

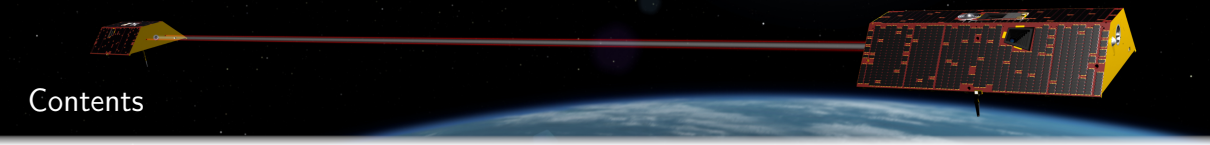
Scale Factor of the Laser Ranging Interferometer in GRACE Follow-On

Malte Misfeldt,
Vitali Müller, Henry Wegener, Laura Müller and Gerhard Heinzel

Max-Planck Institute for Gravitational Physics, Hannover
Institut für Gravitationsphysik, Leibniz Universität Hannover

GRACE Science Team Meeting 2020

Image Credit: NASA/JPL-Caltech



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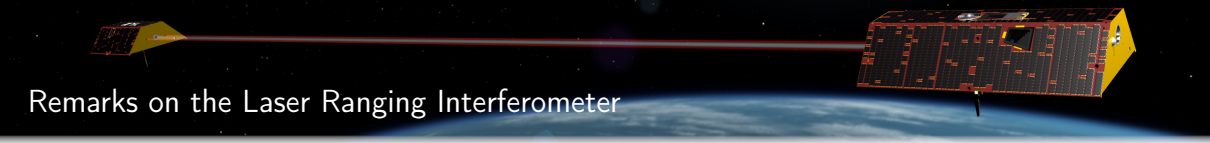
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Remarks on the Laser Ranging Interferometer

The Laser Ranging Interferometer (LRI) onboard the GRACE-FO mission successfully demonstrated the **feasibility of inter-satellite laser ranging**.

The LRI provides **precise ranging data** with a noise level at about $200\,\mathrm{pm}/\sqrt{\mathrm{Hz}}$ at frequencies of 5 Hz.

Long uninterrupted measurements, e.g. for more than 108 days, which is approximately 1650 orbital revolutions.

The LRI team at the AEI Hannover is **analyzing the LRI data** in detail and **developing processing algorithms** to improve the data quality.

LRI Signal and Noise

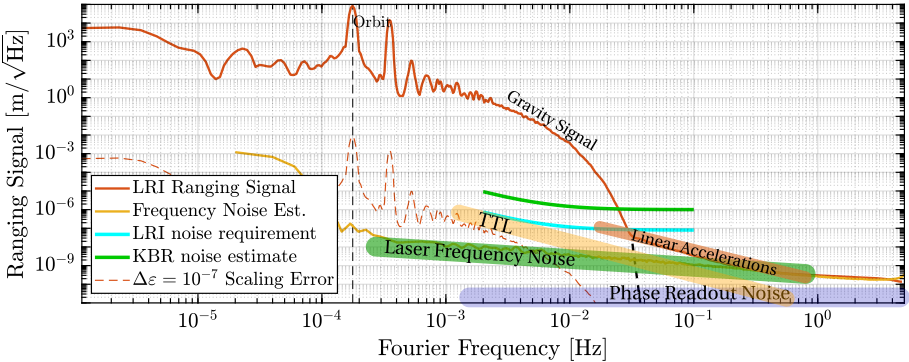
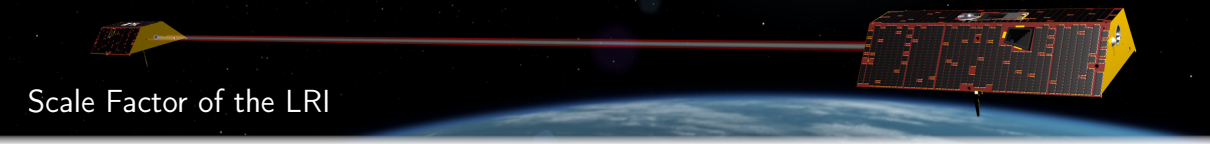


Figure: Spectrum of the LRI ranging signal with some contributors marked.

- See offline display by *Henry Wegener* for more on TTL and linear accelerations.
- But how accurate is the scale factor ε and do we have an estimate on scale factor uncertainties $\Delta\varepsilon$?



Scale Factor of the LRI

- LRI Phase Measurement (after removing phase jumps and conversion from LRI timetags to GPS time):

$$\varphi_{\mathrm{LRI}} = \frac{1}{2}\left(\varphi_T(t-\Delta t) - \varphi_M(t)\right) \quad \text{[cycles]}$$

- Wavelength $\lambda = c_0/\nu_M$ sets scale for converting from phase to meter
  - SDS approach: Compute a daily **cross-correlation between KBR and LRI** measurements LRI scale referenced to KBR/GPS

$$\nu_M \approx 281\,\mathrm{THz} \quad \Rightarrow \quad \rho_{\mathrm{LRI}} = (1+\varepsilon)\lambda_0 \cdot \varphi_{\mathrm{LRI}} \quad \text{[meter]}$$

  - Time dependent approach: estimate a **continuous quantity** $\nu_M(t)$, such that

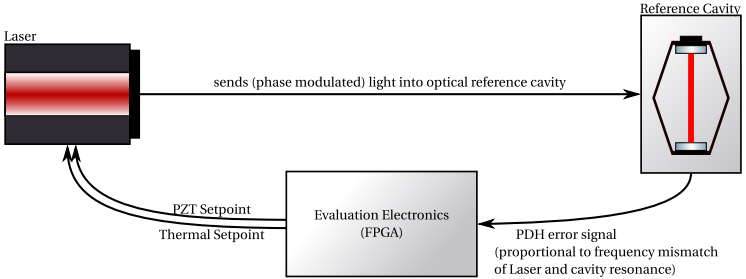
$$\rho_{\mathrm{LRI}} = \int_0^t \frac{c_0}{\nu_M(t')}\ \dot{\varphi}_{\mathrm{LRI}}(t')\ \mathrm{d}t' \quad \text{[meter]}$$

But how can we determine the absolute laser frequency $\nu_M(t)$?

φ..phase, ρ..range, ν..optical frequency, ε..scale factor, c_0..speed of light, M/T..master/transponder

Optical Frequency Estimate for the LRI

Reference Cavity



- On the LRI master side, the laser's frequency is tuned to match the resonance frequency of an optical reference cavity.
- The **frequency mismatch is measured and compensated** using the Pund-Drever-Hall scheme.
- The laser **frequency ν can be controlled** using a thermal and a piezo-electric actuator.
- The actuator setpoints are downlinked in the **Laser Telemetry Packet**.



Optical Frequency Estimate for the LRI

Calibrated Ground Model

- Before launch, a **calibration measurement** was performed to determine the relation between the actuator's control loop setpoints `pztIL`, `pztOOL`, `thermIL`, `thermOOL`[1] and the laser's temperature `lasTRP` to the absolute laser frequency. We use a simple linear model:

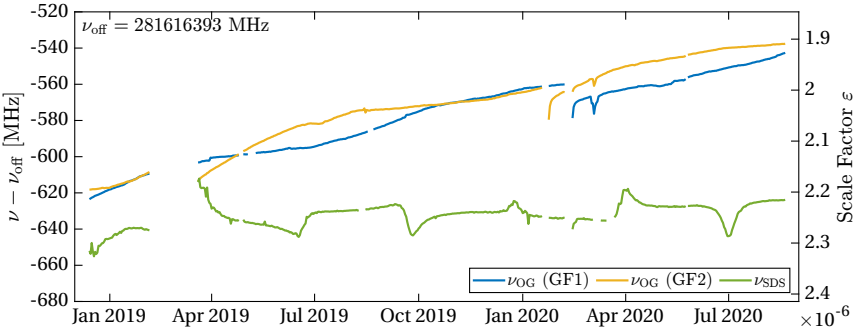
$$\nu_{\mathrm{OG}}(t) = \begin{pmatrix} c_{\mathtt{pztIL}} \\ c_{\mathtt{pztOOL}} \\ c_{\mathtt{thermIL}} \\ c_{\mathtt{thermOOL}} \\ c_{\mathtt{TRP}} \end{pmatrix} \cdot \begin{pmatrix} \mathtt{pztIL}(t) \\ \mathtt{pztOOL}(t) \\ \mathtt{thermIL}(t) \\ \mathtt{thermOOL}(t) \\ \mathtt{lasTRP}(t - 520\,\mathrm{s}) - 26\,^{\circ}\mathrm{C} \end{pmatrix} + \nu_{0,\mathrm{air}}$$

- Numerical values for coupling factors in appendix

[1] The laser telemetry has to be converted from uint32 to int32 using the 2's complement and divided by the bit-depth of 2^{32}.

TRP..thermal reference point, IL..In-Loop, OOL..Out-Of-Loop

PZT and Thermal setpoints in LHK data product, Laser TRP temperature not yet officially released

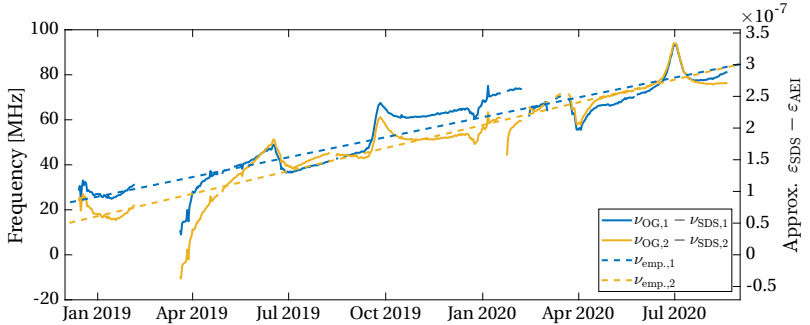


- Application on flight data and comparison with the SDS-derived frequency (LRI1B v04) reveals **a trend and recurring features**.
- Deviations are below 100 MHz ($\Delta\varepsilon < 3.55 \times 10^{-7}$)

Outliers removed for plotting

Optical Frequency Estimate for the LRI

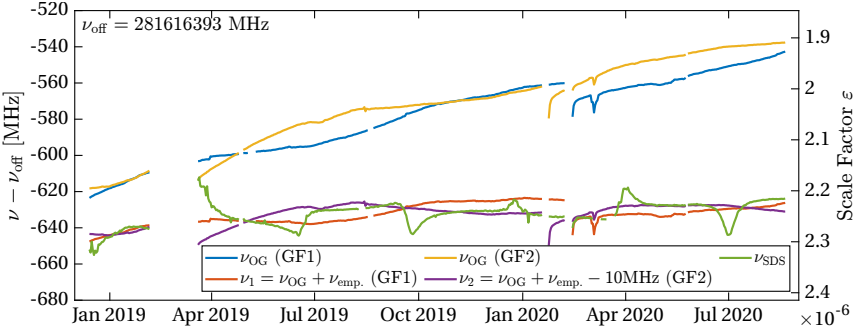
Linear Trend



- We apply a linear empirical model to the calibrated laser frequency to account for the trend in the residuals. The drift in ν_{OG} is smaller than 1.5 Hz/s.
- Numerical values in appendix

Optical Frequency Estimate for the LRI

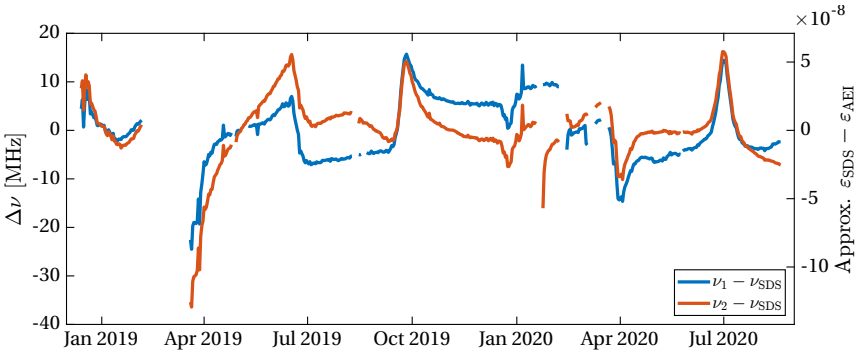
Empirical Correction



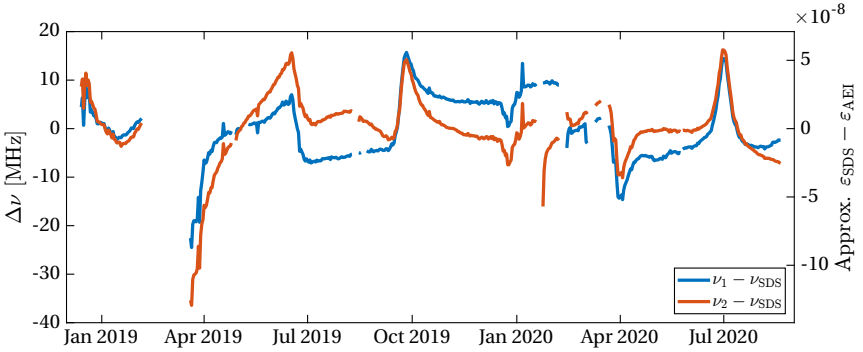
- After detrending, AEI and SDS frequency estimates match closely.

Optical Frequency Estimate for the LRI

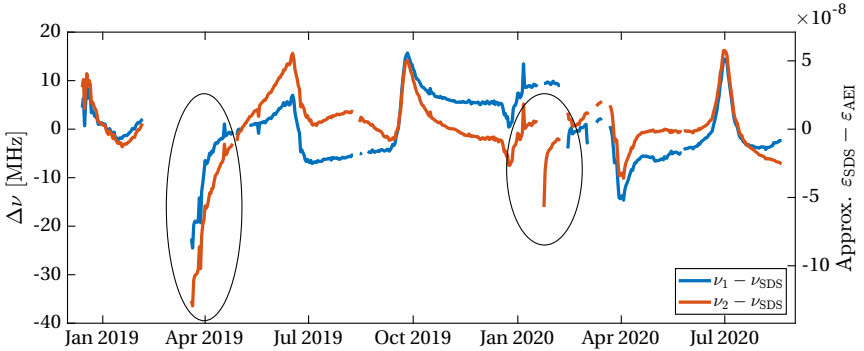
Frequency Residuals



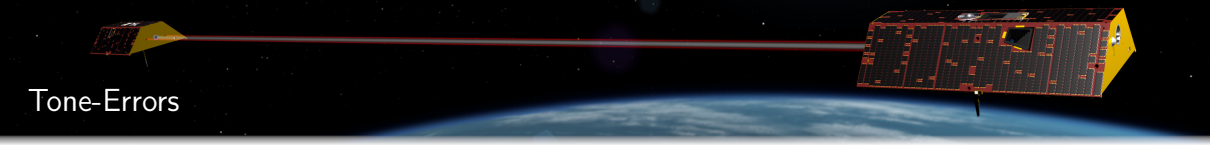
- Frequency Residuals mostly within $\pm 20\,\mathrm{MHz}$ band ($\Delta\varepsilon < \pm 7 \times 10^{-8}$).



- Frequency Residuals mostly within $\pm 20\,\mathrm{MHz}$ band ($\Delta\varepsilon < \pm 7 \times 10^{-8}$).
- We are trying to understand features occurring approx. every three month in the residuals.

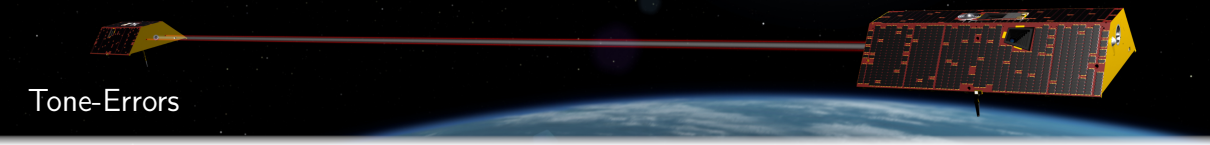


- Frequency Residuals mostly within ± 20 MHz band ($\Delta\varepsilon < \pm 7 \times 10^{-8}$).
- Transient into nominal thermal S/C state after non-nominal mode due to S/C anomalies in Feb/Mar 2019 and Feb 2020.



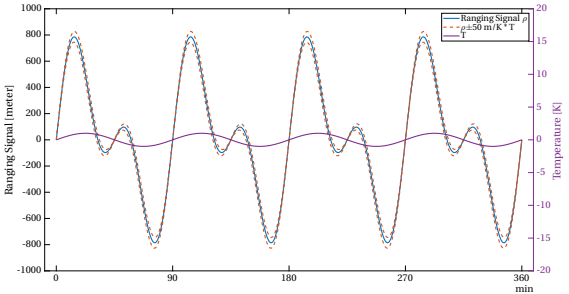
Tone-Errors

- Periodic variations of environmental conditions, like temperature, magnetic field, atmosphere, ... may cause so-called **tone errors**.
- Tone errors are **periodic modulations of the instrument output**, occurring mostly at 1/rev and 2/rev.
  - The ranging signal also has dominant 1/rev and 2/rev oscillations
  - If driving tone & ranging signal oscillate in-phase, the scale factor appears to be different!



Tone-Errors

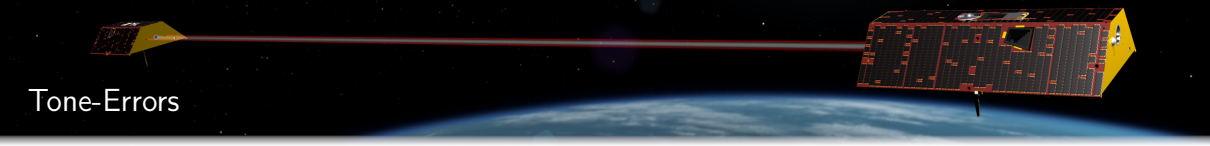
- Example: 1/rev and 2/rev ranging signal, thermal coupling at 1/rev (highly exaggerated)
- Typical values are in the order of a few µm/K.



$$\rho = 500\,\mathrm{m} \cdot \sin\left(\frac{2\pi}{90\,\mathrm{min}} \cdot t\right) + 200\,\mathrm{m} \cdot \sin\left(\frac{2\pi}{45\,\mathrm{min}} \cdot t\right)$$

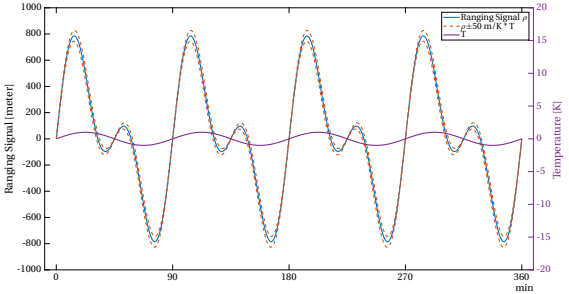
$$T = 1\,\mathrm{K} \cdot \sin\left(\frac{2\pi}{90\,\mathrm{min}} \cdot t\right)$$

$$c_{T,\rho} = 0.92$$



Tone-Errors

- Example: 1/rev and 2/rev ranging signal, thermal coupling at 1/rev (highly exaggerated)
- Typical values are in the order of a few µm/K.
- In the following, we focus on **temperature variations** onboard the GFO satellites by computing a **daily correlation coefficient** $c_{T_i,\varphi} = \mathrm{xcorr}(T_i, \varphi_{\mathrm{LRI}})$ of the LRI Phase φ_{LRI} and various temperature sensors T_i from S/C offred telemetry.



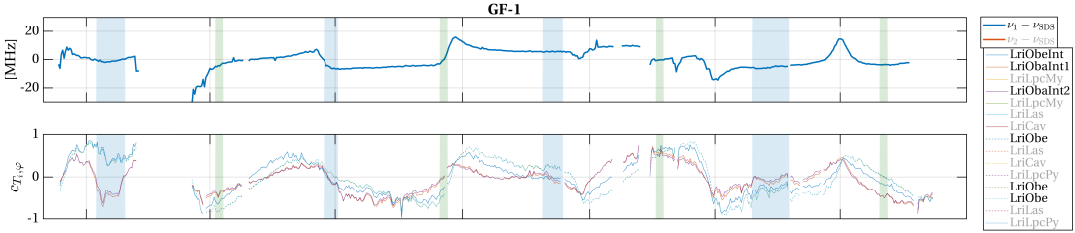
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Tone-Errors

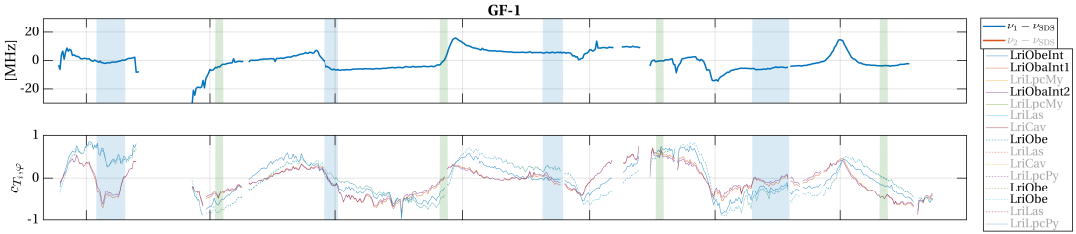
LRI Temperatures with $c_{T_i,\varphi} > 40\%$



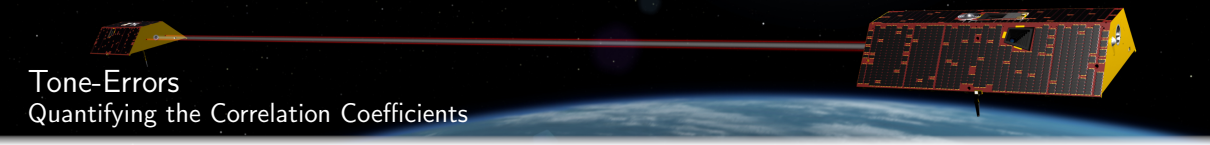
- OBE and OBA temperatures' in-phase correlation $c_{T_i,\varphi}$ show high similarity to frequency residuals $\nu_1 - \nu_{\mathrm{SDS}}$ (or equivalent: to $\Delta\varepsilon$).
- Many other sensors show correlation as well

Tone-Errors

LRI Temperatures with $c_{T_i,\varphi} > 40\%$



- OBE and OBA temperatures' in-phase correlation $c_{T_i,\varphi}$ show high similarity to frequency residuals $\nu_1 - \nu_{\mathrm{SDS}}$ (or equivalent: to $\Delta\varepsilon$).
- Many other sensors show correlation as well
- How to translate this similarity into a temperature coupling with units of [m/K]?



- Convert the frequency difference $\nu_M - \nu_{\mathrm{SDS}}$ into a ranging error ρ_{err}:

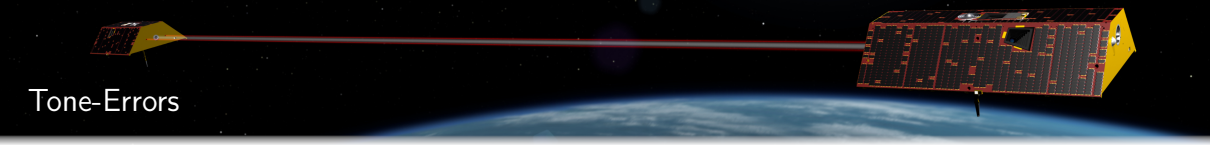
$$\rho_{\mathrm{err}} = \left(\frac{c_0}{\nu_M} - \frac{c_0}{\nu_{\mathrm{SDS}}} \right) \cdot \mathrm{rms}(\varphi_{\mathrm{LRI}}) \quad \text{[rms meter]}$$

- Compute daily in-phase part of the rms temperature variations

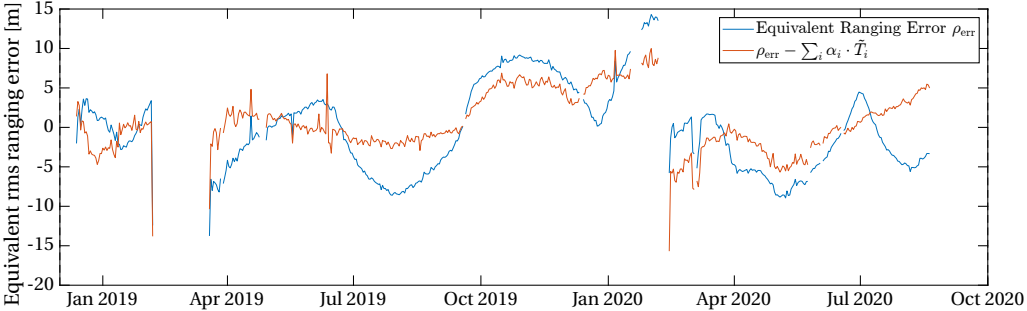
$$\tilde{T}_i = c_{T_i,\varphi} \cdot \mathrm{rms}(T_i) \quad \text{[rms Kelvin]}$$

- Find coupling coefficients α_i [m/K], by minimizing

$$\left| \rho_{\mathrm{err}} - \sum_i \alpha_i \cdot \tilde{T}_i \right|^2 .$$

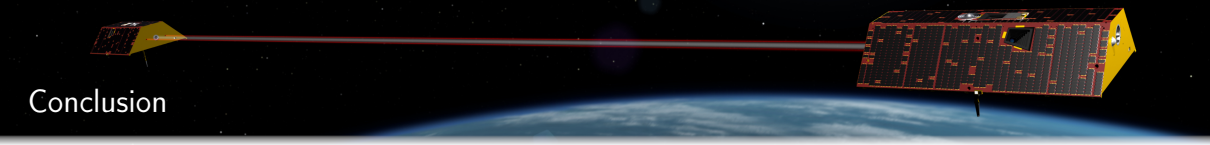


Tone-Errors



- Numerical values in appendix
- $\mathrm{rms}(\rho_{\mathrm{err}}) = 5.41\,\mu\mathrm{m}$
  $\mathrm{rms}(\rho_{\mathrm{err}} - \sum_i \alpha_i \cdot \tilde{T}_i) = 3.72\,\mu\mathrm{m}$
  $\Rightarrow$ Error reduction of approx. $30\,\%$

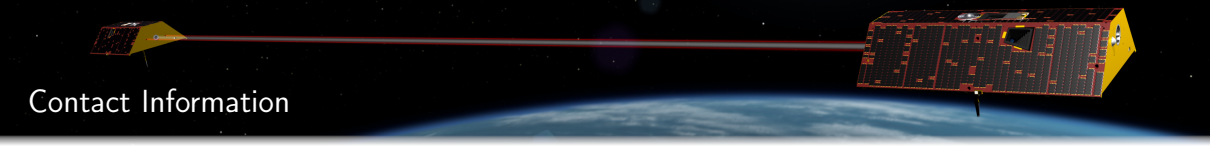
- We use nine LRI temperature sensors
- Coupling coefficients smaller than $\pm 60\,\mu\mathrm{m/K}$



Conclusion

- We showed a relation of the absolute laser frequency of the two LRI units and the LRI telemetry, based on ground calibrations
- An additional empirical (linear) model is needed and derived from cross-calibration to KBR
  - The accuracy of the combined ground & empirical drift model is better than 20 MHz or $\pm 7 \times 10^{-8}$ in scale factor.

- We are investigating, if the remaining variations in the laser frequency could be attributed to tone errors.
  - We have derived the coupling factors from temperature to range.
  - With this temperature coupling applied, the rms ranging error can be reduced by approximately 30 %.
  - However, we have not yet evaluated if these coupling factors can be used to improve the LRI1B range.



Contact Information

Thanks for listening!

Malte Misfeldt
for the LRI team

malte.misfeldt@aei.mpg.de

Max-Planck Institute for Gravitational Physics
(Albert Einstein Institute AEI)

Hannover - Germany

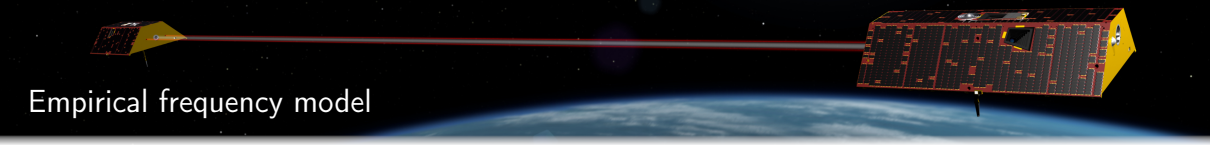


Onground frequency model: Coupling factors

Coupling	Nominal Value	Measured Value	
		GF-1	GF-2
c_{pztIL}	$2\,\mathrm{V} \cdot 5\,\mathrm{MHz/V} = 10\,\mathrm{MHz}$	-	-
c_{pztOOL}	$18\,\mathrm{V} \cdot 5\,\mathrm{MHz/V} = 90\,\mathrm{MHz}$	-	-
c_{thermIL}	$2\,\mathrm{V} \cdot 500\,\mathrm{MHz/V} = 1000\,\mathrm{MHz}$	1060 MHz	1086 MHz
c_{thermOOL}	$18\,\mathrm{V} \cdot 500\,\mathrm{MHz/V} = 9000\,\mathrm{MHz}$	9080 MHz	8859 MHz
c_{TRP}	$-12\,\mathrm{MHz/K}$	-	-
$\nu_{0,\mathrm{air}}$	281 614 840 MHz (GF-1)) 281 614 807 MHz (GF-2)	281 614 693.912 MHz	281 614 714.031 MHz

Table: Nominal values of coupling factors for the two laser units and actual fit results from on-ground measurements. PZT and TRP coupling factors were not refined, since the tests were not sensitive to these couplings.

Note: PZT and thermal setpoints from LHK telemetry have to be converted from uint32 to int32 using the 2's-complement and divided by the bit-depth 2^{32} before applying these coupling factors.

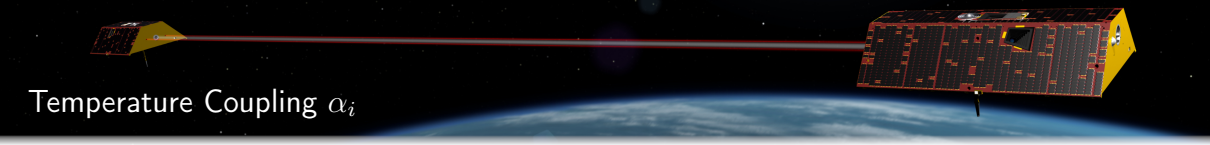


Empirical frequency model

$$\nu(t) = \nu_{\mathrm{OG}}(t) + \nu_{\mathrm{emp.}}(t)$$
$$\nu_{\mathrm{emp.}}(t) = a \cdot (t - t_0) + \Delta\nu$$
$$t_0 = 1210982400 \text{ GPS seconds (2018-May-22 00:00:000Z)}$$

Coupling	GF-1	GF-2
a [Hz/s]	1.1208	1.2860
$\Delta\nu$ [Hz]	4 031 919	−7 629 283

Table: Parameters of the empirical frequency model.

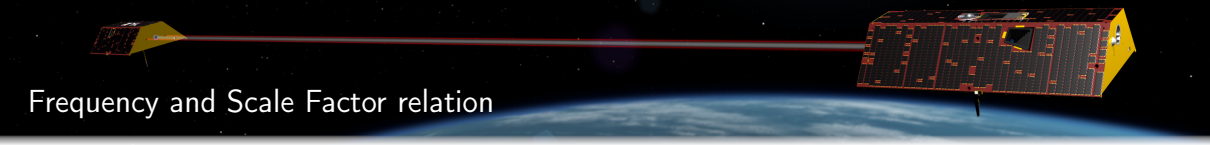


Temperature Coupling α_i

Sensor Name	α_i [µm/K]	$\tilde{T}_i$ [K rms]	$\alpha_i \cdot \tilde{T}_i$ [µm rms]
LriCav A	-29.41	0.24	-7.00
LriCav C	-31.18	0.16	-5.00
LriOba Int1	-57.35	0.06	-3.28
LriLas A	33.57	0.04	1.37
LriCav Int	7.00	0.13	0.91
LriOba A	-4.92	0.16	-0.78
LriObe Int	9.75	0.08	0.75
TmaTrp	58.88	0.01	0.47
LriLrp A	-4.66	0.10	-0.46

Preliminary Values!

- Only LRI temperatures included in this analysis
- If multiple LRI temperatures are highly correlated among each other (e.g. redundant sensors), only one of them is included in this analysis
- The third column gives a rough estimate on the actual tone-error in the ranging measurement (sorted descending)



Frequency and Scale Factor relation

- $\nu_{0,\mathrm{GF}-1} = 281\,616\,393\,\mathrm{MHz}$
  $\nu_{0,\mathrm{GF}-2} = 281\,615\,684\,\mathrm{MHz}$
- Relation Frequency to Scale Factor: $\lambda = (1+\varepsilon)\lambda_0 \quad \Rightarrow \quad \nu = \frac{1}{1+\varepsilon}\nu_0 \quad \Rightarrow \quad \varepsilon = \frac{\nu_0}{\nu} - 1$
- Relation Frequency Difference to relative scale:

$$
\begin{aligned}
\lambda - \lambda' &= (\varepsilon - \varepsilon') \cdot \lambda_0 \\
\Rightarrow \quad \frac{1}{\nu} - \frac{1}{\nu'} &= \frac{\varepsilon - \varepsilon'}{\nu_0} \\
\Rightarrow \quad \frac{\nu_0(\nu' - \nu)}{\nu\nu'} &= \varepsilon - \varepsilon' \\
\Rightarrow \quad \frac{\nu' - \nu}{\nu_0} &\approx \varepsilon - \varepsilon' \quad \text{since } \nu \approx \nu' \approx \nu_0
\end{aligned}
$$