On Use of Electrodynamic Tethers for Saturn Missions II

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1. Saturn versus Jupiter in Tether Operations

All *Giant Planets* have plasma **&** magnetic field **B** allowing *non-conventional* explorations.

Electrodynamic tethers (thermodynamic in character) can *a*) provide propellant-less drag for planetary capture and operation down the gravitational well,
and *b*) generate accompanying power, or store it to later invert tether current

Tethers are effective at Jupiter because its field **B** is high,
but the Saturn field is 20 times smaller and tether operation involves field ***B*** twice:

1. The S/C velocity v' relative to the magnetized planetary plasma induces in it a *motional* electric field $\boldsymbol{E}_m = v'B$, in the S/C reference frame (a XIX century, *Faraday* effect)
2. The field ***B*** exerts *Lorentz* drag per unit length ***IB*** on tether current ***I*** that $\boldsymbol{E}_m$ *drives*

But tethers at Jupiter operate with *tied hands*. Capture is more effective (higher mass-ratio M_{SC} / m_t), the lower the incoming S/C periapsis and the thinner and longer the tape tether.

This leads, however, to too hot tether and to attracted electrons crossing the tape and escaping collection. The maximum mass ratio for effective capture by Jupiter **might be about 3** [1].

Whatever the planet, Lorentz drag work $W_d < 0$ must take the incoming-orbit eccentricity from

$$e_h = 1 + v_\infty^2 r_p/\mu > 1$$

to a value $e_c < 1$ after capture (v_∞ incoming velocity, *periapsis* $r_p \approx$ planet radius R).

Velocity v_∞ is about 5.64 and 5.41 km/s for *Hohmann* transfers from *Earth* to *Jupiter* and *Saturn* respectively, a general energy balance reading

$$\frac{e_h - e_c}{e_h - 1} \times \frac{M_{SC}}{m_t} = \frac{|W_d|}{m_t v_\infty^2/2}$$

Both e_h and e_c being close to unity, drag work may be calculated along a parabolic orbit

$$|W_d| = (m_t v_\infty^2/2) \times \tilde{B}_s^2 \times w_d \qquad \text{(normalizing factor } \tilde{B}_s^2 \equiv \sigma_t B_s^2 a_s v_s / 2^{5/6} \rho_t v_\infty^2 \text{)}.$$

B_s and v_s are values at radius $a_s = (\mu / \Omega^2)^{1/3}$ of a stationary equatorial *circular* orbit where SC and corotating plasma velocities are equal: $a_s \approx 2.24R$ and $1.84R$ for Jupiter and Saturn respectively.

The drag arc extends on both sides of the periapsis, from r_p to $r_M = a_s \sqrt{2a_s/r_p}$, where the relative tangent velocity vanishes [2]

$$w_d \equiv \tilde{r}_M^{8/3} \int_1^{\tilde{r}_M} \frac{(\tilde{r}_M - \tilde{r})\, d\tilde{r}}{\tilde{r}^6 \sqrt{\tilde{r} - 1}} \langle 2 i_{av} \cos^2\phi \rangle_\phi \qquad (\tilde{r} \equiv r/r_p)$$

2. Can a tether at *Saturn capture a S/C with mass ratio of 3?*

For aluminum, $\tilde{B}_s^2 \approx 0.0083$ for Saturn and 2.11 for Jupiter. A gravity-assist from Jupiter in a *Hohmann* route to Saturn could shorten the trip **and** reduce v_∞, increasing $\tilde{B}_s^2$ by a 4.8 factor.

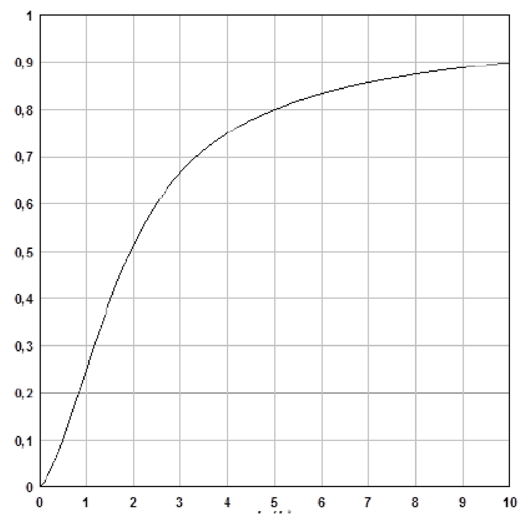
1) Averaging over angle between tether and $\boldsymbol{E}_m$ in w_d, arises from tether spin introduced for Jupiter to limit tether bowing [2]. For the Saturn weak-field no spin is required, w_d increasing by about 2.

2) Work w_d involves length-averaged tether current I_{av}, normalized with the short-circuit value
$i_{av}(L/L^*) \equiv I_{av} / \sigma_t E_m w h$
Length L^* gauges ohmic &
OML-collection impedances:

$$\frac{4}{3} e N_e \frac{2wL^*}{\pi} \sqrt{\frac{2eE_m L^*}{m_e}} \equiv \sigma_t E_m w h$$

$$\rightarrow \quad L^*(r) \propto (v'B/N_e^2)^{1/3}$$

$$i_{av} \approx \begin{cases} 1 - L^*/L, & L/L^* \geq 2 \\ L/4L^*, & L/L^* \leq 2 \\ 0.3 \times (L/L^*)^{3/2}, & L/L^* \ll 1 \end{cases}$$



Saturn and Jupiter operations present an important difference involving i_{av} and the Ref. [1] issues. Consider how maximum tether temperature varies with L and B [2],

$$T_{max} \propto (LB)^{3/8}.$$

Higher field B will require lower length L, **and** higher L^* ($\propto B^{1/3}$). The result is tethers operate in Jupiter at very small values of both L/L^* and current i_{av}, corresponding to negligible ohmic effects.
Saturn **does operate at values i_{av} = O(1)**, making for much larger drag-work value.

3) Prograde capture was implicitly assumed throughout. The short reach of Lorentz drag at Saturn (low a_s and $r_M = 3.53\, r_p$) suggest **retrograde** capture, just changing $r - r_M$ to $r + r_M$ with a gain of 2.

$$w_d \equiv 2\, \tilde{r}_M^{11/3} \times \int_1^{\tilde{r}_\infty} \frac{(1 + \tilde{r}/\tilde{r}_M)\, d\tilde{r}}{\tilde{r}^6 \sqrt{\tilde{r} - 1}} \times i_{av}(\tilde{r})$$

With i_{av} = O(1), the integral converges rapidly due to the r^{-6} power, whatever the upper limit. Write

$$w_d \equiv 2 \tilde{r}_M^{11/3} \times \langle i_{av} \rangle_r \times \int_1^{\tilde{r}_\infty} \frac{(1 + \tilde{r}/\tilde{r}_M)\, d\tilde{r}}{\tilde{r}^6 \sqrt{\tilde{r} - 1}}$$

The above integral increases from 0.971 to 1.008 in taking the upper limit from 1.5 to 2.
Overall, the particular Saturn effects discussed above lead to an energy equation

$$\frac{e_h - e_c}{e_h - 1} \times \frac{M_{SC}}{m_t} = \frac{|W_d|}{m_t v_\infty^2/2} \approx 8.1 \times \langle i_{av} \rangle_r,$$ **which is greater than 3 for $\langle i_{av} \rangle_r > 0.37$**

3. Plasma density modeling issues

Operation analysis at *Saturn* requires models for both magnetic field and plasma density N_e, in some distance-r domain around the planet. The first is represented by its well characterized dipole term. The opposite applies to N_e, in particular for the $r = 1 – 1.5\, R$ case of interest for tethers, which cover the ionosphere (reaching to about 10,000 km or 1.16 R) and the two inner, weaker rings *D* and *C*.

First, because the Coulomb cross section is much larger than the cross-section for collisions of atoms and molecules, the plasma exobase lies higher than the (*Chamberlain*) exobase of the neutral atmosphere. Yet, the electron distribution function may be not quite Maxwellian at the altitude range of interest here. It apparently might present a high energy tail, corresponding to a *Lorentzian* distribution function, characterized by a *kappa* value as low as 2, resulting at given temperature in higher N_e.

At *Orbit Insertion*, *Cassini* reached as close as 1.3 *R*, well within the domain of interest. Unfortunately, however, the plasma it met at Saturn cannot be considered representative because a variety of Saturn conditions favor surprisingly large changes in density, temperature, and ion composition. Its obliquity, about 26º, make for strong seasonal variations, while its sidereal period, 29.5 years, mean Saturn takes almost three 11-year solar cycles to go through the full seasonal variation.

On the other hand, the short, 10.6 hours sidereal day lead to fast plasma changes. The *optically thick* outer rings, *B* and *A*, reaching beyond 2 *R*, lie between Saturn and moon *Enceladus*, at about 4 *R* from Saturn. Day /night plasma parameters correspond to quite different conditions. Observations and modeling suggest that the UV sunlight on the rings dissociates molecules and atoms from the icy surfaces, and these low energy exospheric neutrals then ionize to form a ring exo-ionosphere in that side of the rings. Under unlit conditions the plasma content is anomalously low; on unlit faces, the electron absorbing rings determine the plasma content. The result is a large increase of plasma density from dawn to dusk.

Cassini Orbital Insertion took place past midnight, in mid-2004, when the spacecraft was in the northern hemisphere that had recently passed its winter solstice, and the solar cycle was approaching a minimum in 2008. The mission *Finale*, however, is taking place this very September 2017, at northern solstice, though again close to a solar-cycle minimum in 2019.

At present, after an abundance of experimental data and model attempts, there is no definite acceptable model for plasma density, as tether analysis requires. *Cassini*, now at end-of-life mission, may round up measurements at distances of interest and make basic contribution to modeling [3]-[6].

To escape all this variability and achieve efficient capture, ensuring the highest electron density possible, the need is evident of our tethered spacecraft arrival on solar max conditions, during one of the solstices (and to its respective hemisphere) and with the capture operation occurring on the diurnal side of Saturn.

There is a number of :Saturn missions proposed in the past. On Saturn itself: *Kronos, Saturn Atmospheric Entry Probe, Saturn Ring Observer*. On Titan: *AVIATR, Dragongly, TALISE, Titan Mare Explorer*. On Enceladus: *Enceladus Life Finder, Enceladus Explorer, Life Investigation for Enceladus*. On mixed objectives: *Journey to Enceladus and Titan, Titan Saturn System Mission.*

A new Proposal is sketched here, hoping that data on plasma density from the *Gran Finale* will allow completing it. Emphasis is in the capture and operations approach.

4. Gross estimates for concept of tether use at Saturn

Drag work $|W_d|$ only involves plasma density through the dimensionless length-averaged current i_{av} (inside the normalized work integral w_d), varying with the characteristic length L^*

$$L^*(r) = \left(\frac{3\pi h \sigma_t}{N_e}\right)^{2/3} \times \frac{(m_e v' B)^{1/3}}{2^{7/3} e}$$

as $i_{av}(L/L^*)$ given in the figure above.

Using $B = B_s\, a_s^3 / r^3$ (B_s =3.37 × 10^{-6} Vs/m^2), $v' = (v^2 + \Omega^2 r^2 + 2\Omega r_p v_p)^{1/2}$ (v_s = 26.2 km/s) yields

$$v'B = \frac{\tilde{r}_M^{4/3}}{2^{7/6}} \times v_s B_s \times \tilde{E}_M(\tilde{r}), \qquad \tilde{E}_M \equiv \left(\frac{\tilde{r}^3 + 2\tilde{r}_M \tilde{r} + \tilde{r}_M^2}{\tilde{r}^7}\right)^{1/2},$$

which, when used above, leads to a radial dependence $L^*(r) \propto \tilde{E}_m^{1/3} / \tilde{N}_e^{2/3}$, with the numerator continuously decreasing with increasing r, from 1.65 at r_p.
Writing $N_e = \tilde{N}_e \times 10^2$ cm^{-3}, and setting h = 10 μm, σ_t = 3.5 × 10^7/*ohm×meter* for *Al*, there results

$$L^*(r) \approx \frac{74\,km}{\tilde{N}_e^{2/3}} \tilde{E}_M^{1/3}.$$

Consider a typical density in the lower ionosphere, $N_e(r_p)$ = 10^3 cm^{-3}, leading above to $L^*(\tilde{r}=1) \approx 26\,km$, and select $L = 2L^*(1)$ = 52 km, yielding $i_{av}(2) \approx$ 0.5, well above 0.37.

Away from r_p, drops in density resulting in larger L^* and thus lower i_{av} at selected L can be partially compensated by the drop in $\tilde{E}_M^{1/3}$. Independently, contributions to the radial average, $\langle i_{av} \rangle_r$, become less relevant the farther away from r_p. Also, one may be just select a conveniently longer tape, with no penalty on the captured mass ratio. Farther, from the shape of the figure above, it follows that the effect of decreases in i_{av} values, from density drops, is smaller the larger L.

Finally, from the dependence $L^* \propto h^{2/3}$, a selected thicker tether tape, say moving h from 10 to 20 μm, could be compensated by using a tape longer by a factor $2^{2/3} \approx 1.6$.
Any increments in L and h can be balanced in tape mass m_t by a reduction in tape width w, which has no other effect in the analysis.

Conclusions
Tether capture and apoapsis-down operations are tentatively possible at Saturn.

References

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