic* - the apparent plane where the Sun “revolves” around the Earth -.

The gnomon allows astronomers to also determine the length of the day and the year; noon time; the meridian; the cardinal points; the length of the seasons; the solstices and equinoxes.



The gnomon shadow turns around during any given day, varying in length and thus angle between solar ray and vertical, as it turns.

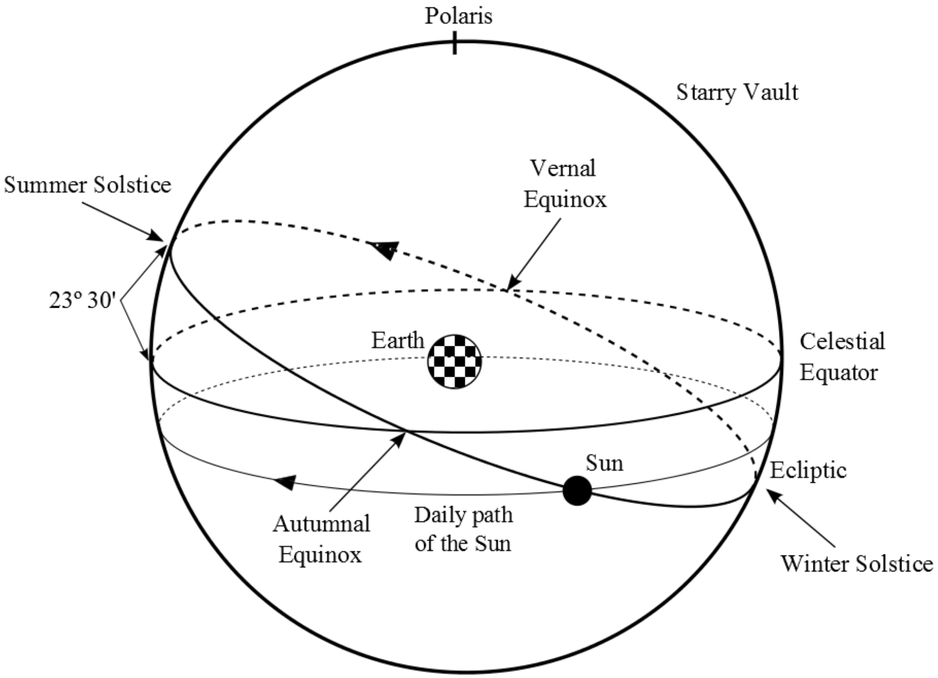
It goes through a minimum at a meridian direction (at noon time) while sweeping some angular range from sunrise to sunset.

Further, the shadow minimum length varies through the year, with times when shortest and Sun closest to vertical, at summer solstice, and times when longest, at winter solstice six months later.

Extreme directions at sunset and sunrise occur at the solstices, angular range swept at summer over 180°, and the opposite at winter.

Equinoxes occur in between, with collinear sunset / sunrise shadow directions, along the East – West line.

The Sun is directly overhead at the respective solstices of the Northern/Southern *Tropics* latitude (23.44°).



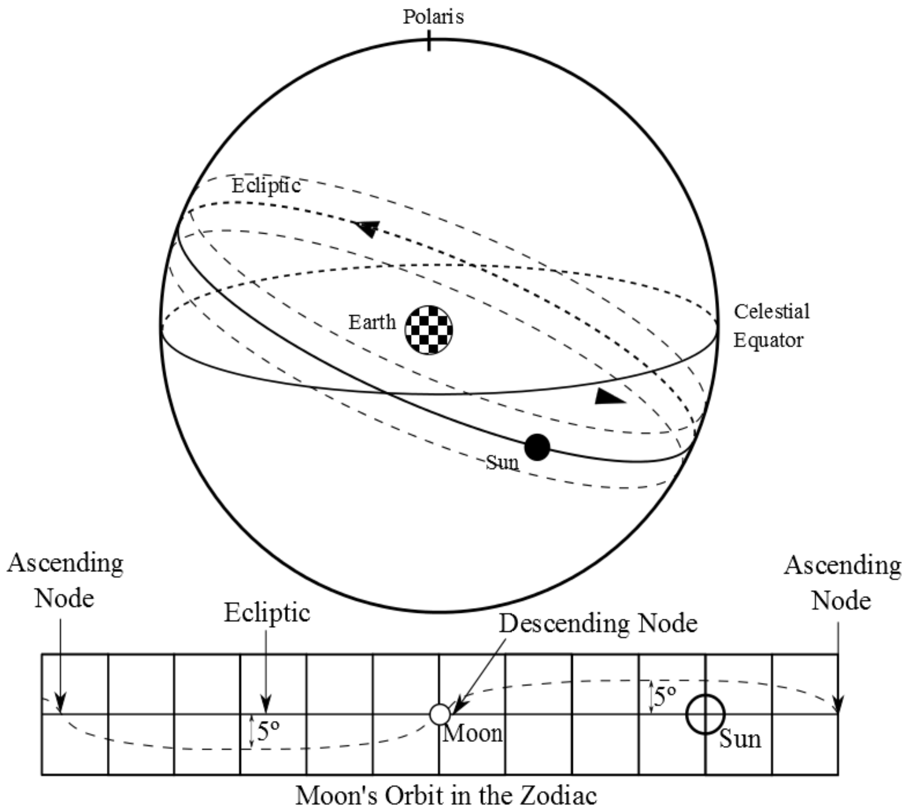
Geocentric picture

The gnomon allows students to determine the latitude of the point of observation, as given by half the sum of the solar distances to the vertical at the solstices.

Half the difference between those distances gives the inclination of the Earth equator to the plane of its orbit around the Sun, about 23.44°.

As the Sun moves slowly in the ecliptic, it also moves, in the opposite direction, making its turn around the Earth, for an observer on Earth.

Day and year periods differing greatly from each other by about two and half orders of magnitude (1 day as against 3.65×10^2 days) helps students to correctly visualize and interpret the observations.



The gnomon also serves to observe the Moon shadow at night, allowing students to determine the inclination of the lunar orbital plane,

as about 5 degrees away from the ecliptic.

This explains why eclipses are infrequent, requiring the moon to be at, or near, the ascending or descending

nodes for the eclipse to occur,

but the more so for solar eclipses (see later)

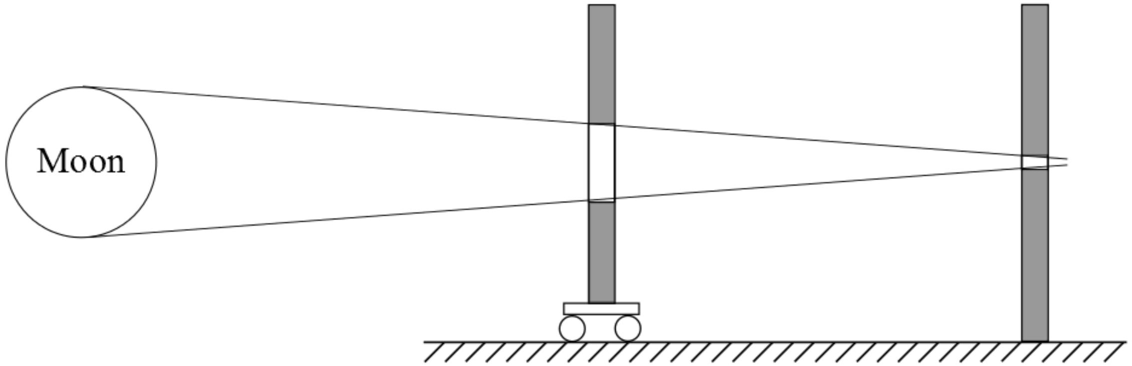
When students confront the difference they observe

in the duration of seasons,

Earth taking longer between spring and fall equinoxes than from fall to spring,

they learn about an old Greek 'explanation' (the *eccentric circle*)

and about Newton's explanation, the elliptic shape with the Sun in one focus.



Lunar angular width

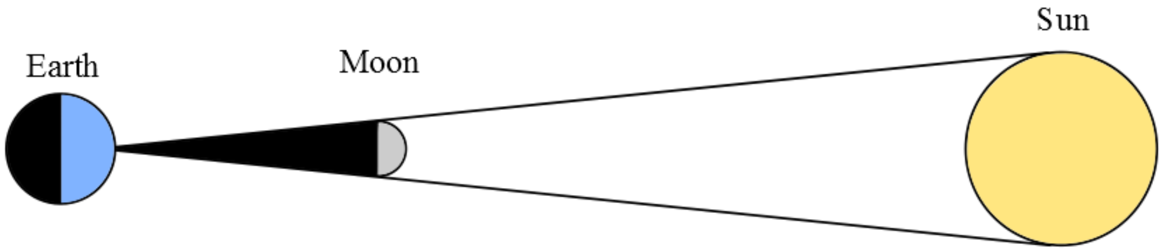
2) Measurements of angles in space, from the ground, though not being so easy, allow determining ratios among the most basic (solar, lunar) characteristic lengths in Astronomy.

Aristarchus of Samos, who apparently was the first author to propose an heliocentric system (18 centuries before Copernicus!), reported, in a classical 3rd century BC work,
observations involving 4 ratios among lunar and solar radii, lunar and solar distances to Earth, and radius of the Earth itself. Measuring this radius on Earth would then yield the other 4 lengths.

The easiest observation, involving the simplest optical "device", determines the angular width of the moon

$$2R_M / D_M \text{ about } 0.5 \textit{ degrees} \quad \rightarrow \quad R_M / D_M \approx \tan 0.25° \approx 4.4 \times 10^{-3}$$

where R_M and D_M are Moon radius and distance to Earth.



Solar eclipse

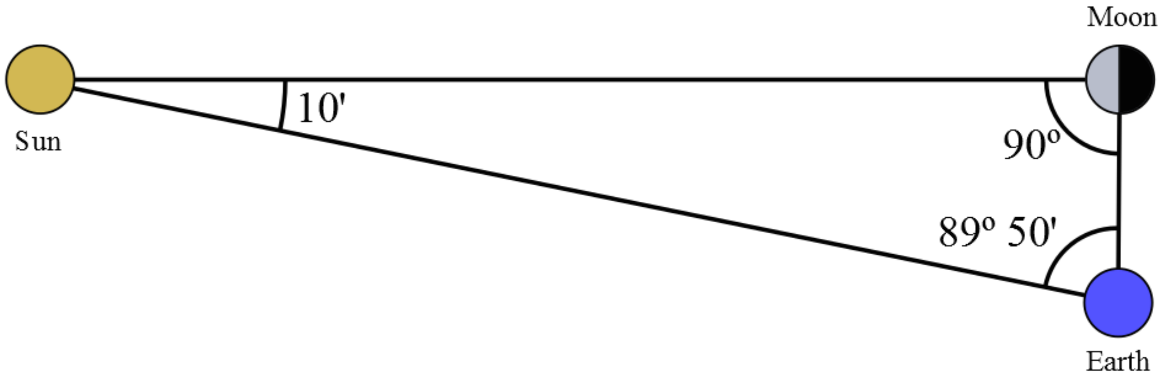
Solar eclipses prove the angular width of the Sun is similar to the angular width of the moon, about half a degree. This observation presents some issues:

1) Eccentricity of lunar orbit is sensible, around 0.055, resulting in not quite a full eclipse at lunar apogee
2) Solar eclipses are infrequent because of the about 5 degrees inclination of lunar orbit to the ecliptic, as previously mentioned
3) Finally, observation of the eclipse itself is a tough job, requiring protection because the Sun may be fully covered over a limited time

Anyway, the similar angular ranges provide a convenient equivalent ratio

$$R_M / R_S = D_M / D_S$$

where R_S and D_S are Sun radius and distance to Earth



Observation at half-moon

Observation at the half-moon condition when the angle Sun-Moon-Earth is 90°, may directly provide the ratio of distances to Sun and Moon.

This is a particularly tough job because the angle at Earth is extremely close to 90°, over 89°50'.

Aristarchus gave a value about 87°.

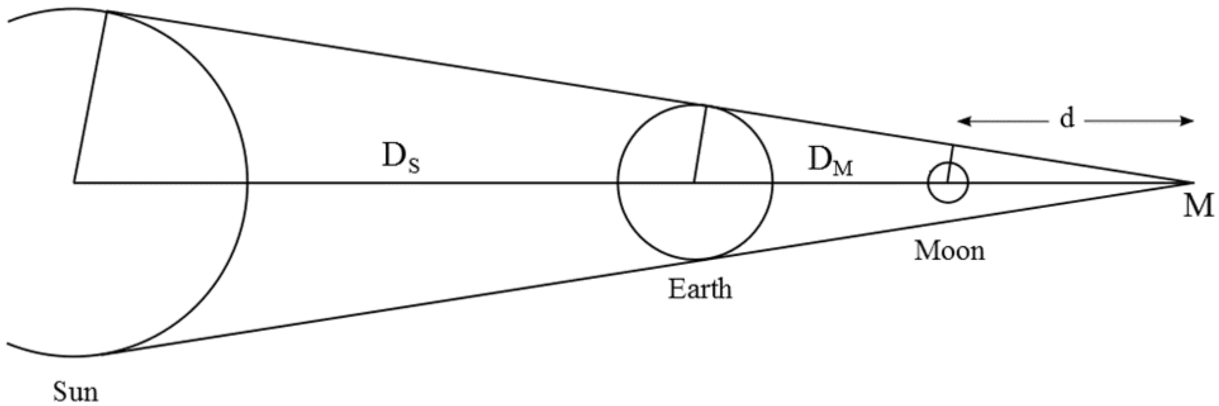
The ratio D_M / D_S is sine of angle at Sun, small in both cases but given by Aristarchus a value roughly

180' / 10' = 18 times too high.

The actual value is

$$D_M / D_S \approx \sin 0.15° \approx 2.6 \times 10^{-3}$$

which, as we saw, is also the value for the respective size ratio R_M / R_S.



In lunar eclipses the shadow cone at the Moon is 2.7 times as wide as the Moon itself.
Lunar eclipses are therefore both conveniently away from the Sun and more often observable.

Angular widths would be equal for Earth and Sun for observers at point M. Observation here relates Earth size to lunar and solar sizes and distances to Earth.

$$\frac{R_E}{D_M + d} = \frac{2.7 R_M}{d} = \frac{R_S}{D_S + D_M + d} \qquad (*)$$

The last equation gives $2.7 \times \frac{D_S + D_M + d}{d} = \frac{R_S}{R_M} = \frac{D_S}{D_M} >> 1 \quad \Rightarrow \quad \frac{2.7}{d} \approx \frac{1}{D_M}$

Equations (*) then yield $\frac{R_E}{R_M} = 2.7 \times \frac{D_M + d}{d} = 3.7$

About 1/3 century after Aristarchus, *Eratosthenes* estimated R_E from 1) distance, taken from Egyptian data, from *Alexandria* to *Aswan* (old *Syene* at the Northern Tropic), supposedly in the same meridian, and, using a gnomon, 2) solar distance to vertical at the summer solstice at Alexandria (null distance at Aswan).

References

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