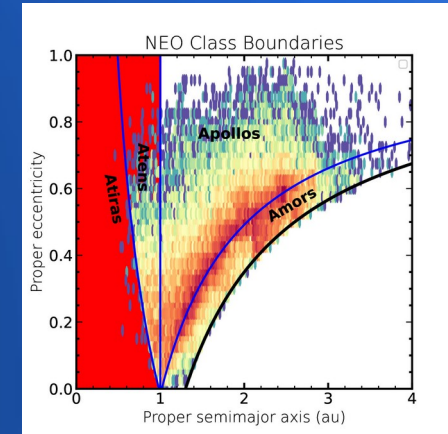
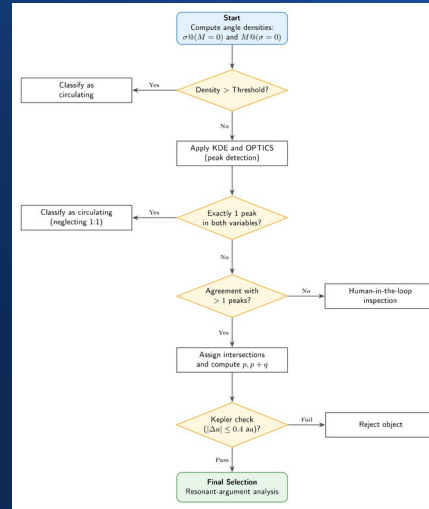
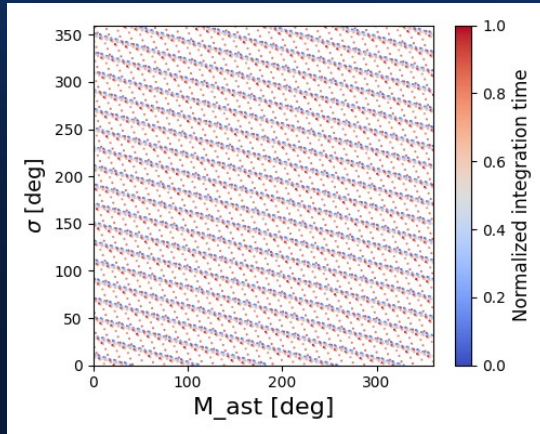


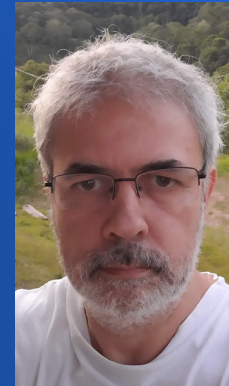
ML-FAIR

A Machine Learning Implementation of the Fast Identification of Mean-Motion Resonances (FAIR) Method



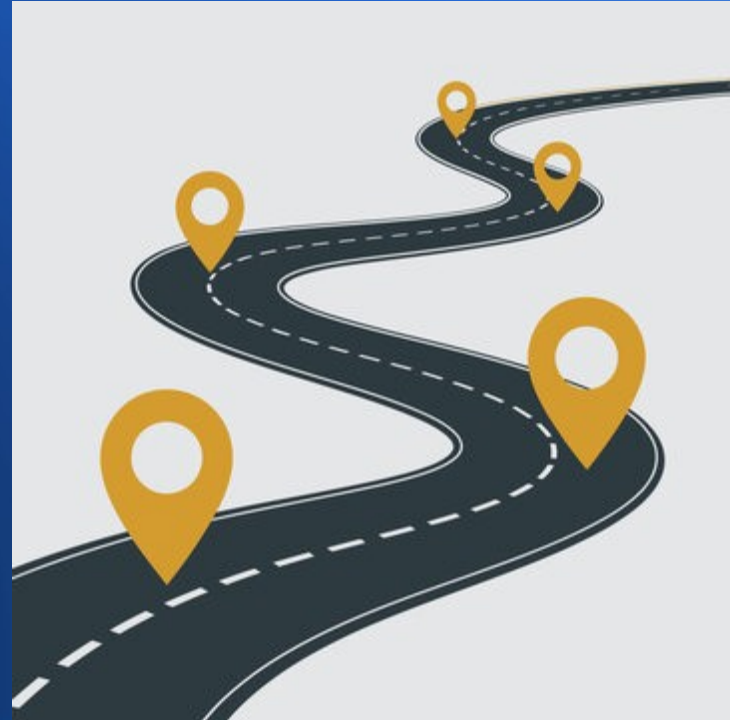
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Roadmap

1. Problem: Resonance identification in NEOs
2. FAIR method (geometric idea)
3. ML-FAIR (automation)
4. Results
5. Conclusions



Introduction

We introduce ML-FAIR (Machine Learning FAIR method)

Automates identification of mean-motion resonances

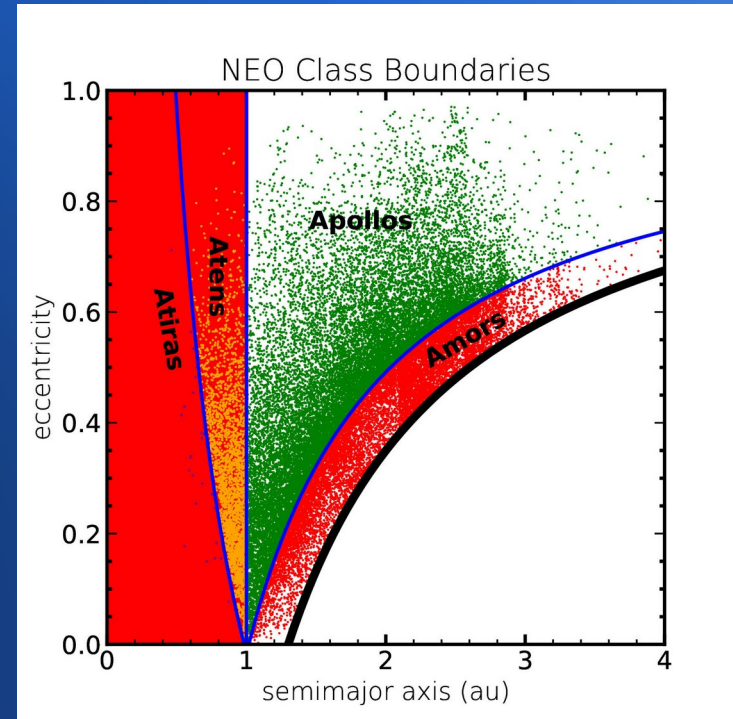
- Uses geometric patterns in angle–angle phase space
- Applies density estimation + clustering
- Eliminates manual inspection of stripe patterns

Applications to Near-Earth Asteroids (NEAs):

- Efficient analysis of large asteroid datasets
- Detects resonances with terrestrial planets
- Works in chaotic orbital regimes (Atens, Atiras)
- Scalable for next-generation surveys (e.g., LSST)

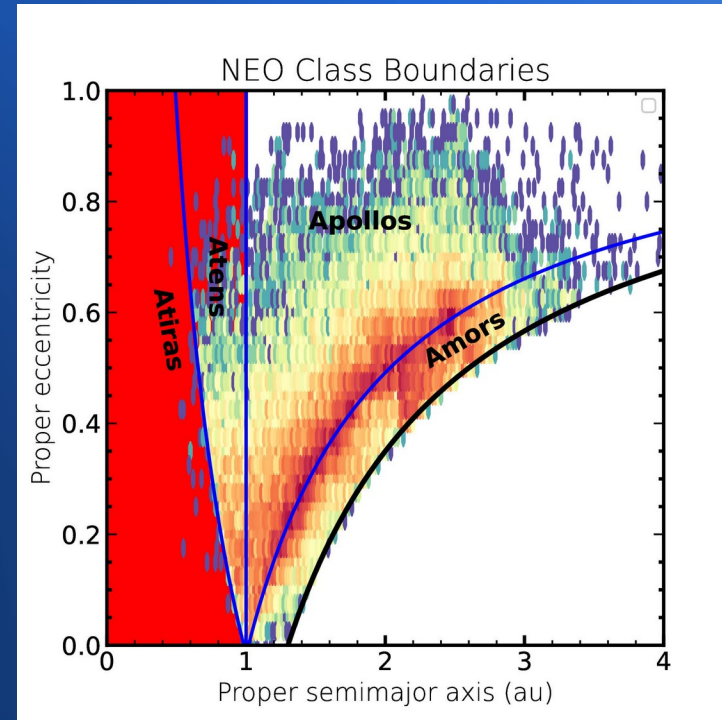
NEOs

- **Classification**
- NEOs are classified based on their orbit relative to Earth:
- **Amors:** $q_e < q < 1.33$ au, Always outside Earth's orbit.
- **Apollos:** $a > a_e$ and $q < Q$, Earth-crossing objects, Larger semi-major axis than Earth.
- **Atens:** $a < a_e$ and $Q > q_e$, Earth-crossing objects, Smaller semi-major axis than Earth.
- **Atiras:** $Q < q_e$, Entire orbit inside Earth's orbit
- **Population fractions (~35k objects)**
- Apollos: 56.6%, Amors: 35.4%, Atens: 7.9%, Atiras: 0.1%



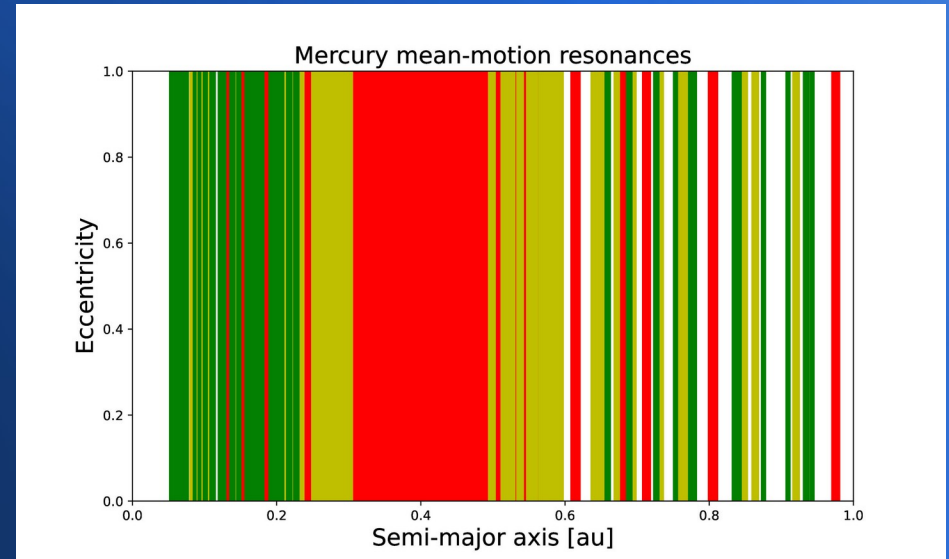
NEOs

- **Key dynamical features:**
- Highly chaotic orbits, Close encounters with terrestrial planets
- Typical Lyapunov timescale: ~150 years
- Standard long-term orbital invariants cannot be computed
- **Implications for resonance studies:**
- Chaotic evolution makes resonance identification difficult
- Long-term orbital classification uncertain
- **Study focuses on:**
- Atens & Atiras ($a < 1$ au)
- Region most relevant for inner Solar System resonances



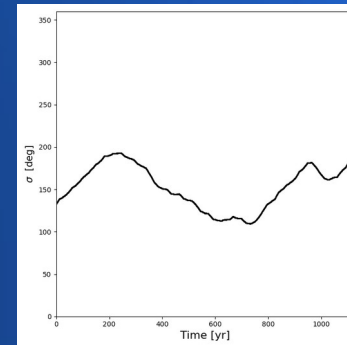
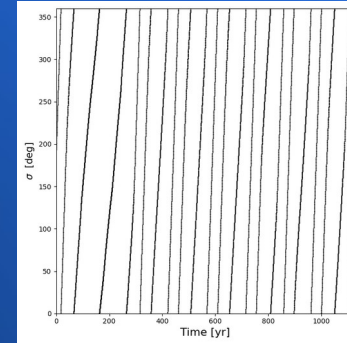
The Forest of Resonances with Terrestrial Planets

- Near-Earth region with $a < 1$ au contains a very large number of mean-motion resonances (MMRs).
- Resonances defined by ratio $p/(p+q)$, where $q = \text{order}$.
- Up to order 20: Mercury (225), Venus (177), Earth (127), Mars (66).
- Inner vs outer resonances depend on relative semi-major axes.



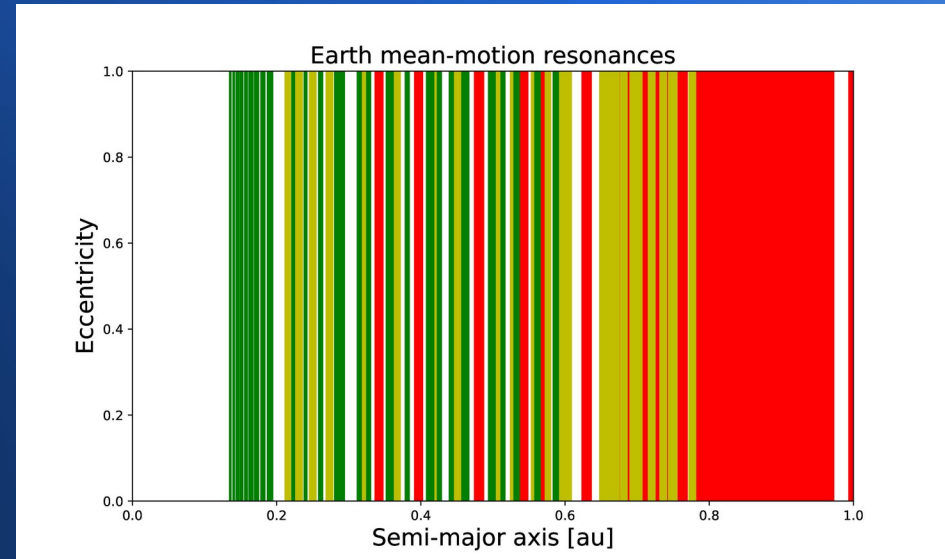
The Forest of Resonances with Terrestrial Planets

- Large number (~595 relevant resonances) makes analysis difficult.
- Traditional method: compute resonant arguments over time.
- Requires checking libration vs circulation behavior.
- Computationally expensive and impractical for large datasets.



The Forest of Resonances with Terrestrial Planets

- Automated pipeline (Smirnov 2023): numerical integration + statistical analysis.
- Uses libration detection and periodograms.
- Reduces false identifications.
- Another possible approach based on an efficient geometric alternative is the FAIR method (Forgács-Dajka et al. 2018).



FAIR method overview

- **FAIR = Fast Identification of Mean-Motion Resonances**
- Key idea:
-  Don't analyze many equations
-  Analyze geometry in angle–angle space: Resonances produce structured patterns (stripes)
-  Replace: Time-series analysis → Pattern recognition, we are replacing time evolution with geometry.
- Resonance condition:

$$\frac{n'}{n} \simeq \frac{p}{p+q}$$

- n and n' are the mean-motions of an asteroid and a planet
- p, q : integers, q = resonance order

FAIR

- For eccentricity-type resonances, we can write the resonant argument for inner resonances as:

$$\sigma = (p+q)\lambda' - p\lambda - q\varpi$$

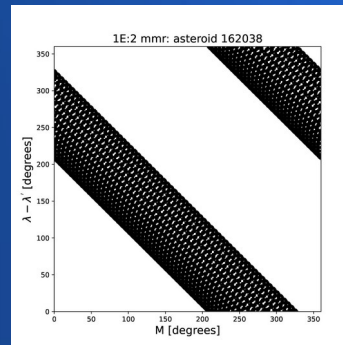
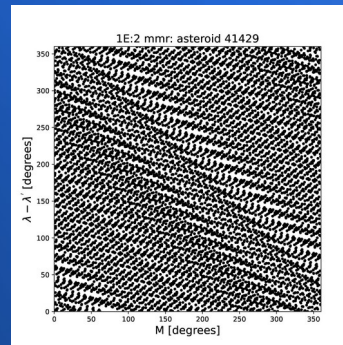
- And, for outer resonances, as:

$$\sigma = (p+q)\lambda - p\lambda' - q\varpi$$

- Here, λ and λ' are the mean longitudes of the asteroid and a planet. ϖ is the longitude of perihelion of the asteroid.
- Resonance $\Rightarrow \sigma$ librates (oscillates).
- Non-resonance $\Rightarrow \sigma$ circulates.

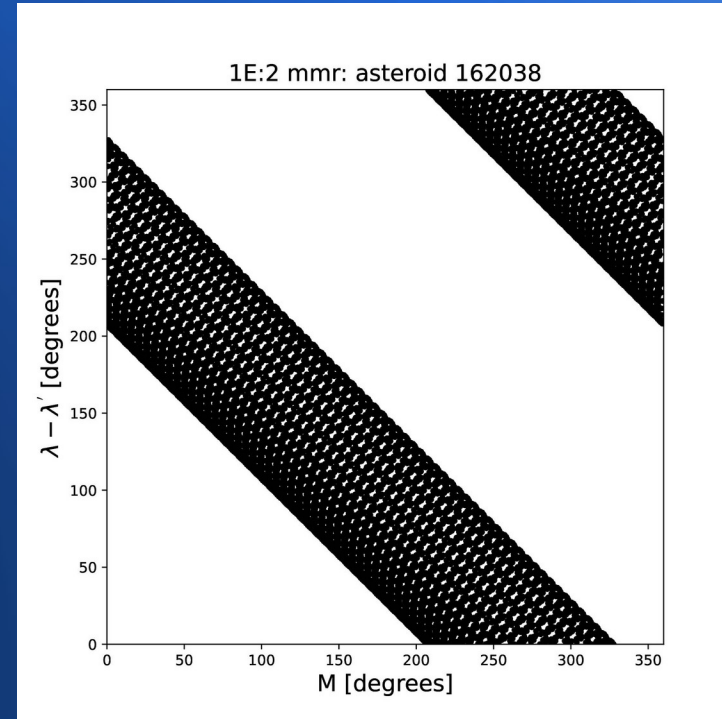
Angle–angle representation

- The FAIR method consists of analyzing:
- $\lambda' - \lambda$ versus M for inner resonances,
- $\lambda - \lambda'$ versus M for outer resonances.
- Circulation: points in these plots fill the plane approximately uniformly.
- Resonant motion, the points align along oblique stripes.



How to read a resonance from stripes

- The number of stripe intersections with the coordinate axes encodes the resonance integers p and q .
- For an outer $p:(p+q)$ resonance:
- Horizontal intersections: q
- Vertical intersections: p
- Example: one horizontal and one vertical intersection correspond to $q=1$ and $p=1$.
- Therefore, $p+q=2$, identifying the 1E:2 resonance.



Angle–angle representation

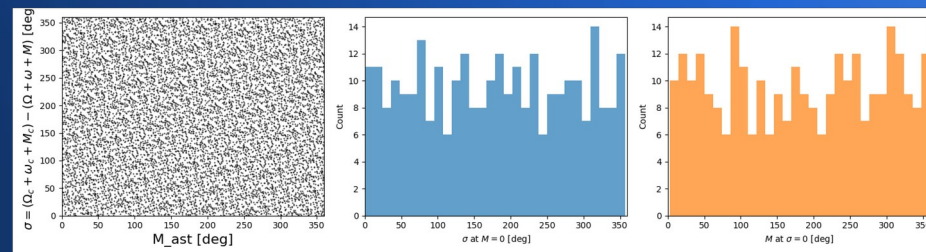
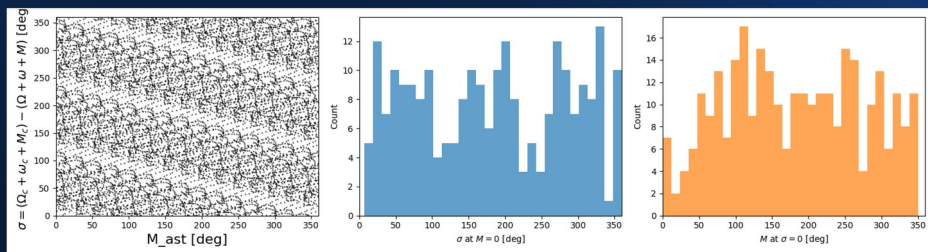
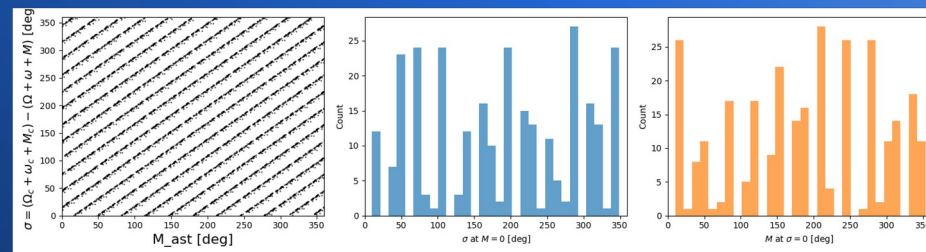
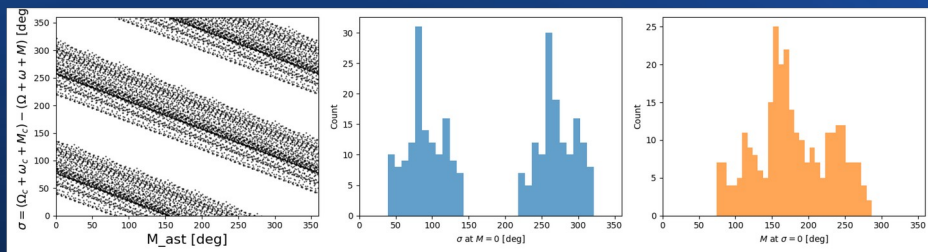
- Why FAIR is Powerful?
- **Content:**
- Only 2 plots per planet needed.
- Works in crowded dynamical regions.
- Converts physics $\rightarrow$ pattern recognition problem.
-  Enables machine learning (ML-FAIR).

Type	Resonance variable	Plot	Hor	Vert
Inner	$(p + q)\lambda' - p\lambda - q\varpi$	$\lambda' - \lambda$ versus M	q	$p+q$
Outer	$(p + q)\lambda - p\lambda' - q\varpi$	$\lambda - \lambda'$ versus M	q	p

ML-FAIR

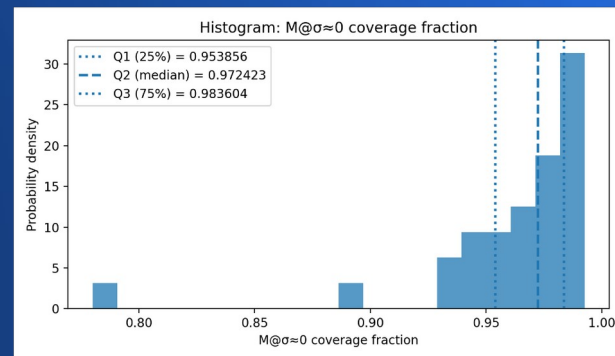
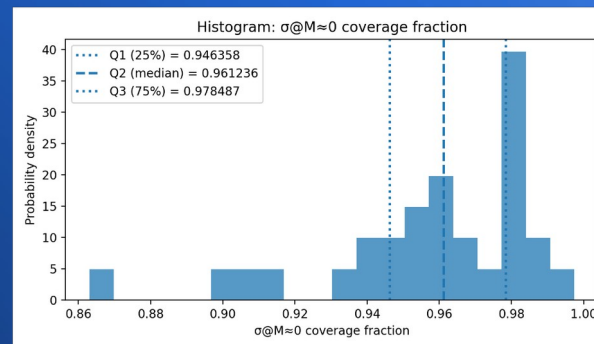
- Goal: Automate FAIR resonance identification using machine learning.
- Key idea: Count stripe intersections in (M, σ) space automatically.
- Transform 2D problem $\rightarrow$ 1D angular distributions.
- Instead of analyzing stripes directly, we sample where they cross the axes — converting geometry into distributions.
- Define $\sigma@ (M=0)$ and $M@ (\sigma=0)$ variables.
- Use tolerance window ($\approx 5^\circ$) to collect angular samples.

ML-FAIR



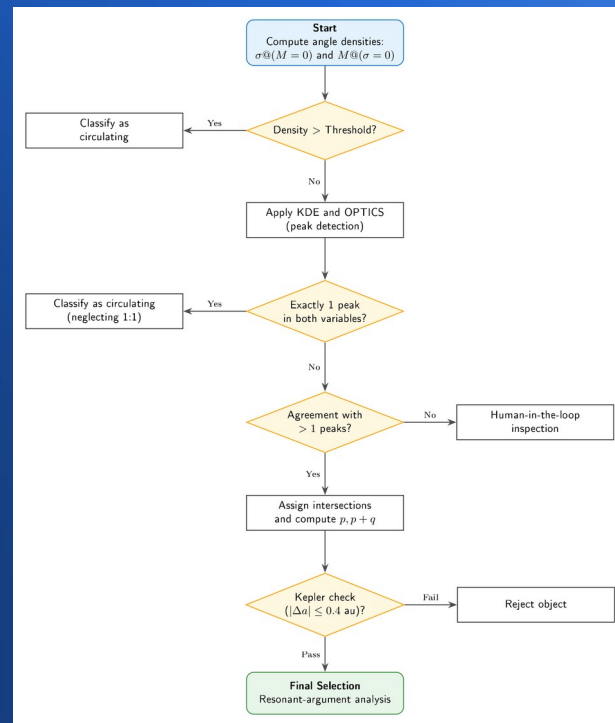
ML-FAIR

- Four behaviors: librating (low/high crossings), switching, circulating.
- Use angular density coverage on circular domain.
- Empirically, circulating orbits fill >95% of the domain.
- Threshold (95%) separates circulating vs others.
- Non-circulating → proceed to clustering analysis.

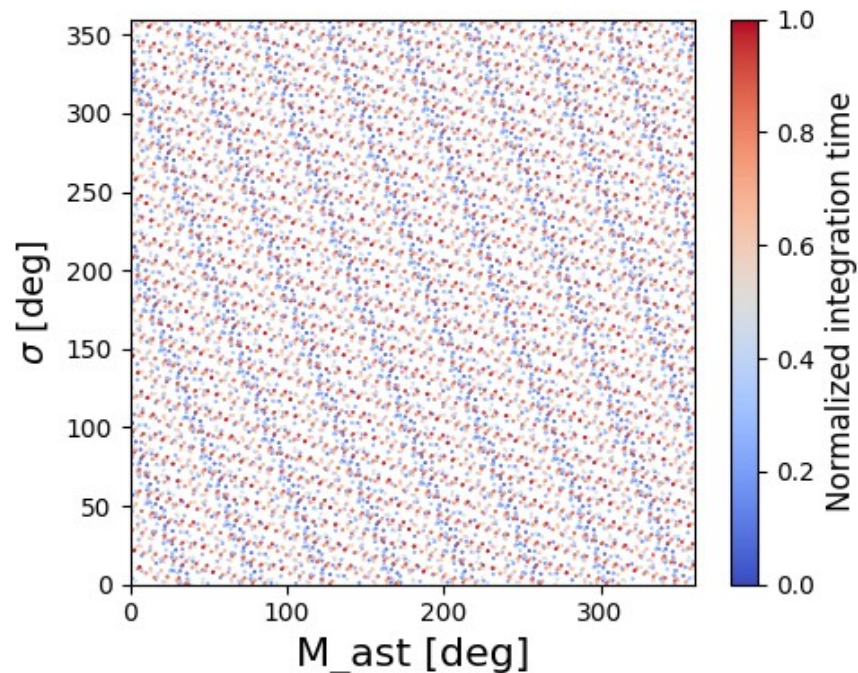
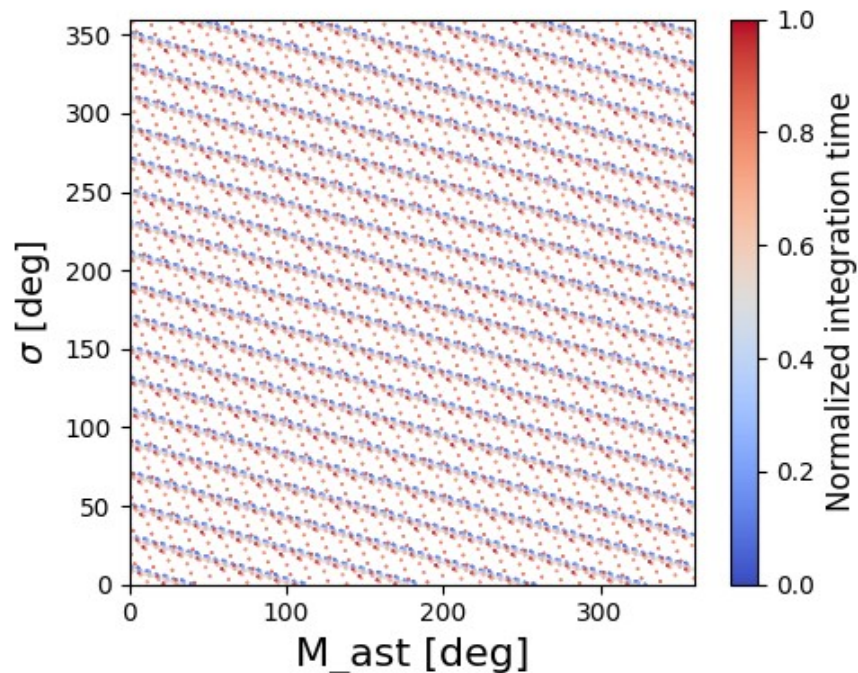


ML-FAIR

- Apply unsupervised methods: OPTICS (Ankerst et al. 1999) and circular KDE (Hall et al. 1987), the best-performing methods.
- Detect peaks in angular distributions → count intersections.
- Ensemble approach improves robustness.
- In case of disagreement, → human-in-the-loop classification.
- Steps: density filter → clustering → decision logic → resonance ID
- Final validation: compare with orbital parameters (physical consistency).



ML-FAIR: human-in-the-loop



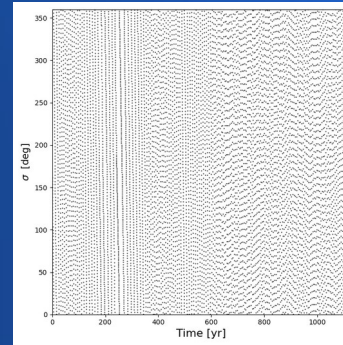
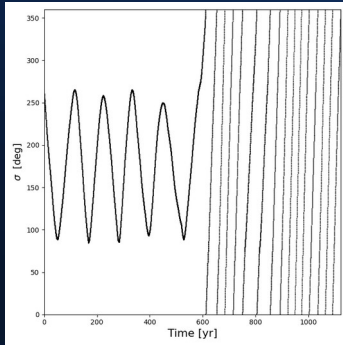
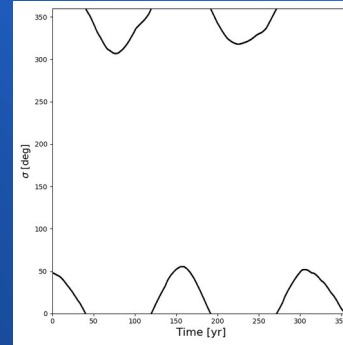
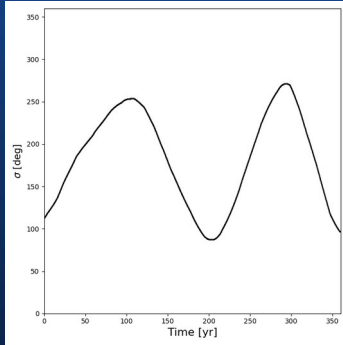
Results: goals

- Focus on Atira and Aten asteroids ($a < 0.98$ au).
- Analysis of mean-motion resonances with terrestrial planets.
- Application of ML-FAIR for automated detection.
- Benchmark comparison with existing methods (Smirnov 2023).

Results

- N-body integrations using REBOUND (Bulirsch–Stoer).
- Integration timescale: 1200 years.
- Includes all planets + Moon.
- Chaotic dynamics: short Lyapunov times:
(~150 yr) ➡ Limits on the classification of a single orbit.
- We plotted the resonant arguments of all candidates:
four possible behaviors.

Results



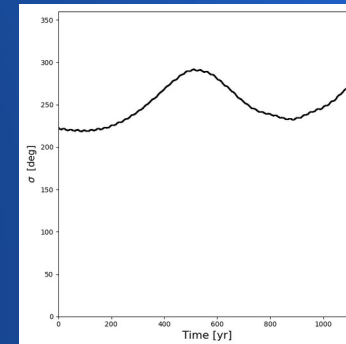
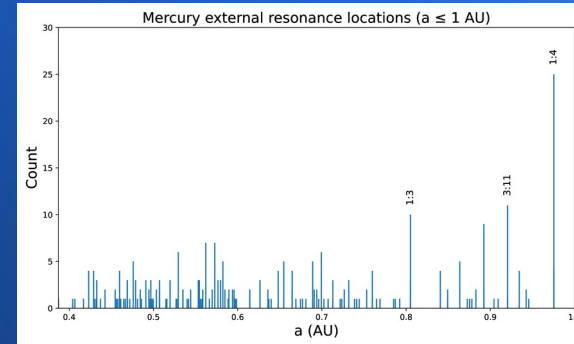
Results

- High agreement with benchmark resonance identification.
- Automates stripe detection and resonance classification.
- Reduces need for manual inspection: 85% of images automatically classified.
- Robust across librating, switching, and circulating cases.

Planet	Flagged Images	Fract. Auth. Recovered
Mercury	366	87.3%
Venus	397	85.4%
Earth	268	90.7%
Mars	243	91.6%

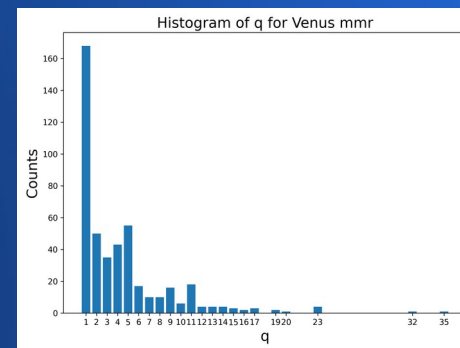
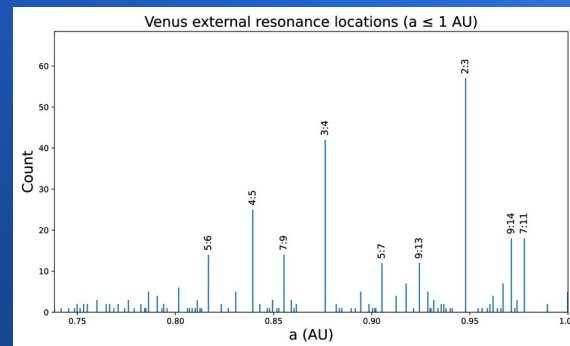
Mercury MMR

- ML-FAIR identified 171 asteroids possibly in MMR with Mercury.
- Only three resonances have a candidate population of 10 asteroids.
- The analysis of the resonant arguments of asteroids in MMRs with Mercury identified only two possible librating objects, in the 1Mer:4 and 1Mer:3 MMRs.
- 466507 (2014 FK33) is in the 1Mer:4 MMR, while 2006 SE6 is in the 1Mer:3 MMR.
- They were found to be in a rare libration orbit around the 270° equilibrium point.
- These two asteroids could be the first-ever objects identified in MMR with Mercury.



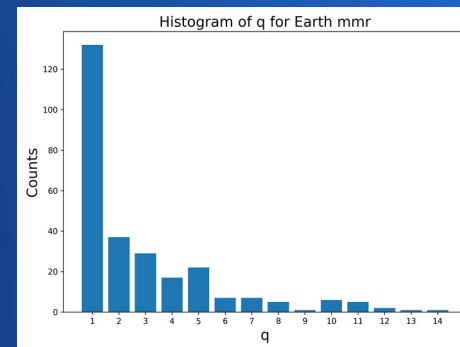
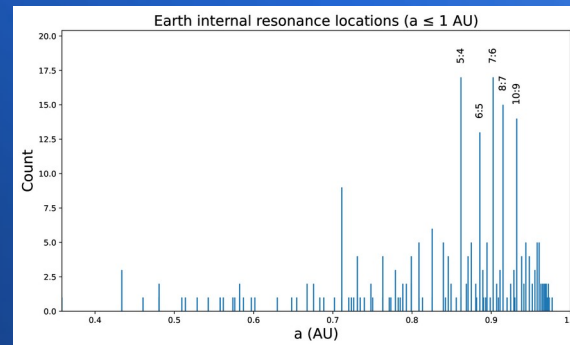
Venus MMR

- ML-FAIR identified 141 possible MMRs with Venus, 9 of which have a population of 10 candidates or more.
- Several high-order resonances with $q > 10$ were identified for Venus.
- Results are in good agreement with Smirnov (2023).



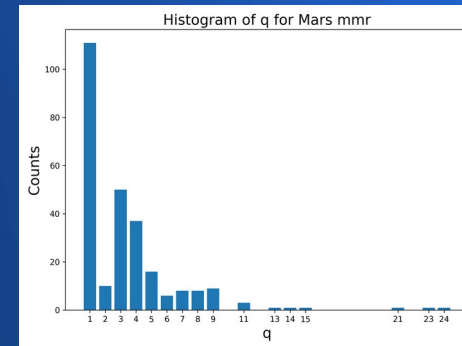
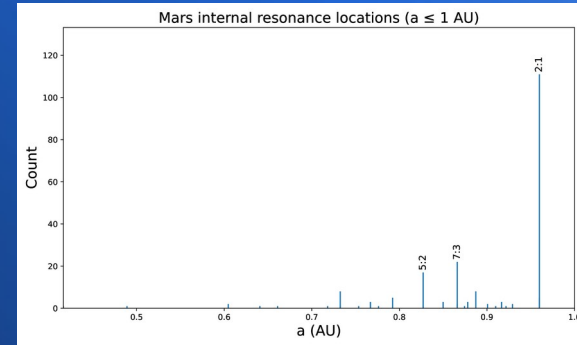
Earth MMR

- Five resonances have 10 candidates or more: the 5E:4 (17 candidates), the 7E:6 (17), the 8E:7 (15), the 10E:9 (14), and the 6E:5 (13).
- ML-FAIR identified only 8 resonances with order q higher than 10.
- The number of asteroids in libration states is fairly limited, with the 8E:7 and 10E:9 MMR having the largest number of librating objects, 6.
- Again, good agreement with results from other methods.



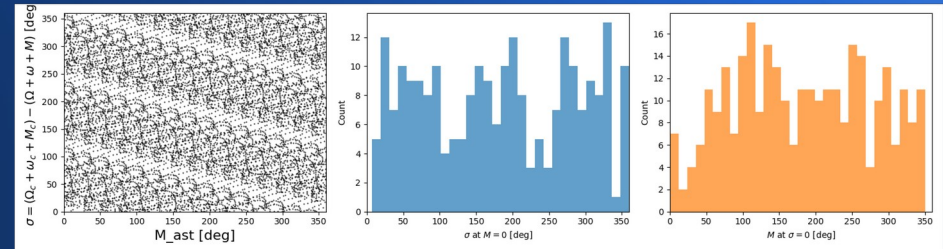
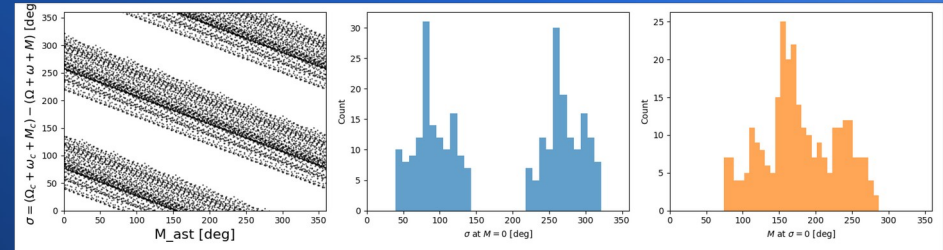
Mars MMR

- There are three resonances with a candidate population of 10 asteroids or more: the 2M:1, 7M:3, and 5M:2.
- Just 9 high-order resonances with one candidate asteroid or more.
- Again, good agreement with results from previous methods.



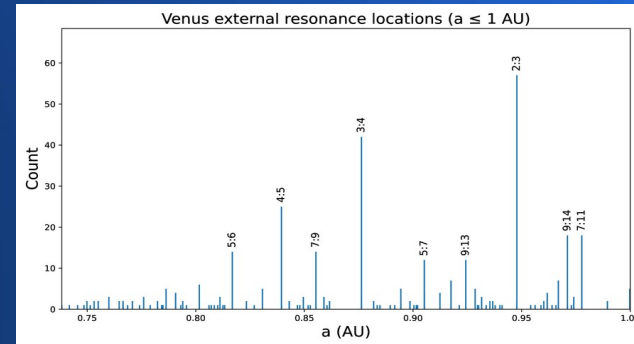
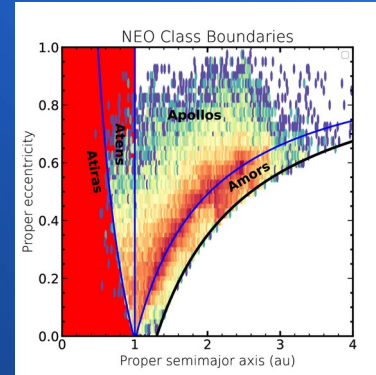
Conclusions

- ML-FAIR successfully automates FAIR resonance identification.
- Detects mean-motion resonances using geometric pattern recognition.
- Achieves high agreement with traditional methods.
- Greatly reduces need for manual inspection.
- Efficient for large asteroid datasets.



Conclusions

- Enables scalable analysis of NEAs in the big-data era.
- Particularly effective for chaotic populations (Atens, Atiras).
- Applicable to next-generation surveys (e.g., LSST).
- Can be extended to higher-order and multi-body resonances.
- Future: improved ML models and real-time resonance detection pipelines.



Thank you for your attention

- Take-away message: “**ML-FAIR bridges celestial mechanics and machine learning, opening the door to real-time dynamical classification in next-generation asteroid surveys.**”
- More information on this work can be found at:
- **Carruba, V., Aljbaae, S., Caritá, G., Domingos, R. C., Bala, M. M., Pedroso, R. D. Z., and Delfino, E. M. D. S. (2026), “ML-FAIR: An Automated Machine-Learning Framework for the Fast Identification of Eccentricity-Type Mean-Motion Resonances,” *Celestial Mechanics and Dynamical Astronomy*, 138, 39.**
- The codes developed for this work are available at:
- **<https://github.com/valeriocarruba/ML-FAIR>**

