Coupling of pointing angles and angular rates in the GRACE Follow-On Laser Ranging Interferometer

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Overview

The GRACE Follow-On Laser Ranging Interferometer (LRI) was switched on for the first time on 14 June 2018. Since then, it has been providing high quality ranging data, as well as S/C pitch and yaw angles, with very good reliability. This presentation focuses on the sensitivity and noise sources of the LRI range.

The following aspects will be covered:

- Tilt-to-length (TTL) coupling
- Angular rate coupling (ARC)
- Updated TTL results
- Center-of-mass (CM) offsets from LRI
- LRI sensitivity investigations

LRI noise sources

The LRI has two main noise sources:

- Laser frequency noise (LFN) (at high frequencies)
- Tilt-to-length (TTL) coupling (at low frequencies)

TTL coupling has been measured successfully and its contribution to the range spectrum is **within the requirement**. It is unlikely to significantly influence gravity field recovery, since it is of lower magnitude than other error contributors such as accelerometer noise.

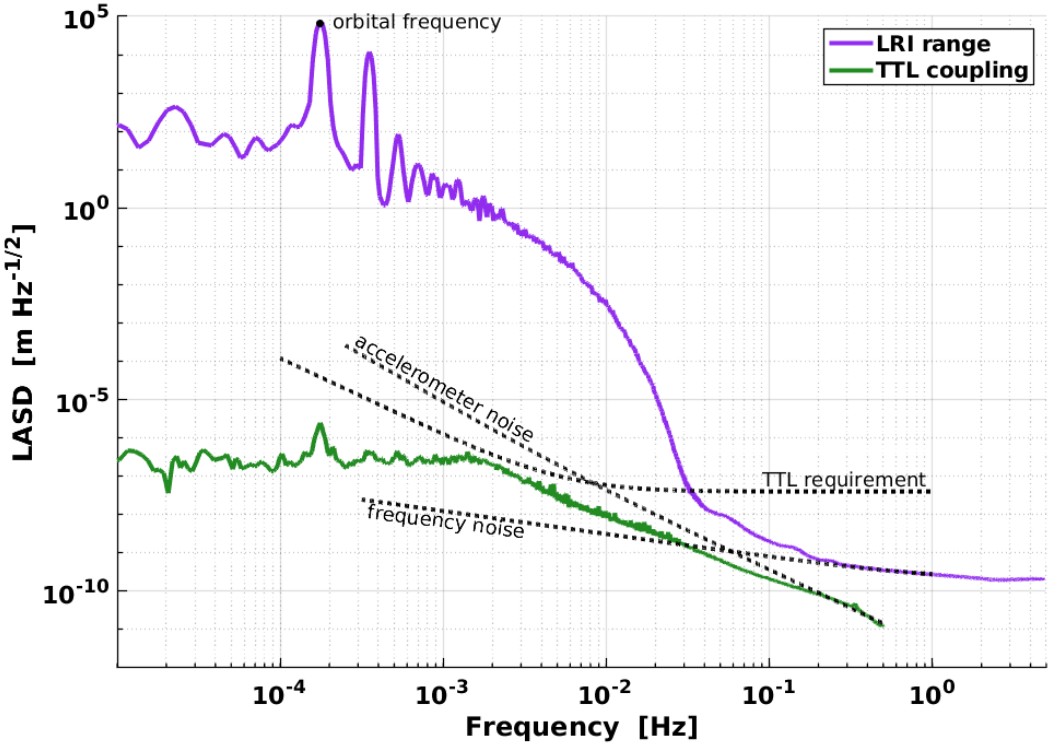


Figure: Amplitude spectral density of LRI range and TTL coupling, measured in April 2019.

Vertex point offset

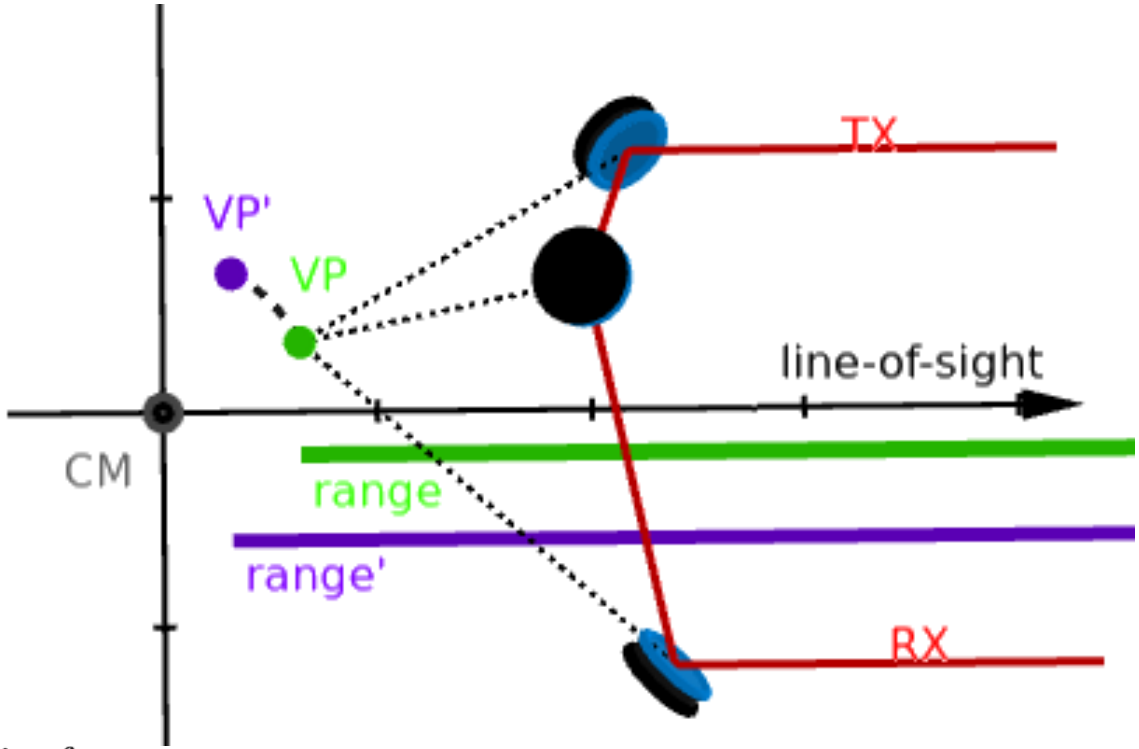
The LRI reference point is the Vertex Point (VP). There is a small offset between VP and the center-of-mass (CM), of the order of 100 µm. When the S/C rotates, the VP location varies. This causes a TTL error, which is approximated by the LoS-projection of the vector from VP to CM. The first order approximation is:

$$\delta\rho_{\mathrm{TTL}} \approx \vec{e}_{\mathrm{LoS}}^{T} \cdot \vec{V} \approx V_z\theta_y - V_y\theta_z$$

θ_y : pitch

θ_z : yaw

$\vec{V} = (V_x, V_y, V_z)^T$: vector VP $\rightarrow$ CM in satellite frame



Therefore, pitch and yaw coupling factors are equivalent to y and z CM offsets.

Measuring TTL:

- Center-of-mass calibration (CMCal) maneuvers are used as stimuli
- Three pointing angles per S/C: roll, pitch, yaw
- The yaw angle on GF2 is the one with the largest TTL coupling factor
- The figure on the right shows the LRI range and estimated TTL during a yaw maneuver on GF2, performed in January 2019

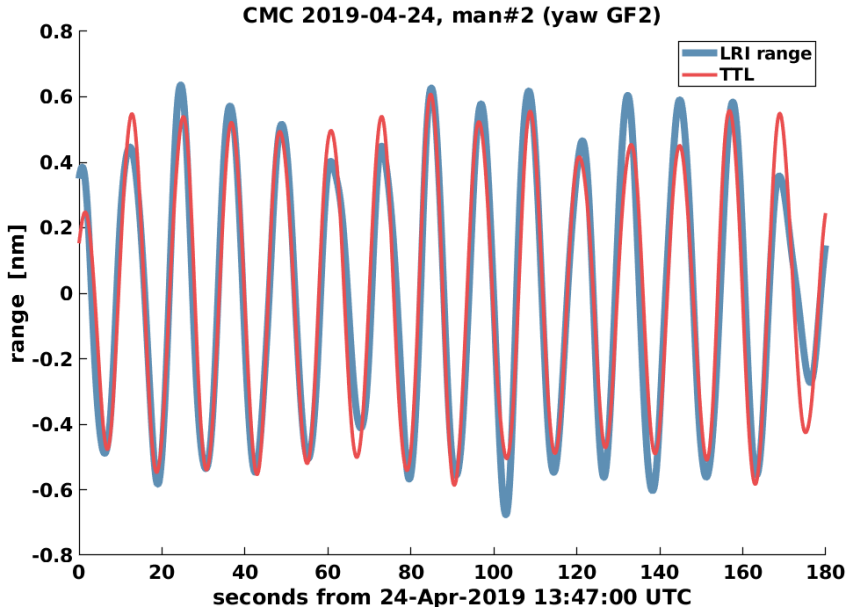


Figure: TTL fit for a yaw maneuver on GF2, in January 2019.

The estimation of the LRI TTL error is described in:

Henry Wegener et al. “Tilt-to-Length Coupling in the GRACE Follow-On Laser Ranging Interferometer”. In: Journal of Spacecraft and Rockets 0.0 (0), pp. 1–10. doi: 10.2514/1.A34790.

Riddle: unexpected artifact

We encountered unexpected behavior during the yaw maneuvers on GF1. There seemed to be a phase shift between the stimulus (yaw angle) and its response in the LRI range.

The true explanation of this observation turns out to be a coupling of the rate of change of the yaw angle of the main (transmitter) S/C into the LRI range, which adds to the TTL error. The effect is sketched on the following slide.

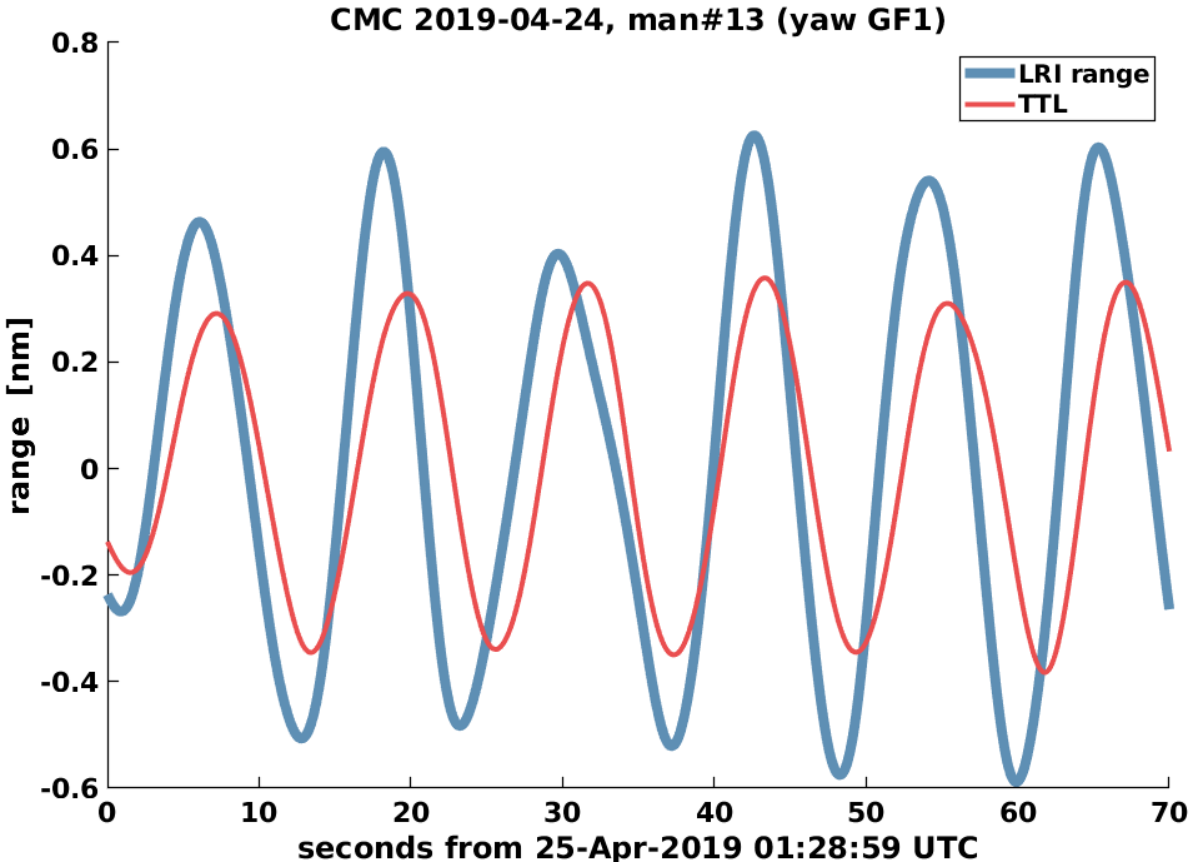


Figure: TTL fit for a yaw maneuver on GF1, in January 2019.

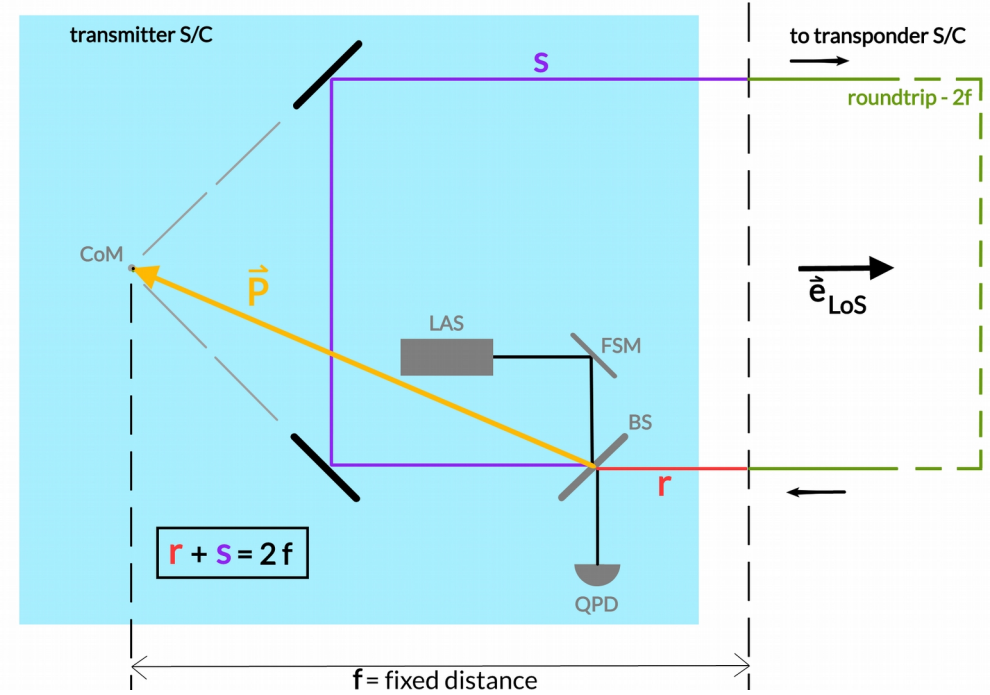
Solution: angular rate coupling

The roundtrip path length changes during the light travel time. The consequence is an **angular rate coupling (ARC)**. This effect had not been expected, because the light travel time had been neglected.

The difference between measured and instantaneous roundtrip path is approximated by the change of the length **r** during the roundtrip time τ. The error in the range (half the roundtrip path) can be modelled as

$$\begin{aligned}\delta\rho_{\mathrm{ARC}} &\approx \frac{\tau}{2}\cdot\dot{r}_1(t) \approx \frac{\tau}{2}\cdot\frac{d}{dt}\left(e_{\mathrm{LoS}}^T\cdot\vec{P}\right)\\ &\approx \frac{L}{c}\cdot\left(24\ \mathrm{mm\,rad^{-1}}\dot{\theta}_y(t)+300\ \mathrm{mm\,rad^{-1}}\dot{\theta}_z(t)\right)\\ &\approx 15\ \mu\mathrm{m\,rad^{-1}}\dot{\theta}_y(t)+175\ \mu\mathrm{m\,rad^{-1}}\dot{\theta}_z(t)\end{aligned}$$

for a S/C separation of L = 175 km.



Note: ARC occurs only on the main (transmitter) S/C, not on the transponder S/C.

Measuring ARC

ARC can be measured together with the TTL coupling, using CMCal maneuvers.

In summary:

- We reanalyzed the CMCal maneuvers, fitting not only the pointing angles, but also their derivatives
- Estimated coupling factors **agree with the model** within the uncertainty
- ARC is mainly a **yaw rate coupling** of the transmitter S/C
- This resolves the aforementioned riddle

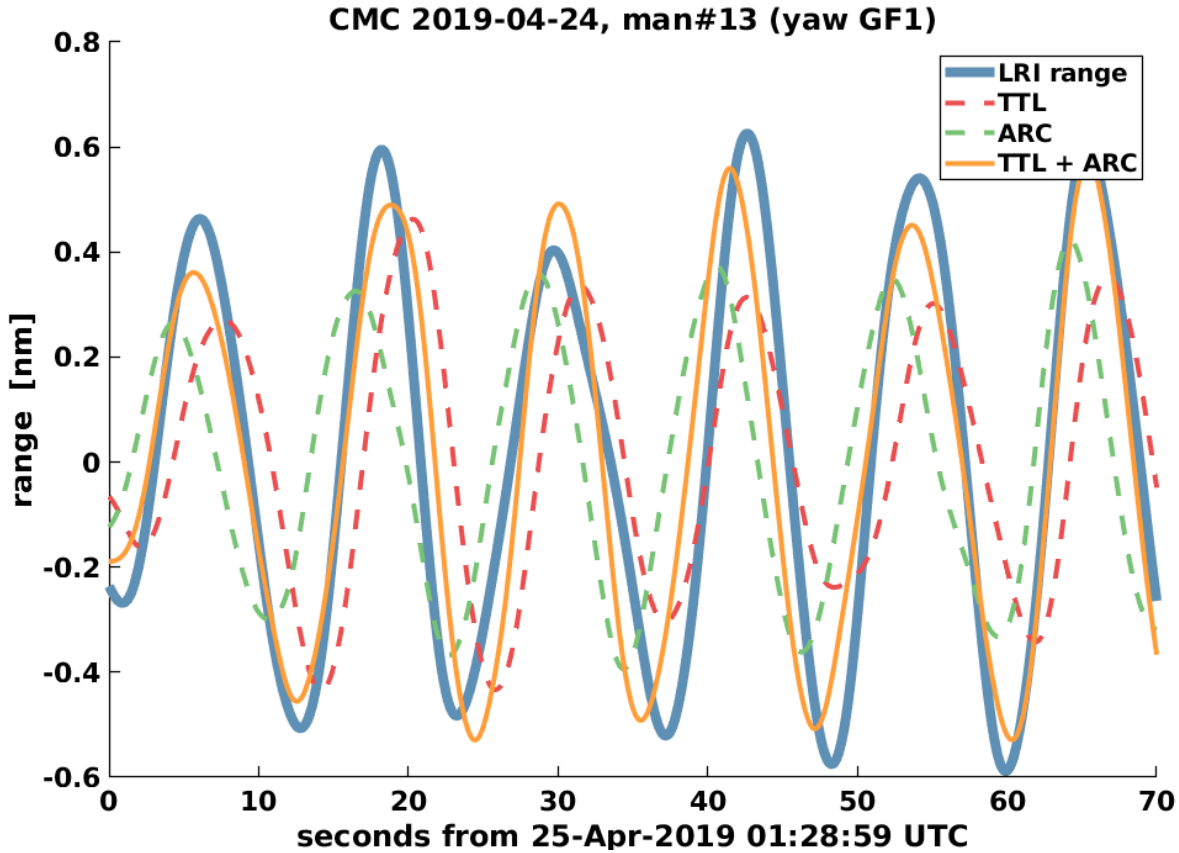


Figure: TTL and ARC fit for a yaw maneuver on GF1.

Conclusions:

- ARC is likely to be negligible for most science applications
- The erroneous effect of ARC on the range is lower than that of TTL, for low to mid Fourier frequencies
- For the frequencies above 100 mHz, ARC and TTL are similar in magnitude
- ARC becomes relevant during CMCal maneuvers (at 83.3 mHz)

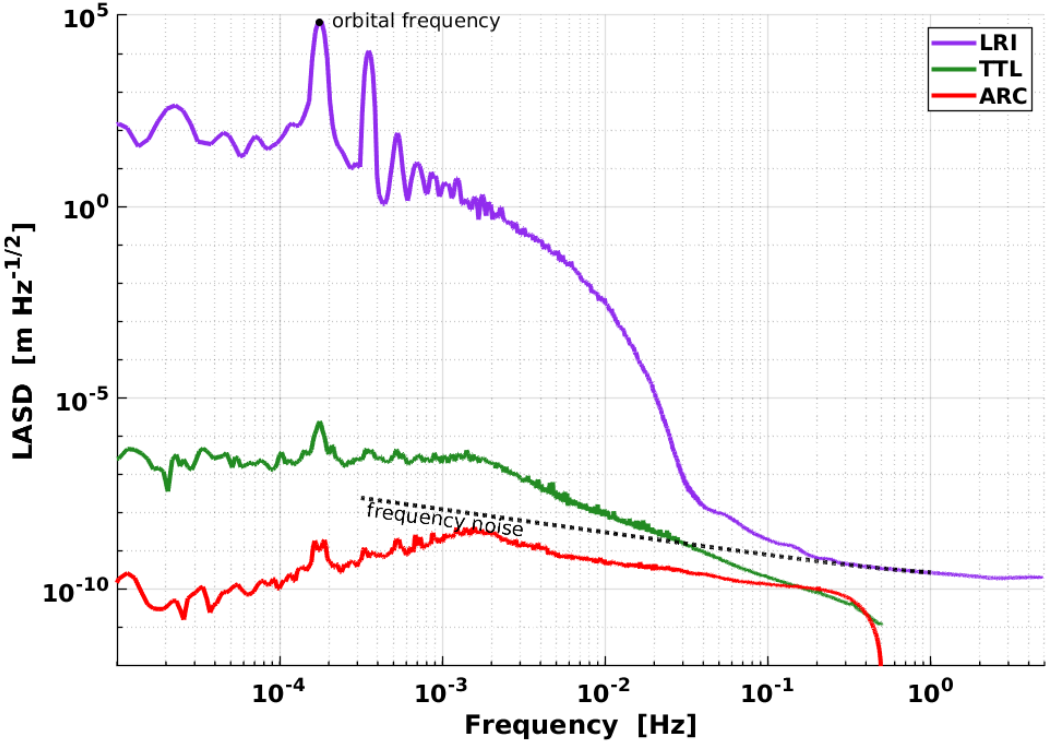


Figure: Amplitude spectral density of LRI range, TTL coupling, and ARC, measured in April 2019.

Updated TTL results

date	TTL [µm rad^{-1}]: pitch GF1	yaw GF1	pitch GF2	yaw GF2	ARC [µm rad^{-1} s]: $\frac{d}{dt}$ pitch	$\frac{d}{dt}$ yaw
GF2 transmitter:				GF2:	GF2:	
2018-06-21	133.0 ± 4.3	166.0 ± 14.0	67.9 ± 6.6	90.1 ± 15.9	-10.1 ± 8.0	130.8 ± 23.8
GF1 transmitter:				GF1:	GF1:	
2019-01-16	96.5 ± 3.2	46.0 ± 5.0	77.2 ± 4.5	149.3 ± 8.6	14.1 ± 2.9	188.8 ± 13.1
2019-03-26	88.0 ± 1.6	52.4 ± 2.3	82.1 ± 7.0	139.6 ± 8.9	6.9 ± 2.6	180.1 ± 9.1
2019-04-24	101.2 ± 2.3	69.5 ± 4.5	84.6 ± 4.1	131.6 ± 6.5	20.8 ± 3.9	164.7 ± 13.3
2019-05-23	89.8 ± 2.0	71.0 ± 5.1	49.6 ± 4.7	100.9 ± 7.5	14.9 ± 3.4	158.5 ± 10.3
2019-06-22	97.0 ± 2.4	91.0 ± 7.3	33.7 ± 3.9	92.4 ± 8.5	1.6 ± 4.3	180.5 ± 5.2
2019-07-24	104.4 ± 2.5	82.8 ± 4.6	77.8 ± 4.0	129.9 ± 8.0	-8.4 ± 6.8	171.0 ± 12.3
2019-08-26	104.5 ± 2.0	82.4 ± 3.7	97.8 ± 5.1	139.9 ± 8.1	6.2 ± 3.1	180.8 ± 10.9
2019-09-28	111.6 ± 2.3	62.1 ± 3.2	100.4 ± 6.0	131.0 ± 8.2	-9.6 ± 7.5	159.3 ± 10.2
2020-02-01	93.9 ± 2.7	62.4 ± 3.6	–	–	16.8 ± 2.4	181.3 ± 5.1
2020-02-14	100.7 ± 3.5	74.2 ± 3.7	–	–	-1.0 ± 4.0	189.2 ± 12.1
2020-05-09	107.7 ± 3.6	72.6 ± 2.6	–	–	27.2 ± 5.7	188.0 ± 10.2
2020-05-15	87.7 ± 2.3	93.8 ± 2.8	–	–	11.6 ± 2.9	181.3 ± 8.2
2020-10-20	63.9 ± 3.1	87.3 ± 9.1	86.7 ± 3.2	119.2 ± 5.2	-37.6 ± 6.0	94.2 ± 17.0

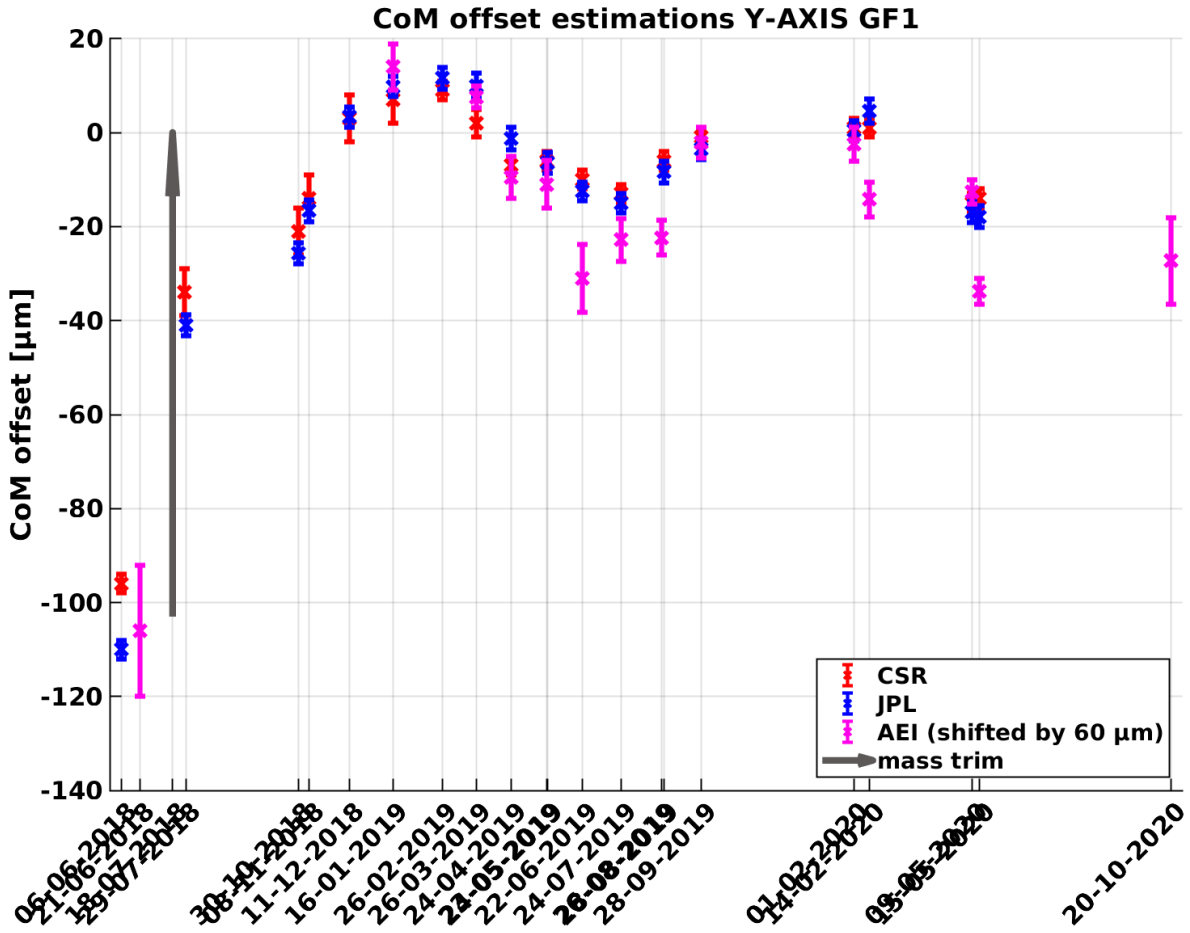
Table: updated TTL and ARC estimation results in terms of linear coupling factors. Pitch and yaw coupling factors correspond to offsets between VP and CM in cross-track (y) and nadir (z) directions. Roll coupling is negligible and not shown here. Results for 20 October 2020 are preliminary.

Tracking of CM movement (y GF1)

GF1 y (cross track component):

- Good agreement of long term CM tracking
- The values reflect well the mass trim maneuver performed in July 2018 (indicated by gray arrow)
- CM offset as well as long term stability within requirements

- The measured TTL coupling factors can be converted to y and z components of the VP-CM offset.
- AEI values are based on LRI data, and refer to the LRI reference point (=VP)
- CSR / JPL values are based on ACC data, and refer to the ACC reference point

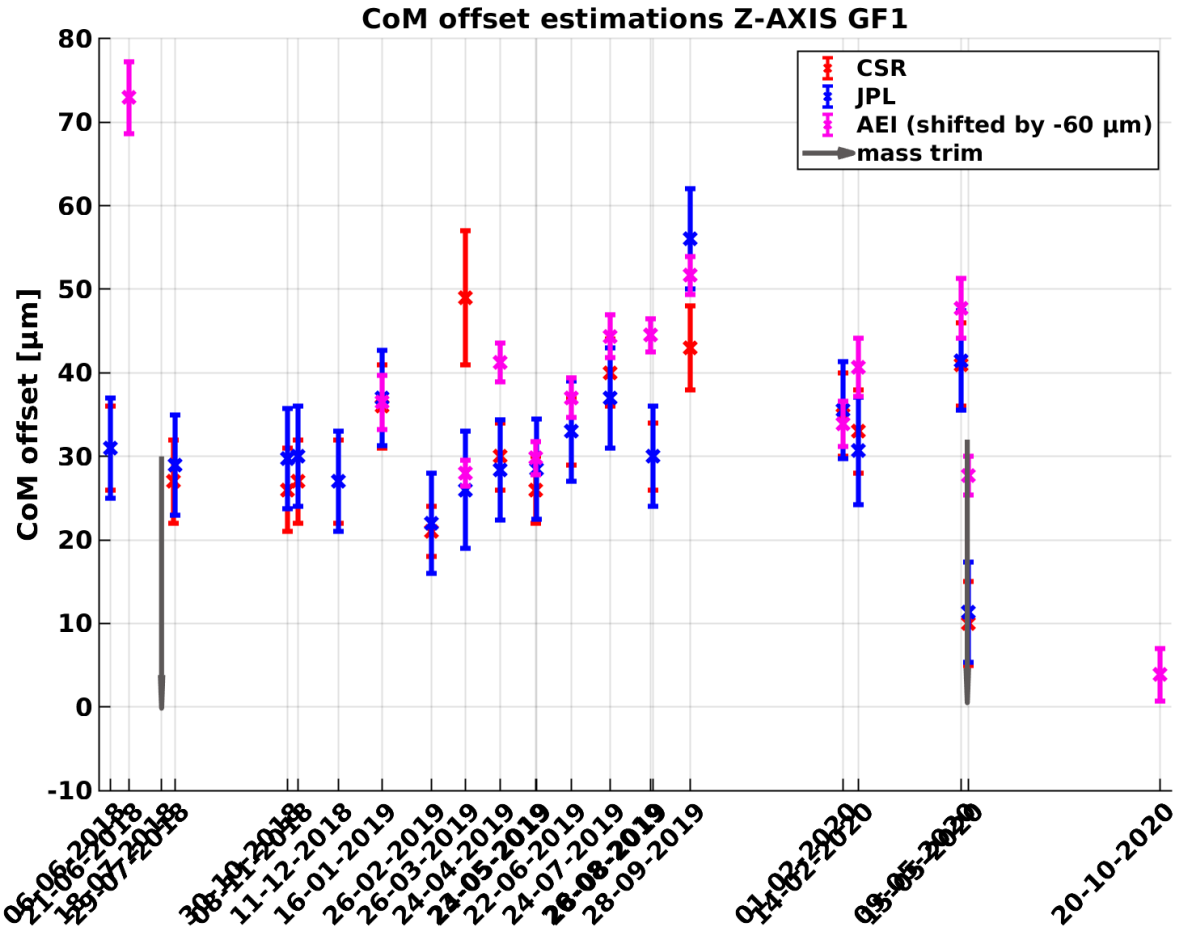


Tracking of CM movement (z GF1)

GF1 z (nadir component):

- Good agreement of long term CM tracking
- AEI values reflect well the CM trim maneuvers
- CM offset as well as long term stability within requirements

- The measured TTL coupling factors can be converted to y and z components of the VP-CM offset.
- AEI values are based on LRI data, and refer to the LRI reference point (=VP)
- CSR / JPL values are based on ACC data, and refer to the ACC reference point

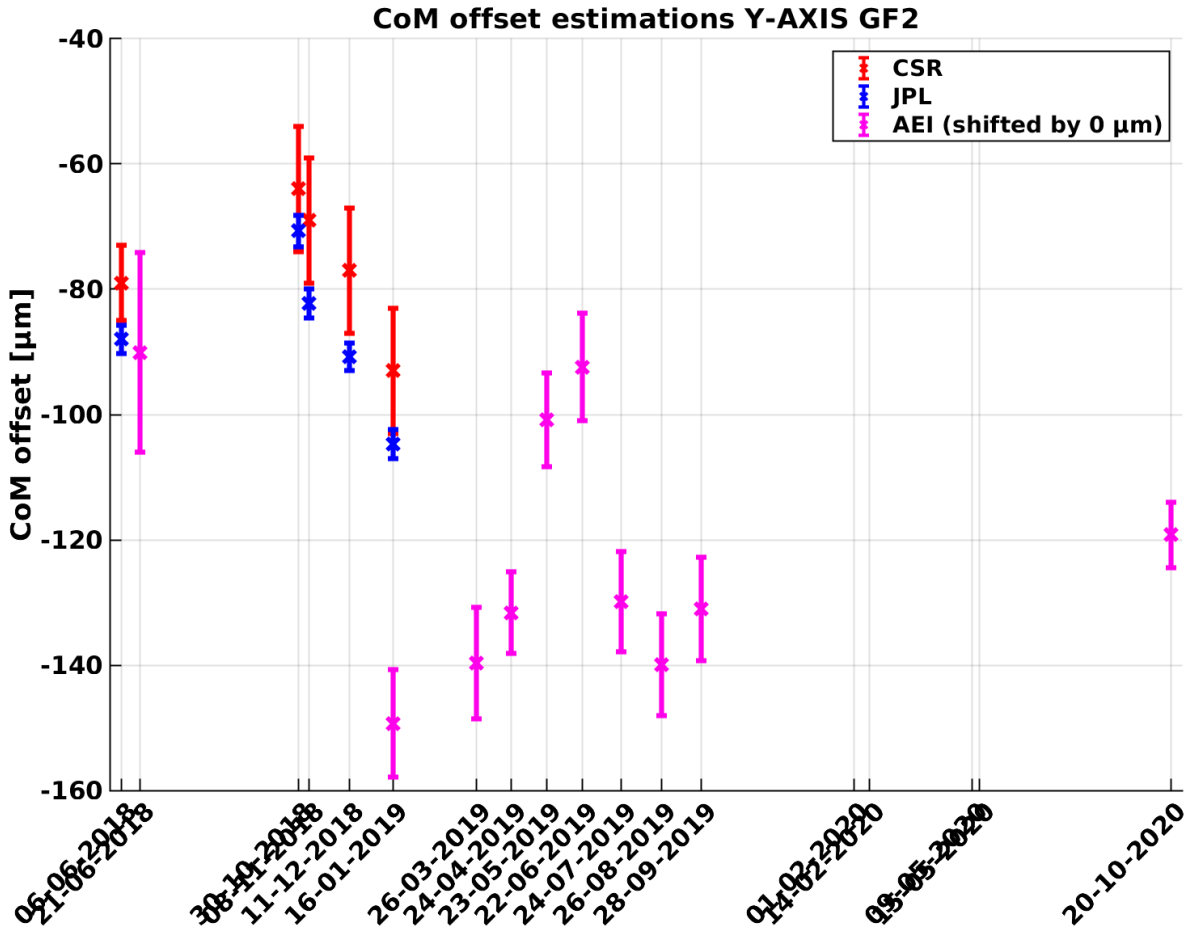


Tracking of CM movement (y GF2)

GF2 y (cross track component):

- Values between May and July 2019 can be viewed as outliers; those values likely do not reflect actual changes of the CM location
- CM offset as well as long term stability within requirements

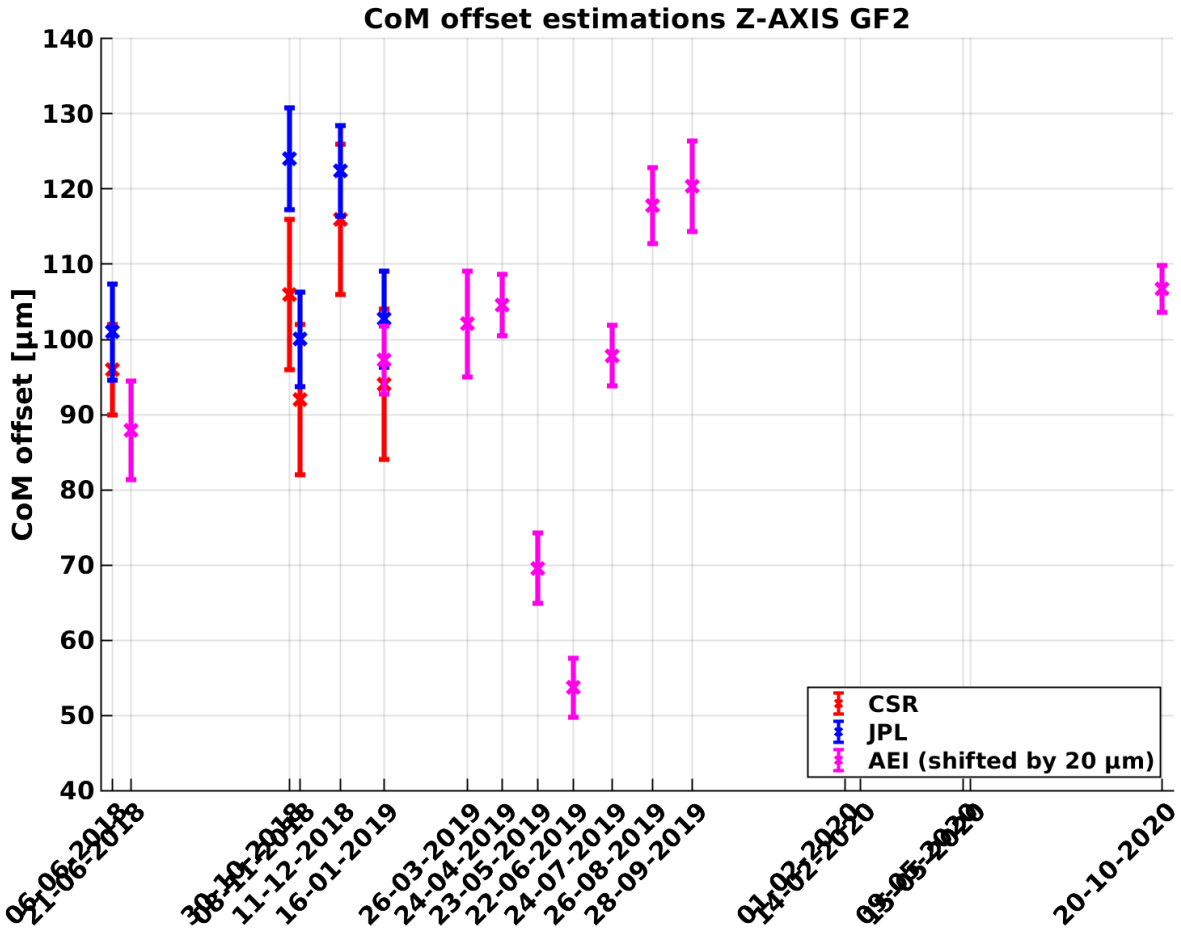
- The measured TTL coupling factors can be converted to y and z components of the VP-CM offset.
- AEI values are based on LRI data, and refer to the LRI reference point (=VP)
- CSR / JPL values are based on ACC data, and refer to the ACC reference point



Tracking of CM movement (z GF2)

GF2 z (nadir component):

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- The measured TTL coupling factors can be converted to y and z components of the VP-CM offset.
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Understanding the LRI spectrum

The two largest expected noise sources, TTL and laser frequency noise (LFN), are well understood. However, the spectrum of the inter-satellite range is noise-dominated only at Fourier frequencies above 200 mHz. For frequencies below 35-40 mHz, the prominent signal is the one containing the gravity field information. Above 35 mHz, the microwave ranging system measures almost only noise. What does the LRI measure in this frequency range between 35 and 200 mHz?

Above 40 mHz, one does not expect to see any gravity signal in the inter-satellite range. It is thus likely that the LRI is measuring range variations which are caused by non-gravitational forces acting on the S/C. In order to verify this hypothesis, we attempt to compute these variations and subtract them from the LRI range. If the spectrum goes down to the level of the instrument noise, we can conclude that our noise model is in fact correct and defines the true sensitivity of the LRI in this frequency region. We can then exclude the presence of other significant noise sources, and improve our understanding of the instrument performance. Furthermore, we can potentially gain new insights from the signal caused by these non-gravitational forces.

Understanding the LRI spectrum

Here, we will focus on the frequency region between 35 and 80 mHz. We describe two approaches to remove part of the non-gravitational signal from the LRI range.

- Using **ACT1A** data: non-gravitational accelerations are taken from the ACT1A data product. For each S/C, the component of the acceleration, which is directed along the line-of-sight, is determined with the help of SCA1B and GNV1B data. From that, the differential acceleration is determined, which contributes to the range acceleration. This yields a time series of range accelerations, which we abbreviate by **DLN** (differential linear non-gravitational) in the following. It can be subtracted from the LRI range accelerations, the respective spectra of the time series computed, and converted to range in meters by integration.

- Using **THR1B** data: Attitude control thrusters firings also produce undesired small linear accelerations. These are in fact measured by the accelerometers, and also by the LRI, if they are directed along the line-of-sight. We used LRI data to model these responses of thruster firings in the range. Based on that model, a time series of linear accelerations can be generated by taking timings and durations of thruster activations from the THR1B data product. We will abbreviate it by **THR**. In the same manner as described above, the thruster responses can be removed from the LRI data, and the respective spectra can be computed and compared.

Understanding the LRI spectrum

For each single day, an ASD is computed for the different time series. The RMS variations of these spectra, restricted to the frequency range from 35 to 80 mHz, are computed and compared here.

Observations:

- January 2019 period: elevated spectrum caused by different AOCS setting, i.e. longer thruster durations (1 second)
- Dominated by DLN, sometimes even by THR alone
- beta' dependency due to solar radiation pressure (SRP)
- Values predicted by a simplistic SRP model do not perfectly match the data, but show the same profile

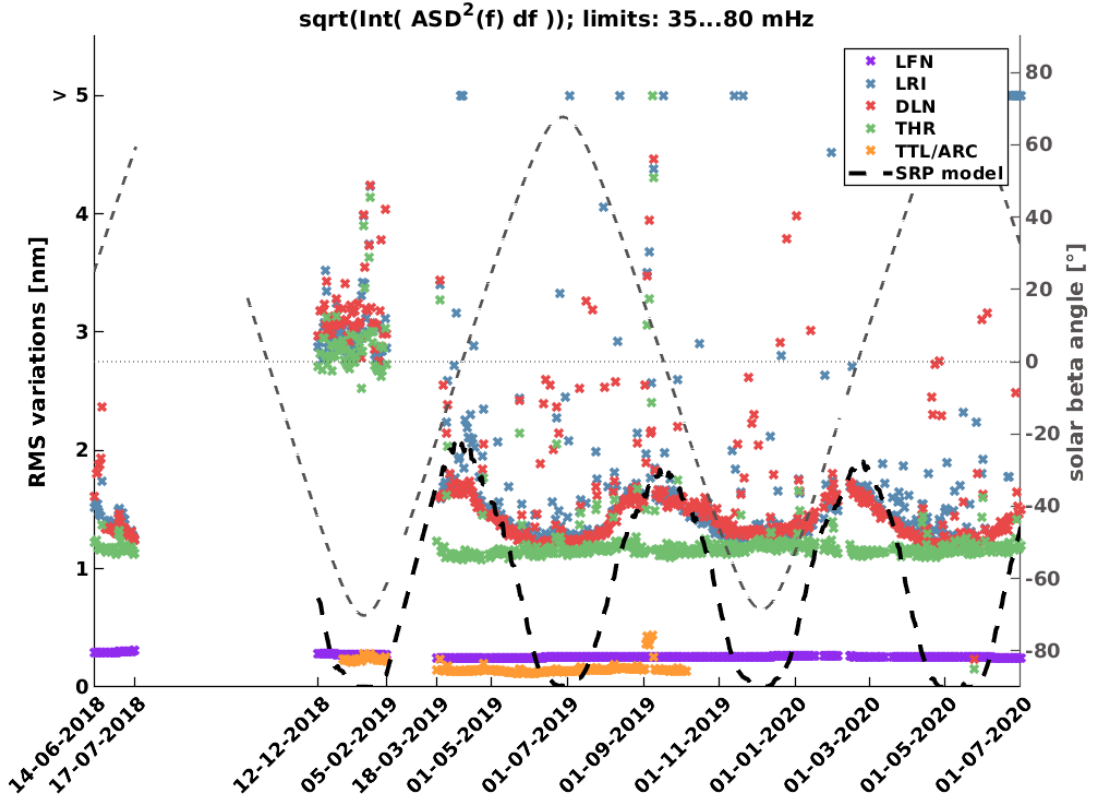


Figure: RMS variations of the ASD of LRI range and some of its contributors.

Understanding the LRI spectrum

RMS variations of the ASD of LRI range after subtracting non-gravitational contributions using ACT1A or THR1B

Observations:

- The level of TTL+ARC is never reached
- Thus, subtracting TTL makes no visible difference above 35 mHz
- During periods when thruster responses constitute the largest contribution, the modelled time series using THR1B data works best and can reduce the spectrum down to the level of LFN
- During periods when SRP is dominating, ACT1A data works better, however not reaching LFN level

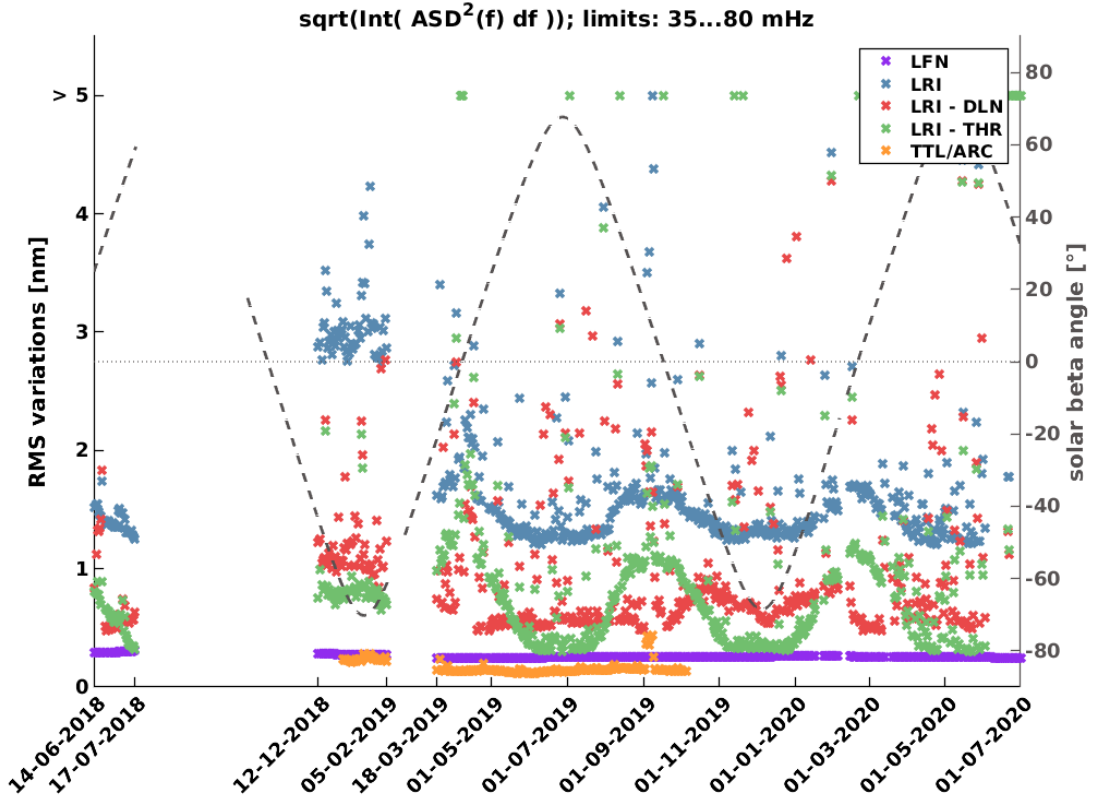


Figure: RMS variations of the ASD of LRI range and some “reduced” time series.

Summary:

- Tilt-to-length (TTL) coupling has been measured and all requirements are fulfilled
- TTL is unlikely to significantly impact gravity field recovery from LRI ranging data
- Coupling factors can be used to determine CM offsets in y and z axes
- CM offsets for GF2 are especially useful, since the GF2 accelerometer data is degraded
- So far, the measured CM offsets seem to be relatively stable over time
- Angular rate coupling (ARC) has been measured and fits well with the model
- ARC is negligible in low and mid frequencies, but shows up in the CMCal data
- LRI spectrum can be reduced by subtracting non-gravitational accelerations
- The CBE on the total ranging noise is given by the sum of frequency noise, TTL, and (almost negligible) ARC