Cassini states and librations of the Galilean satellites: a numerical approach in Euler angles

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INTRODUCTION

The problem of determining the correct initial conditions for the numerical integration of the rotational motion of a celestial body is a fundamental one. Indeed, even a slightly off initial guess can excite free rotational modes (i.e., free precession and free libration) with amplitudes much larger than the forced amplitudes. Tidal dissipation effects over long timescale have most likely damped all free modes, although small amplitudes at the free frequencies are still possible due to recent excitation mechanisms. The problem of the initial conditions for the librations can be avoided entirely by using linearized models, which allow to separate the solution to the differential equations as the superposition of a free libration term, that depend on the initial conditions, and a forced libration term that is independent of the initial conditions, allowing to determine an approximate solution in closed form. A similar linearized approach can be used to model obliquity. We employ the full triaxial rotational equations for the Euler angles that define the orientation of the Galilean satellites' principal axes in an inertial frame. To solve the problem of the initial conditions we propose an approach that makes use of the differential state transition matrix to determine initial conditions that lead to zero amplitudes of the free modes. We determine a set of three libration parameters for each of the four Galilean satellites. These results are useful as an accurate reference rotation model for a solid satellite, and can serve as a basis for comparison with the libration and obliquity estimates that will be obtained from the Juice and Europa Clipper data in the 2030s, which will study in detail Ganymede and Europa, respectively.

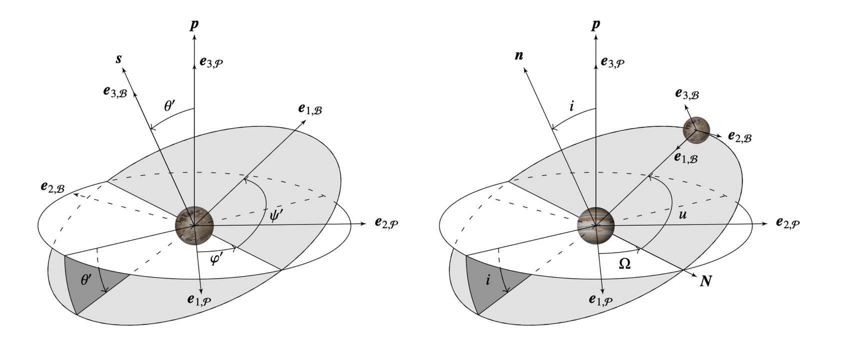


Fig. 1: Schematic representation of the relations between the Euler angles, Laplace frame $\mathscr{P}$, principal axis frame $\mathscr{B}$ (left). Representation of the relationship between the orbit, and the orientation of these reference frames (right).

CASSINI STATES

The empirical laws that elegantly describe the Moon's rotation, commonly known as Cassini's laws, first appeared in a treatise by Giovanni Domenico Cassini in 1693 [1]. They can be summarized as follows [1–3]:

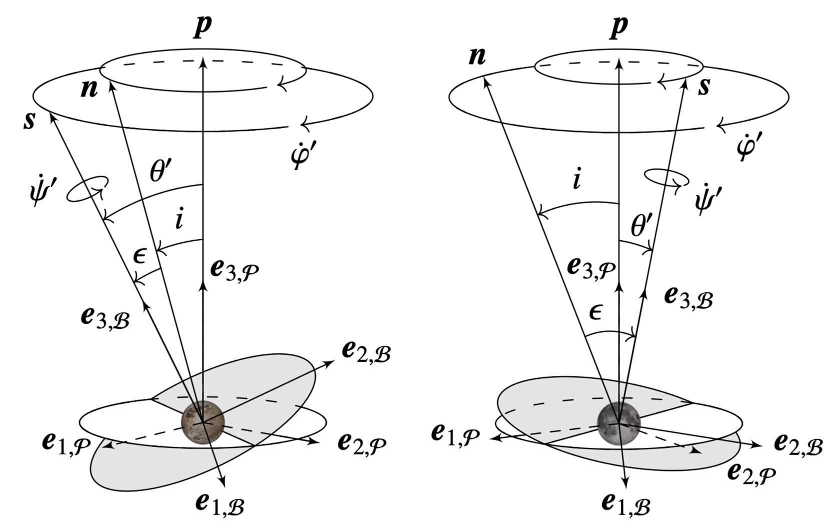
1. The period of the Moon's rotation about its polar axis equals its mean sidereal orbital period around Earth.
2. The descending node of the lunar equator on the ecliptic precesses at the same rate as the ascending node of the lunar orbit on the ecliptic.
3. The inclination of the lunar equator with respect to the ecliptic remains constant.

We define the perturbations to the Euler angles relative to the exact Cassini laws as three parameters τ, σ and ϱ, such that (see, e.g., [6-7])

$\theta' = \iota + \varrho,$

$\phi' = \Omega + \sigma,$

$\psi' = u + \pi - \sigma + \tau,$



(a) Cassini state I (b) Cassini state II

Fig. 2: Cassini states I and II differ based on whether ω and n are on the same side of p, or on opposite sides, respectively (see e.g., [4, 5]). All the Galilean satellites, contrary to Earth's Moon are expected to occupy a Cassini state I.

where, ι is the mean inclination of the satellite's equator with respect to the Laplace plane normal (p). Then, the corresponding librational parameter (ϱ) is the departure of θ′ from ι. We denoted by u the mean argument of latitude of the satellite and by Ω the mean longitude of the ascending node of the satellite's orbit.

Satellite	T_Ω (days)	T_ϖ (days)	T_n (days)
Io	2752.8597	486.7647	1.7691
Europa	10982.6531	486.8374	3.5512
Ganymede	48362.3384	51565.0101	7.1545
Callisto	165843.2841	215545.7111	16.6891

Table 1: Periods of the nodal precession (T_Ω), apsidal precession (T_ϖ) and orbital period (T_n) of the Galilean satellites.

FREE MODES

Even without Jupiter exerting a time-variable torque, the satellites would oscillate regardless at discrete frequencies with amplitudes dependent on the initial conditions [6]. There are three free modes which correspond to the eigenfrequencies of the linearized equation [7]. These are a free longitudinal libration mode, a free wobble, and a free precession (e.g., [8]). The amplitudes of the three modes depend on the initial conditions and are expected to be small due to dissipative effects, although they may have been excited in the recent past due to impacts, or other effects not modeled here. We aim to determine the initial conditions corresponding to the forced equilibrium, that is, the state in which the amplitudes of the free modes are minimized. Indeed, unless a recent excitation mechanism is present, it is reasonable to assume that these free modes have been damped by dissipative effects acting over long timescales. In order to determine the initial conditions that result in no free librations and precession we employ a differential correction technique based on the dynamical state transition matrix [9]. To this aim, the free frequencies components need to be identified. First an initial guess for the amplitudes and frequencies is obtained by performing a fast Fourier transform (FFT), to identify the peaks in the frequency spectrum corresponding to the free modes of the linearized equations. Then these frequencies and the corresponding amplitudes are used as an initial guess for a nonlinear least-squares fitting [10, 11]. Then the residuals are computed, and the same method is applied to the residuals. Lastly the frequencies are joined together.

RESULTS

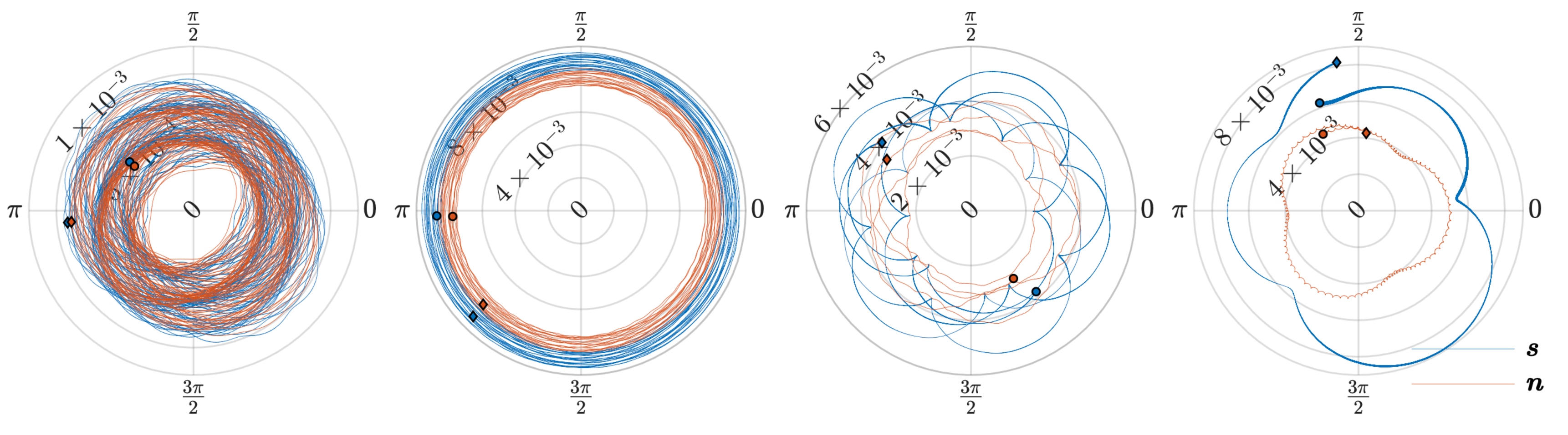
We determined the initial conditions needed to numerically integrate the triaxial rotational equations. Three parameters, corresponding to the latitude and

Satellite	$T_{f,\varphi'}$ (days)	$T_{f,\theta'}$ (days)	$T_{f,\psi'}$ (days)	T_{f,w_+} (days)	T_{f,w_-} (days)
Io	149.4568	0.8794	13.2828	1.7553	1.7832
Europa	1172.1270	1.7729	52.5845	3.5441	3.5583
Ganymede	7291.8584	3.5755	186.5077	7.1499	7.1592
Callisto	74170.8315	8.3436	897.5149	16.6866	16.6914

Table 2: Free modes periods of the Galilean satellites assuming a rigid body model.

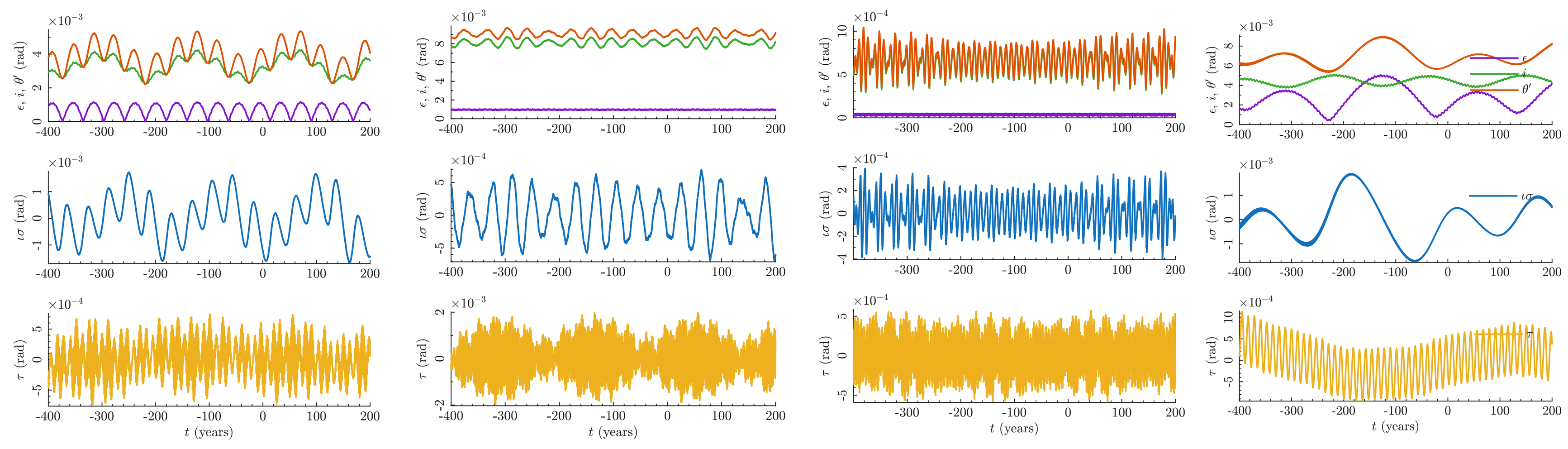
longitudinal librations, that is the deviation from the Cassini state I (Fig. 2a), are determined and a frequency analysis (not shown here) is performed.

In Fig. 3 we show the spectra of the Euler angles before and after the differential correction, showing that the free modes are effectively removed from the



(a) Io (b) Europa (c) Ganymede (d) Callisto

Fig. 4: Co-precession of the spin axis (**s**) and orbit normal (**n**) over a time period of 600 years. Units are radians.



(a) Io (b) Europa (c) Ganymede (d) Callisto

Fig. 5: Evolution of the obliquity (ϵ) in purple, orbital inclination (i) relative to the Laplace plane in green, and the libration parameters $\iota\sigma$in blue, τ in yellow, and $\theta' = \iota + \rho$ in red, of each Galilean satellites.

spectra. In Fig. 4 we show the co-precession of the spin axis and orbit normal corresponding to the Cassini state II for each of the four Galilean satellites. In Fig. 5 we show the evolution of the libration parameter for 600 years starting January 1st 1600.

For Ganymede, we find a value of ι of about 0.226°. The obliquity ϵ of Ganymede varies between 0.0005° and 0.06658°, while the orbital inclination i ranges from 0.13053° to 0.24332°. The orbital recession period corresponds to 137.411 years and the rotation period is 7.155 days. The free precession of the spin axis has a period of 19.964 years. The period of free longitudinal libration is 186.508 days. The faster free nutation mode occurs with a period of 3.576 days.

TOWARD A MORE REALISTIC MODEL

Future work should focus on extending the current framework to incorporate different plausible interior structure models to properly evaluate the effects of coupling mechanisms between the solid and liquid layers. Nevertheless, the results presented here provide an accurate rigid-body reference model that can be used to interpret future high-precision measurements from missions like Juice and Europa Clipper.

For the time being we model an internal ocean by adding a Rayleigh dissipation term in the Lagrangian formulation to penalize differences in angular velocity between the shell and the ocean. At this preliminary stage the ocean is assumed to be uniformly rotating like a solid.

In Fig. 6 we show a comparison of the librational parameters under two scenarios: a rigid body model and a simplified model consisting of a thin icy shell overlying a subsurface ocean. A striking difference is observed in the obliquity, which is significantly smaller in the presence of an ocean. The expected sensitivity of the GALA laser altimeter on board Juice to the rotational state of Ganymede's icy shell suggests that precise obliquity measurements may provide a direct means to confirm or rule out the long-hypothesized presence of an internal ocean.

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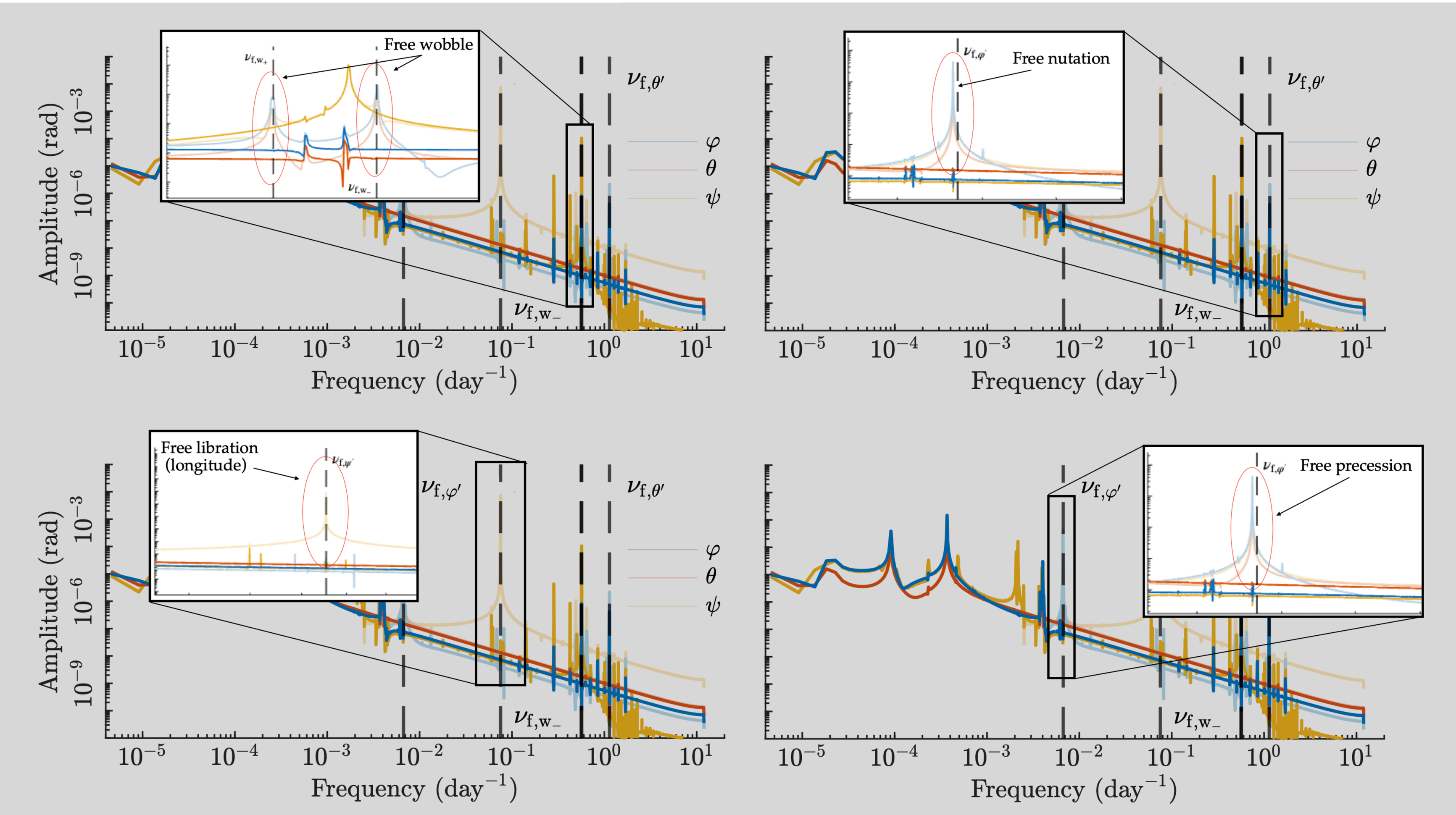
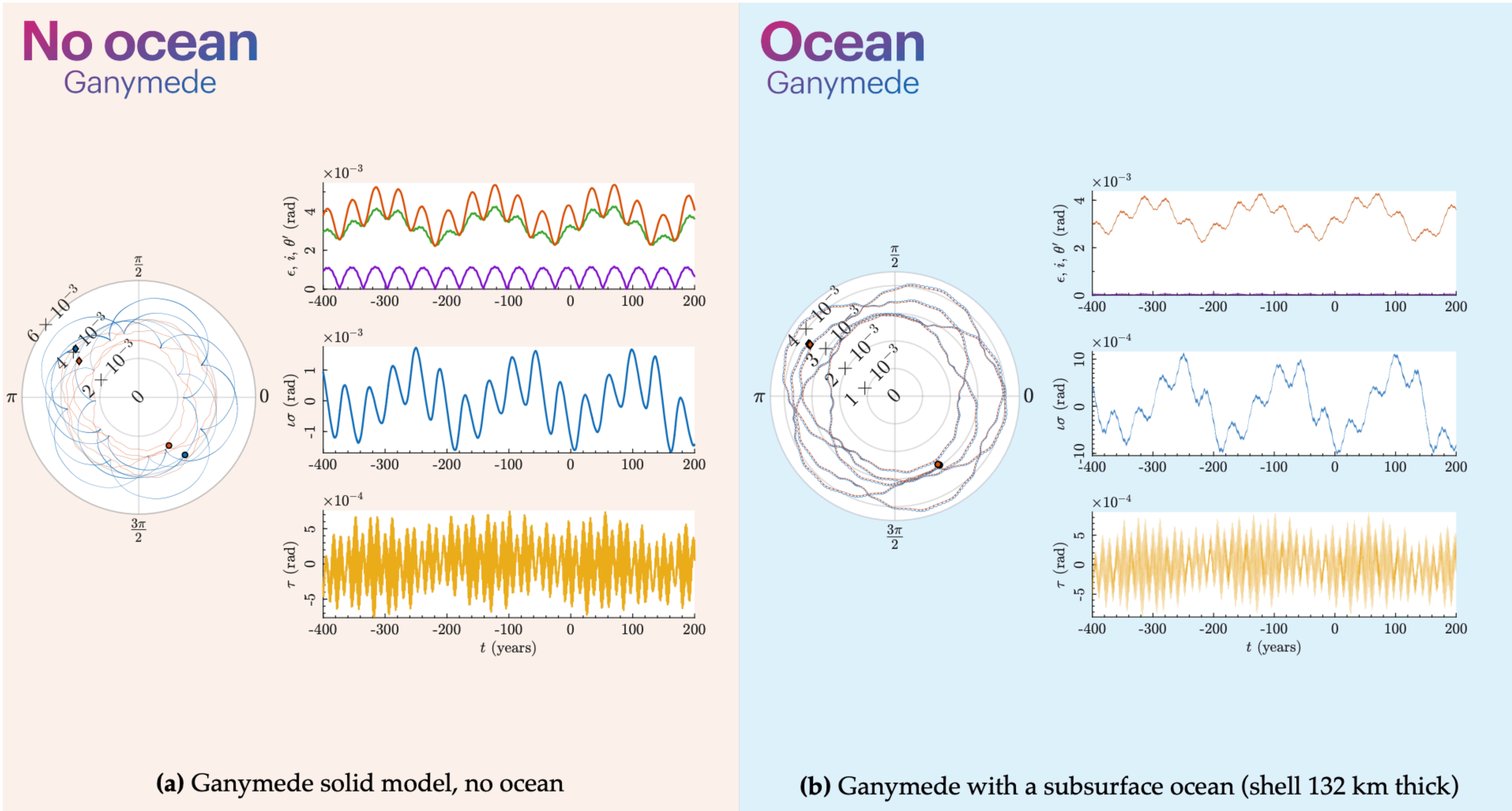


Fig. 3: In a faint color the spectrum of the Euler angles obtained integrating the rotational equations with the *guess* initial conditions. In a solid color the spectrum of the same Euler angles obtained with the converged initial conditions after the removal of the free frequencies using our differential correction method.



(a) Ganymede solid model, no ocean **(b)** Ganymede with a subsurface ocean (shell 132 km thick)

Fig. 6: Comparison of the co-precessional motion of the spin axis and orbit norma, and libration parameters for Ganymede as a solid body (right) and a Ganymede model with a subsurface ocean and outer shell 132 km thick, corresponding to 5% of Ganymede radius.