

Global Gravity-based Groundwater Product

Gravity Field Recovery and Observation Noise Separation from Simultaneous Laser Ranging Interferometer and K-Band Ranging System Measurements

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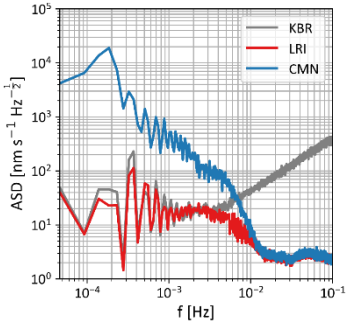
GSTM 2020, October 27 2020

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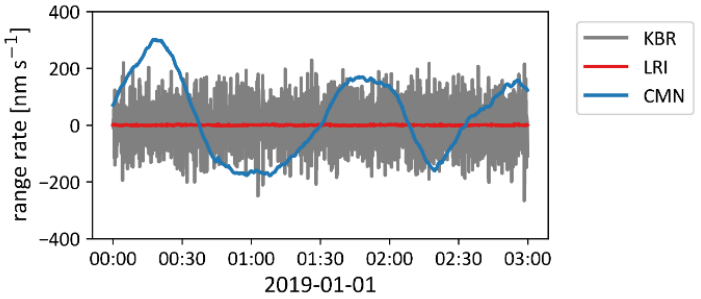
Outline

Stochastic Models



- Stochastic models properly weight observations in a least-squares adjustment
- Using KBR and LRI together allows to determine the properties of different noise sources

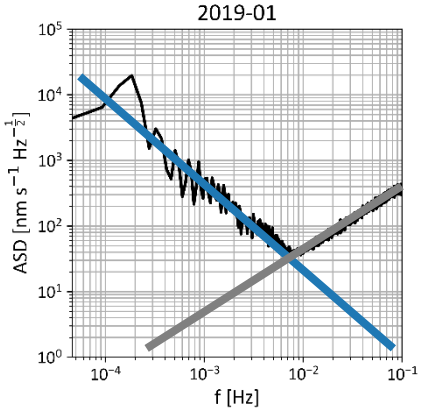
De-mixing of Residuals



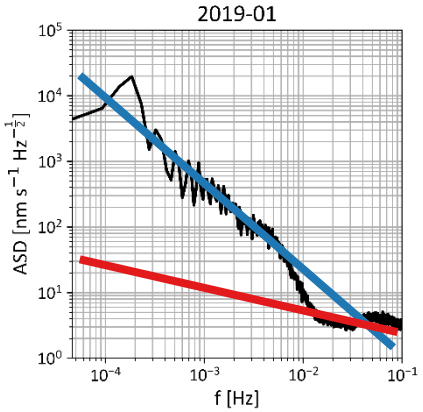
- Post-fit residuals can be split into individual components using their stochastic properties
- Result: individual time series for KBR, LRI, common noise
- De-mixed residuals can be used to closer study instrument related effects

Stochastic Observation Model

$\Sigma(\Delta l)$ KBR-only



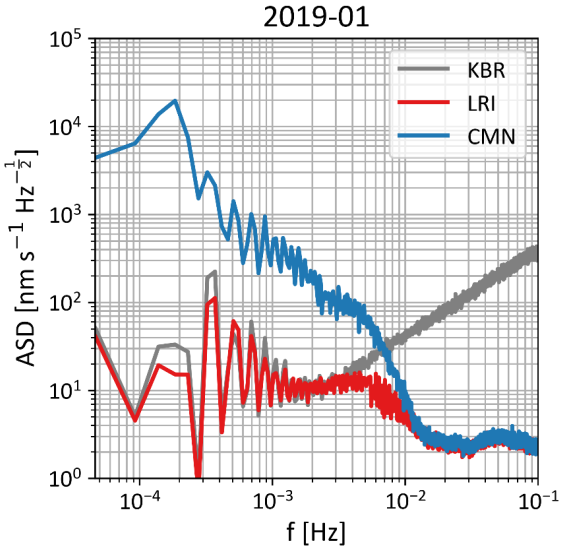
$\Sigma(\Delta l)$ LRI-only



$$\Delta l = l_{KBR} - f_0$$

f_0 common to both, can be separated

$$\Delta l = l_{LRI} - f_0$$



f_0 … forward modelled observations which contain ACC/ACT and background model errors

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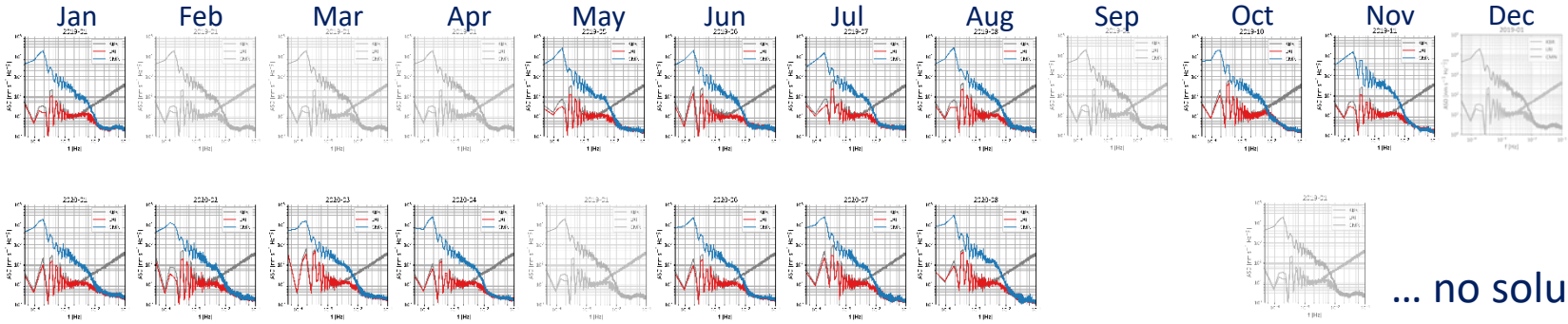
Stochastic Observation Model

- Covariance model for simultaneous ranging observations
  - Determined with variance component estimation (VCE, e.g., Koch 1980, Ellmer 2018)

$$\Delta l = \begin{bmatrix} l_{KBR} \\ l_{LRI} \end{bmatrix} - \begin{bmatrix} f_0 \\ f_0 \end{bmatrix} \quad \Sigma(\Delta l) = \begin{bmatrix} \Sigma_{KBR} & \\ & \Sigma_{LRI} \end{bmatrix} + \begin{bmatrix} \Sigma_{CMN} & \Sigma_{CMN} \\ \Sigma_{CMN} & \Sigma_{CMN} \end{bmatrix}$$

off-diagonal block enables separation

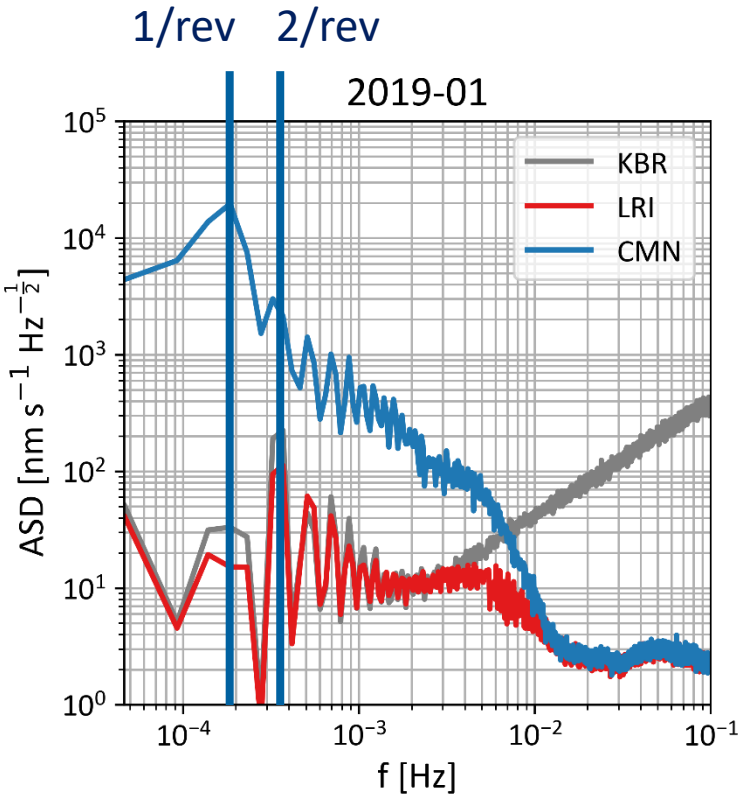
- Result of the VCE are 3 stationary covariance functions per month
  - In spectral domain they can be represented as amplitude spectral density (ASD)



… no solution

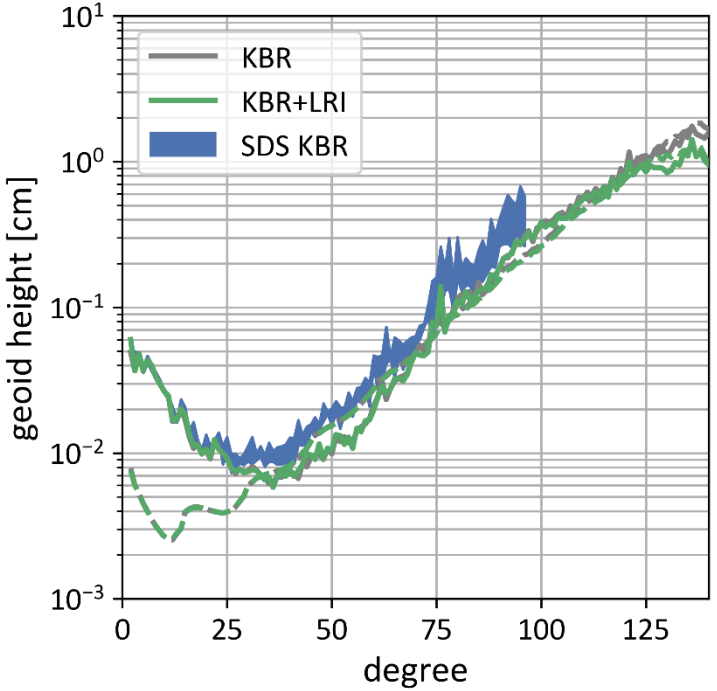
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Stochastic Observation model



stochastic model

realistic formal errors



Separation of KBR, LRI, and CMN Components

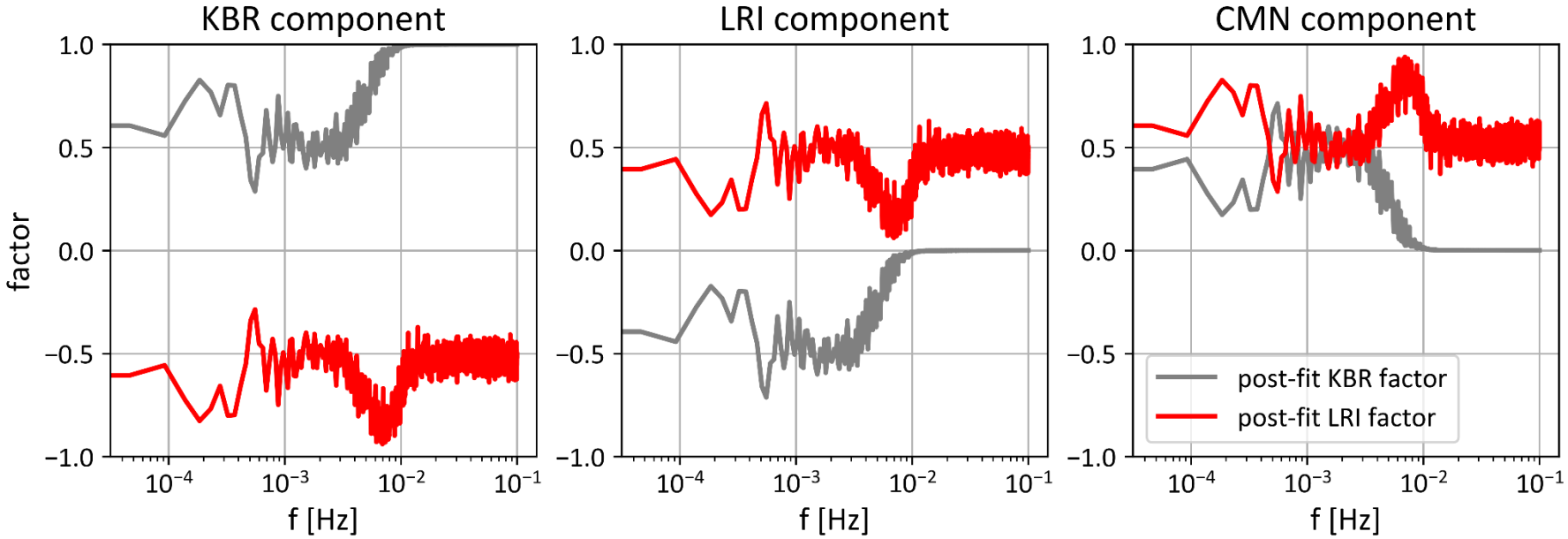
Separation of KBR, LRI, and CMN Residuals

- We can use the determined KBR, LRI, and CMN covariance functions to disentangle residuals through collocation (see Appendix):



- Individual components are linear transformations of post-fit residuals
  - Approximately this is a frequency-wise linear combination

Separation of KBR, LRI, and CMN Residuals



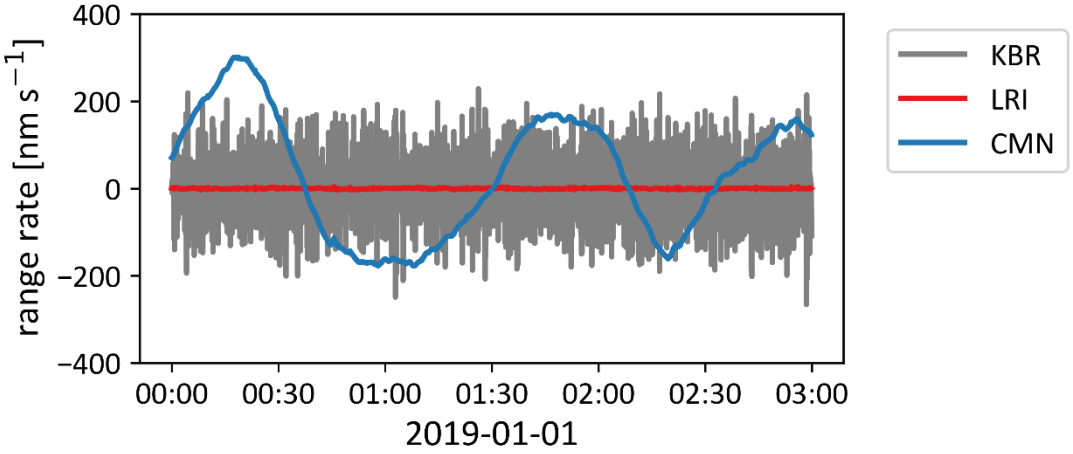
Ranging components are weighted differences: common features cancel out

Common component is a weighted average: different features average out

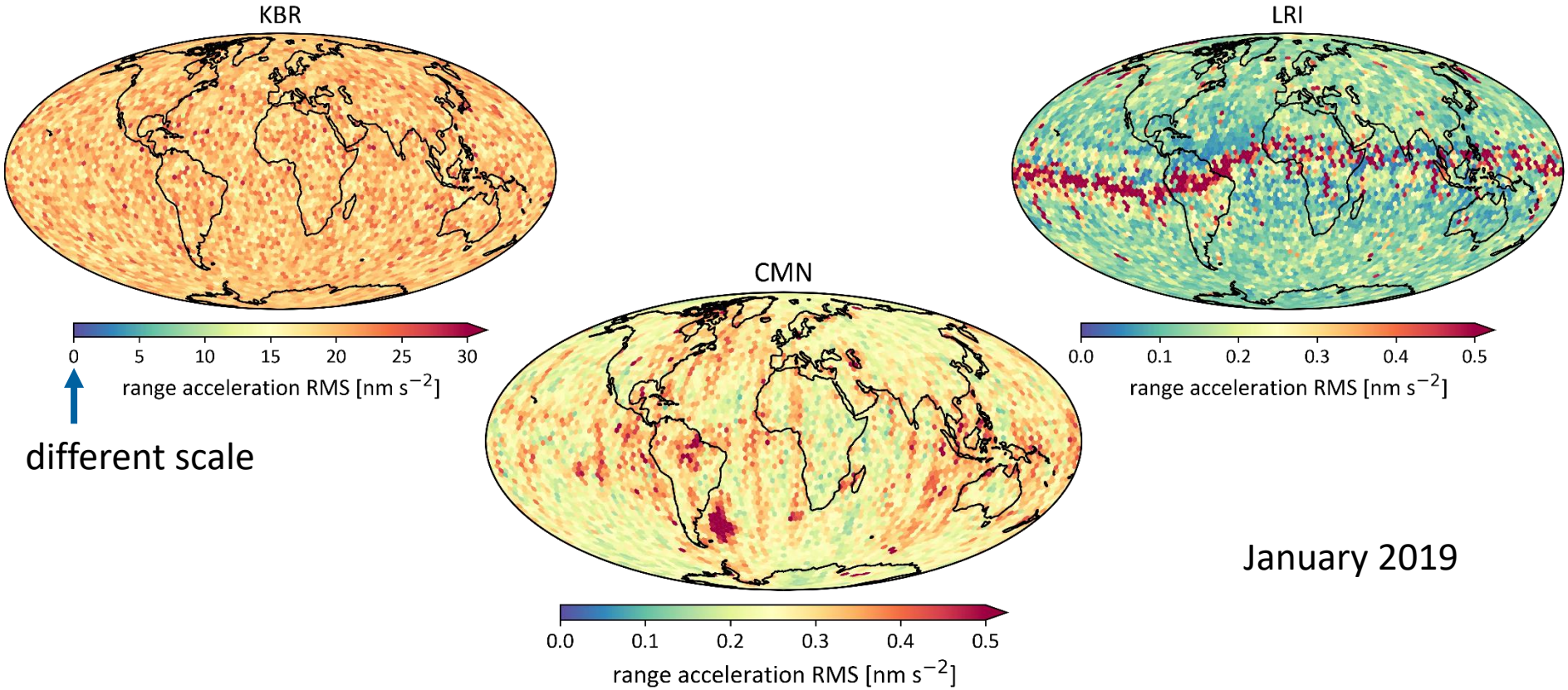
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Separation of KBR, LRI, and CMN Residuals – Time Domain

- De-mixed residuals in time domain:
  - Spectral properties are reflected in time series

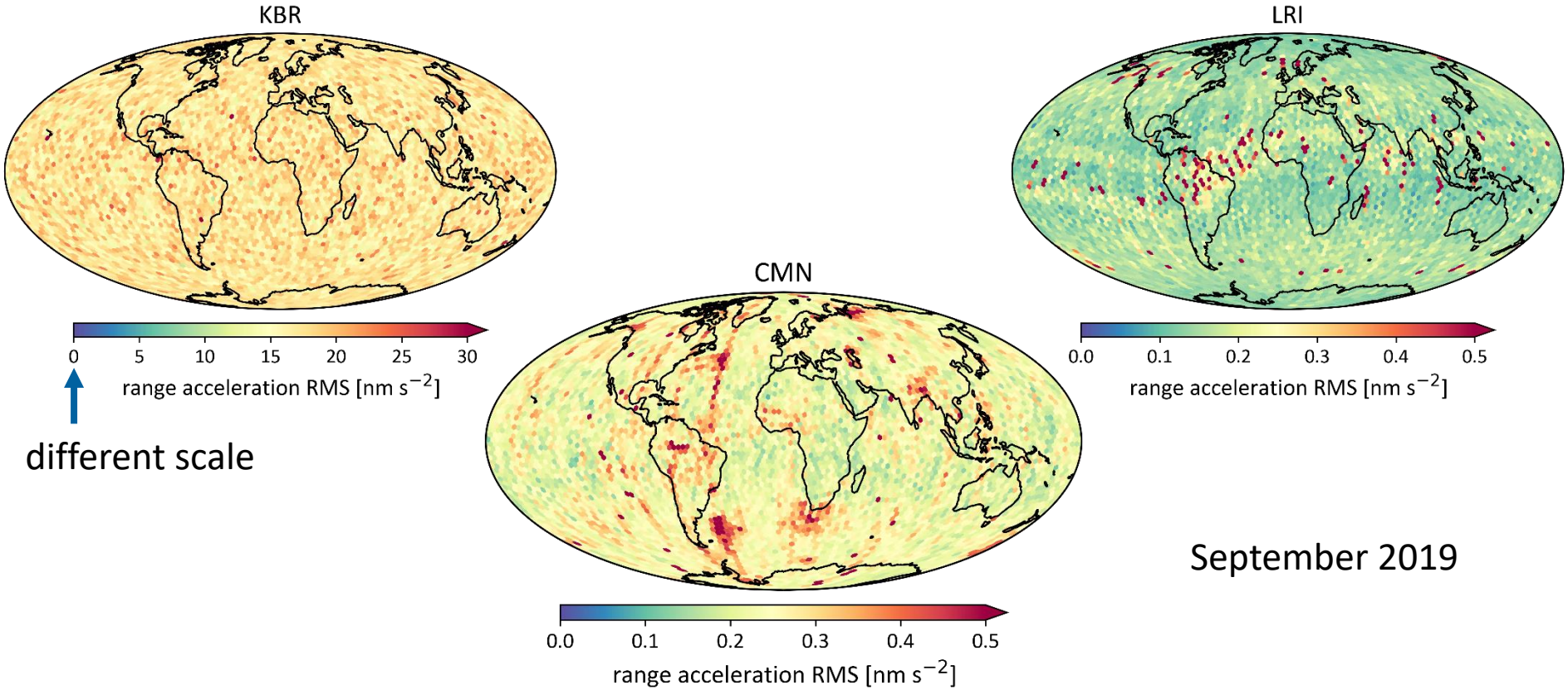


Separation of KBR, LRI, and CMN Residuals – Space Domain



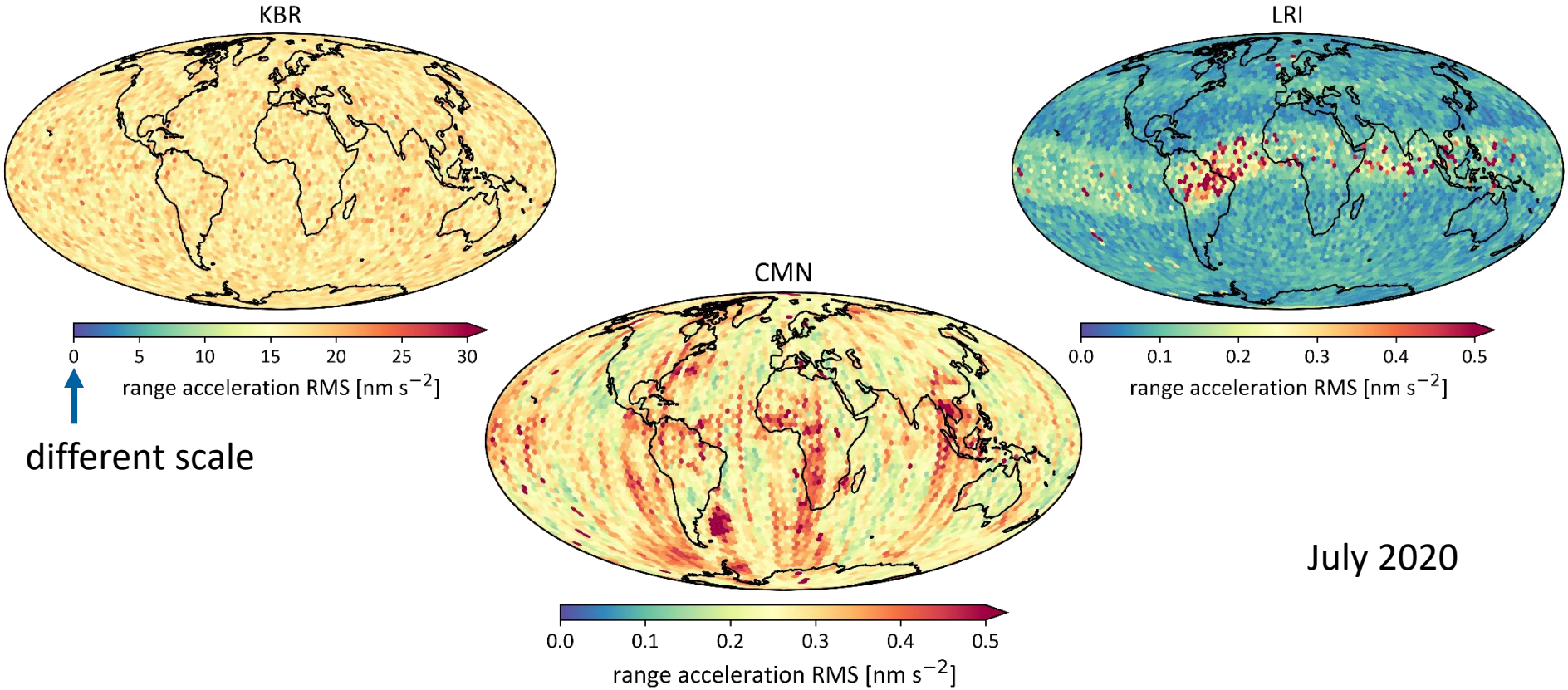
January 2019

Separation of KBR, LRI, and CMN Residuals – Space Domain



September 2019

Separation of KBR, LRI, and CMN Residuals – Space Domain



July 2020

Summary

Summary

- Using KBR and LRI together requires a stochastic model which takes the correlations introduced by ACC/ACT and background models into account

- With such a stochastic model, we can separate common noise sources from instrument specific ones
  - ACC/ACT and background model errors can be disentangled from ranging errors
  - Contrast is not perfect so there is leakage between the noise sources

- The separation of post-fit residuals may enable a closer inspection of LRI peculiarities
  - Range-rate residuals for KBR, LRI, and CMN are available on our FTP server: ftp://ftp.tugraz.at/outgoing/ITSG/temp/gracefo_residuals

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Thank you!

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Appendix

Separation of KBR, LRI, and Common Noise Sources

Stochastic Observation Model

- We estimate the gravity field in a non-linear least squares adjustment:

SST observations

SST residuals

$$\Delta l = l_{SST} - f(x_0, y_m, a_{ACC}, a_{ACT}, \dots) = A\Delta x + e_{SST}$$

forward-modelled observations

$$l_{KBR} - f_0 = A\Delta x + e_{KBR}$$

$$l_{LRI} - f_0 = A\Delta x + e_{LRI}$$

- Forward modelled observations f_0 are present in reduced observations of KBR and LRI
  - They contain ACC/ACT noise, background model errors, SCA noise, …

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Stochastic Observation model

- We estimate the gravity field in a non-linear least squares adjustment:
  - We need to find $\Sigma(e) \equiv \Sigma(\Delta l)$ to minimize $e^T \Sigma(e)^{-1} e$

$$\Delta l = \begin{bmatrix} l_{KBR} \\ l_{LRI} \end{bmatrix} - \begin{bmatrix} f_0 \\ f_0 \end{bmatrix} = \begin{bmatrix} I & & -I \\ & I & -I \end{bmatrix} \begin{bmatrix} l_{KBR} \\ l_{LRI} \\ f_0 \end{bmatrix}$$

$$l_{KBR} \sim \Sigma_{KBR}$$

$$l_{LRI} \sim \Sigma_{LRI}$$

$$f_0 \sim \Sigma_{CMN}$$

covariance matrix of common noise sources

- Through covariance propagation we we obtain the covariance matrix of the reduced observations:

$$\Sigma(\Delta l) = \begin{bmatrix} \Sigma_{KBR} & \\ & \Sigma_{LRI} \end{bmatrix} + \begin{bmatrix} \Sigma_{CMN} & \Sigma_{CMN} \\ \Sigma_{CMN} & \Sigma_{CMN} \end{bmatrix}$$

off-diagonal block enables separation

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Stochastic Observation model

- $\Sigma(\Delta l)$ can be determined together with the gravity field through variance component estimation (VCE)
  - See Koch (1980) for general theory or Ellmer (2018) for application to GRACE/KBR

$$\Sigma(\Delta l) = \begin{bmatrix} \Sigma_{KBR} & \\ & \Sigma_{LRI} \end{bmatrix} + \begin{bmatrix} \Sigma_{CMN} & \Sigma_{CMN} \\ \Sigma_{CMN} & \Sigma_{CMN} \end{bmatrix} = \sum_k \sigma_k V_k$$

variance components

- We assume KBR, LRI, and CMN to be temporally stationary noise sources
  - Background model uncertainties (stationary in space and time) are treated according to Kvas and Mayer-Gürr (2019)

Separation of KBR, LRI, and CMN Residuals

- We can use the determined KBR, LRI, and CMN covariance functions to disentangle residuals through collocation:

$$\begin{bmatrix} e_{KBR} \\ e_{LRI} \end{bmatrix} = \begin{bmatrix} I & & I \\ & I & I \end{bmatrix} \begin{bmatrix} s_{KBR} \\ s_{LRI} \\ s_{CMN} \end{bmatrix}$$

De-mixing matrix

$$\begin{bmatrix} s_{KBR} \\ s_{LRI} \\ s_{CMN} \end{bmatrix} = \begin{bmatrix} \Sigma_{KBR} & & \\ & \Sigma_{LRI} & \\ & & \Sigma_{CMN} \end{bmatrix} \begin{bmatrix} I & \\ & I \\ I & I \end{bmatrix} \begin{bmatrix} \Sigma_{KBR} + \Sigma_{CMN} & \Sigma_{CMN} \\ \Sigma_{CMN} & \Sigma_{LRI} + \Sigma_{CMN} \end{bmatrix}^{-1} \begin{bmatrix} e_{KBR} \\ e_{LRI} \end{bmatrix}$$

Separation of KBR, LRI, and CMN Residuals

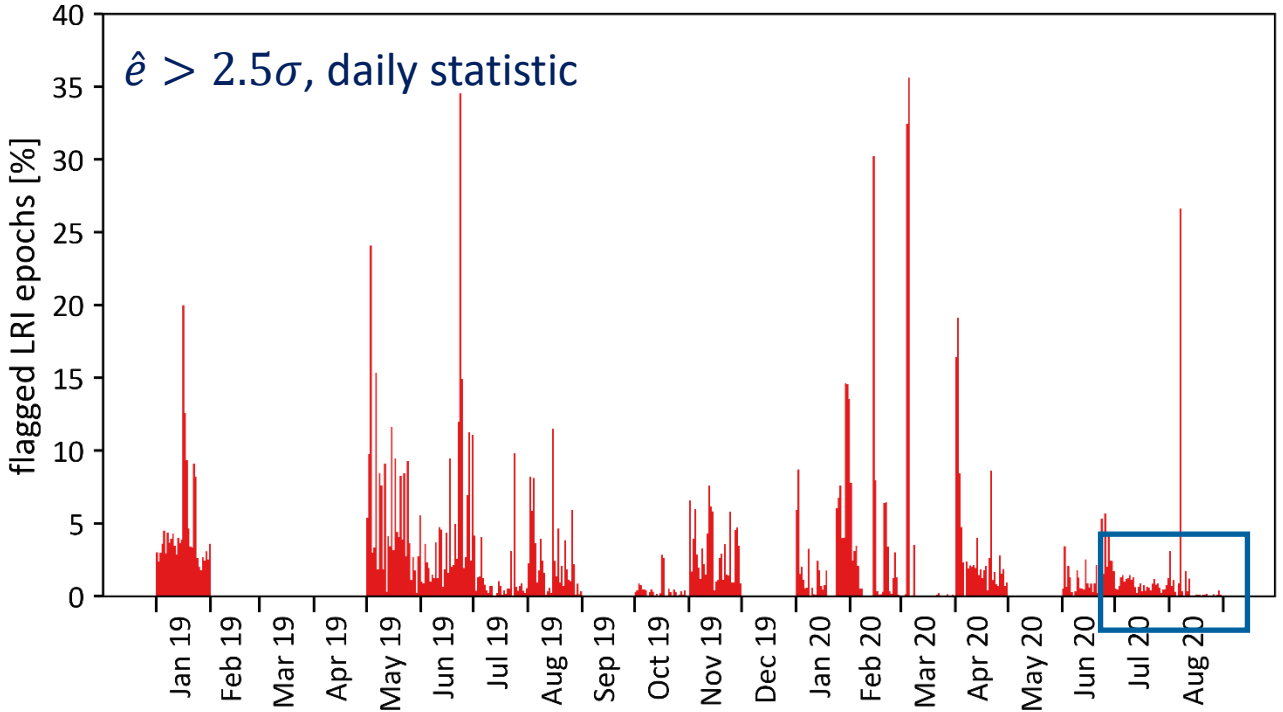
- We can use the determined KBR, LRI, and CMN covariance functions to disentangle residuals through collocation:



- Individual components are linear transformations of post-fit residuals
  - Approximately this is a frequency-wise linear combination (see Appendix)

Flagged LRI Epochs

- Threshold based outlier detection based on separated LRI residuals



July and especially August show improvement

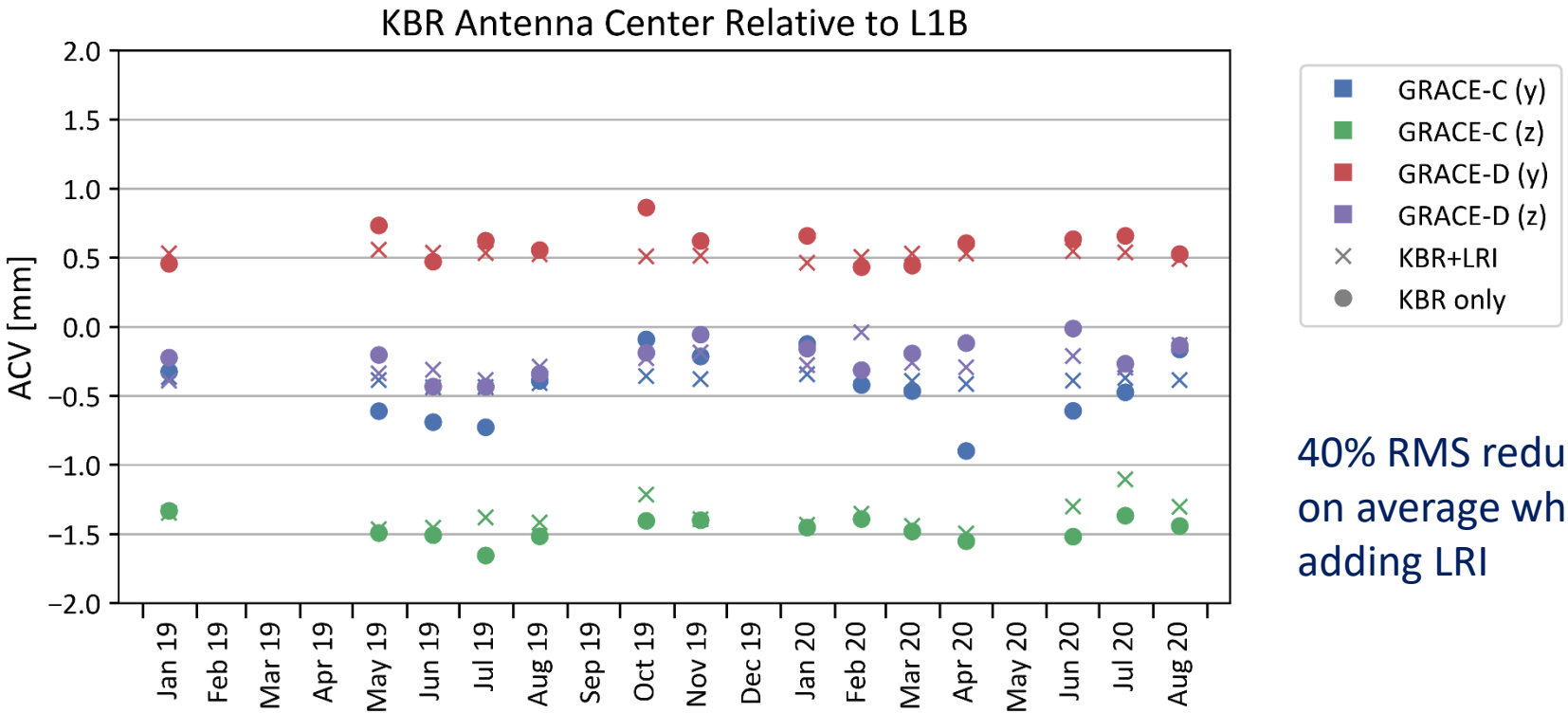
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Estimated Ranging Calibration Parameters

Estimated Ranging Calibration Parameters

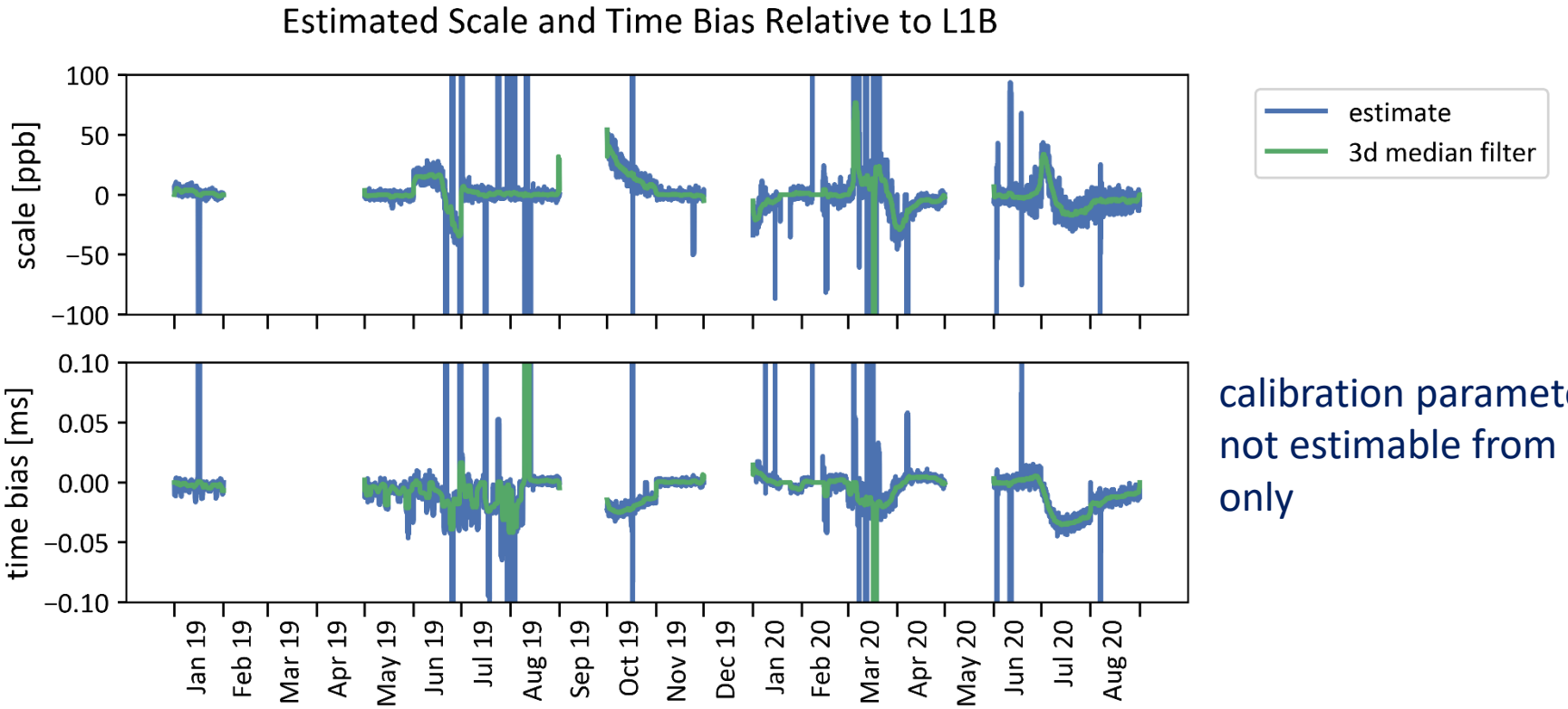
- Calibration parameters for both ranging instruments are co-estimated with the gravity field
- KBR: antenna center variations (y, z)
- LRI: time bias, scale (frequency difference), linear splines with 1.5 hours for each day

Estimated KBR Antenna Center



40% RMS reduction on average when adding LRI

Estimated LRI Scale (Laser Frequency Difference) and Time Bias



calibration parameters not estimable from LRI only