APPLYING THE FORMULA FOR MASS DISTRIBUTION OF FRAGMENTS OF DISRUPTED BODY TO METEORITE SHOWERS

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Abstract

Formula for the cumulative mass distribution of fragments of a disrupted body is obtained as a function of the fragment mass, mass fraction of the largest fragment(s), the number of largest fragments, and the power exponent under the assumption of a power law for the probability density function.

This formula is used to describe the mass distribution of the recovered meteorites for eight meteorite showers: Tsarev, Sikhote-Alin, Mbale, Bassikounou, Almahata Sitta, Košice, Sutter’s Mill, and Chelyabinsk.

Based on a comparison of the empirical and theoretical fragment mass distributions, the most probable values of the power exponent are found.
Introduction & Background

To model the independent motion of meteoroid fragments it is necessary to know their mass distribution. An analogy can be drawn with impact experiments on high-speed collisions which were made to model asteroid destruction. In many experiments [1–4, and others], it was noted that cumulative mass distribution curve is described by a power law, but the whole curve usually cannot be represented by a single exponent in a power law and is divided into two or three segments.

Here, power law is used not for cumulative, but for probability density distribution. In this case, cumulative distribution enables to adequately describe results of impact experiments by single curve, i.e. using a single exponent. We compare the proposed cumulative distribution with mass distributions of recovered meteorites when a large number of them was collected.

Power law distribution in a discrete form was used for grain mass distribution in studies of small meteoroids [5, 6, and others]. Various approaches were applied to approximate mass distribution of found meteorites [7–9, and others].

Formula for cumulative mass distribution

We assume a power law for the probability density function and found the normalizing coefficient with use the equation of conservation of the total mass of all fragments $M$ (mass of the meteoroid just before breakup, mass of the target in experiments, mass of all meteorites).

Then probability density function $n_m$ is

$$n_m = M \frac{1-\beta}{m_l^{1-\beta}} m^{-\beta-1}$$  \hspace{1cm} (1)

$m$ is the fragment mass, $m_l$ is mass of the largest fragment, the power exponent $\beta$ is constant ($\beta < 1$).

The cumulative number of fragments $N_m$ with masses larger than or equal to $m$ is found by integration of equation (1) from $m$ to $m_l$

$$N_m = \frac{1-\beta}{\beta l^{1-\beta}} \left( \frac{m}{M} \right)^{-\beta} l^{-\beta} + c$$  \hspace{1cm} (2)

$l = m_l/M$ is the mass fraction of the largest fragment(s), $c$ is the number of the largest fragments.
MASS DISTRIBUTION OF RECOVERED METEORITES

**Tsarev**
December 6, 1922, USSR

- **69 fragments**
- $M = 1325.2$ kg
- $m_i = 283.8$ kg

- **dots** – data from catalog [10]

**Sikhote–Alin**
February 12, 1947, USSR

- **582 fragments**
- $M = 20177.6$ kg
- $m_i = 1745$ kg

- **lines** – formula (2):
  - $\beta = 0.5$
  - $\beta = 0.7$
  - $\beta = 0.75$

**Mbale**
August 14, 1992, Uganda

- **53 fragments**
- $M = 110.6$ kg
- $m_i = 27.4$ kg

- **dots** – data from catalog [12]
MASS DISTRIBUTION OF RECOVERED METEORITES

**Bassikounou**
October 16, 2006, Mauritania

- **Almahata Sitta**
(Asteroid 2008 TC₃)
October 7, 2008, Sudan

- **Sutter’s Mill**
April 22, 2012, USA

- **dots – data from catalog [13]**
  - 108 Fragments
  - $M = 62.38$ kg
  - $m_i = 6.1$ kg

- **dots – data from catalog [14]**
  - 662 fragments
  - $M = 10.5$ kg
  - $m_i = 0.379$ kg

- **dots – data from catalog [15]**
  - 77 fragments
  - $M = 942.7$ g
  - $m_i = 204.6$ g

Lines – formula (2):
- $\beta = 0.5$
- $\beta = 0.4$

Lines – formula (2):
- $\beta = 0.67$
- $\beta = 0.6$

Line – formula (2):
- $\beta = 0.5$
MASS DISTRIBUTION OF RECOVERED METEORITES

Košice
28 February 2010, Slovakia

- dots – data [17, 18]
- 218 + 5 hypothetical fragments (total mass of 2–2.5 kg) between two largest fragments ($m_f \approx 2.3$ kg) and the third one (0.318 kg)
  $M = 13.3$–$13.8$ kg

Chelyabinsk
15 February 2013, Russia

- dots – data [17, 18], not registered fragment [19], and 4 terminal masses of observed fragments [20]
- 1711 fragments
  $M = 144.19$ kg
  $m_f = 24.3$ kg
Discussion & Conclusions

Formula for the cumulative mass distribution of fragments of disrupted body is proposed as a function of fragment mass, mass fraction of the largest fragment(s), number of largest fragments, and power exponent. Cumulative mass distributions of recovered meteorites are constructed for eight meteorite showers: Tsarev, Sikhote-Alin, Mbale, Bassikounou, Almahata Sitta, Košice, Sutter's Mill, and Chelyabinsk. Comparison of these distributions with the proposed analytical distribution show that formula (2) adequately describes meteorite mass distribution in cases of uniform change of fragment masses without gaps.

In cases where there is the largest fragment(s), which is several times larger than the next one, formula (2) satisfactorily describes meteorite mass distribution starting from the next fragment.

Difference between analytical and empirical distributions of found meteorites at very small masses is natural and should be, because, unlike laboratory experiments, it is problematic to find most small particles.

Preliminary estimate of the most probable range of exponent $\beta$ for meteorite distributions is from 0.5 to 0.7, but further research is needed to determine more accurately the range of possible $\beta$ values.

Formula for the probability density function can be used to model meteoroid fragmentation; the total mass and energy deposition can be found by integration over all fragment initial masses.

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