Optimization and assessment of high-resolution regression mascons for multiple time intervals within the GRACE/GRACE-FO record

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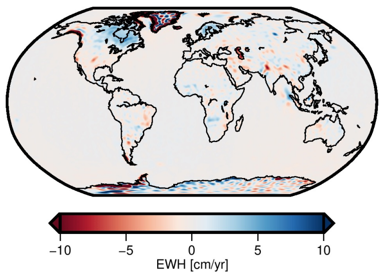
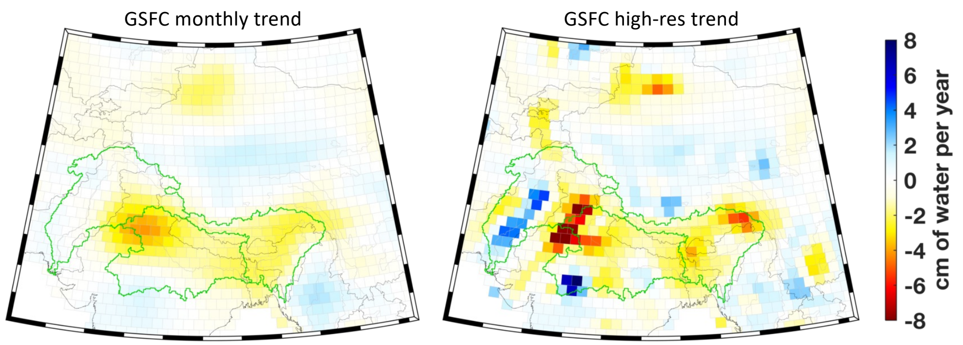
Background

- Multiple studies have demonstrated enhanced spatial resolution in mean field, trends, annual signal by stacking normal equations, e.g., series of GOCO (TU Graz) & EIGEN (GFZ/GRGS) models:

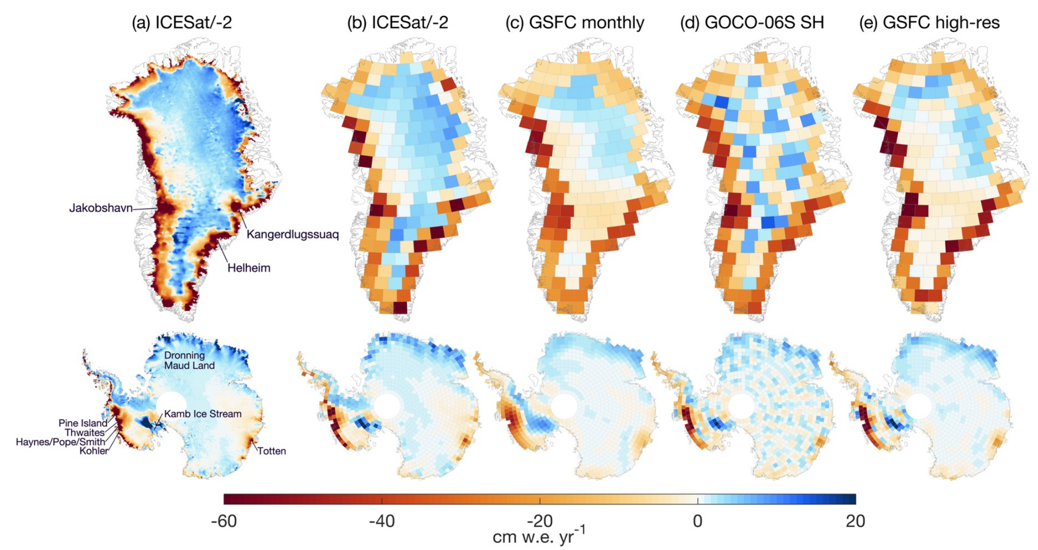


Kvas et al., (2020), Earth System Science Data, https://doi.org/10.5194/essd-13-99-2021

- Signal-to-noise improvements from global stacked high-resolution mascon trends:



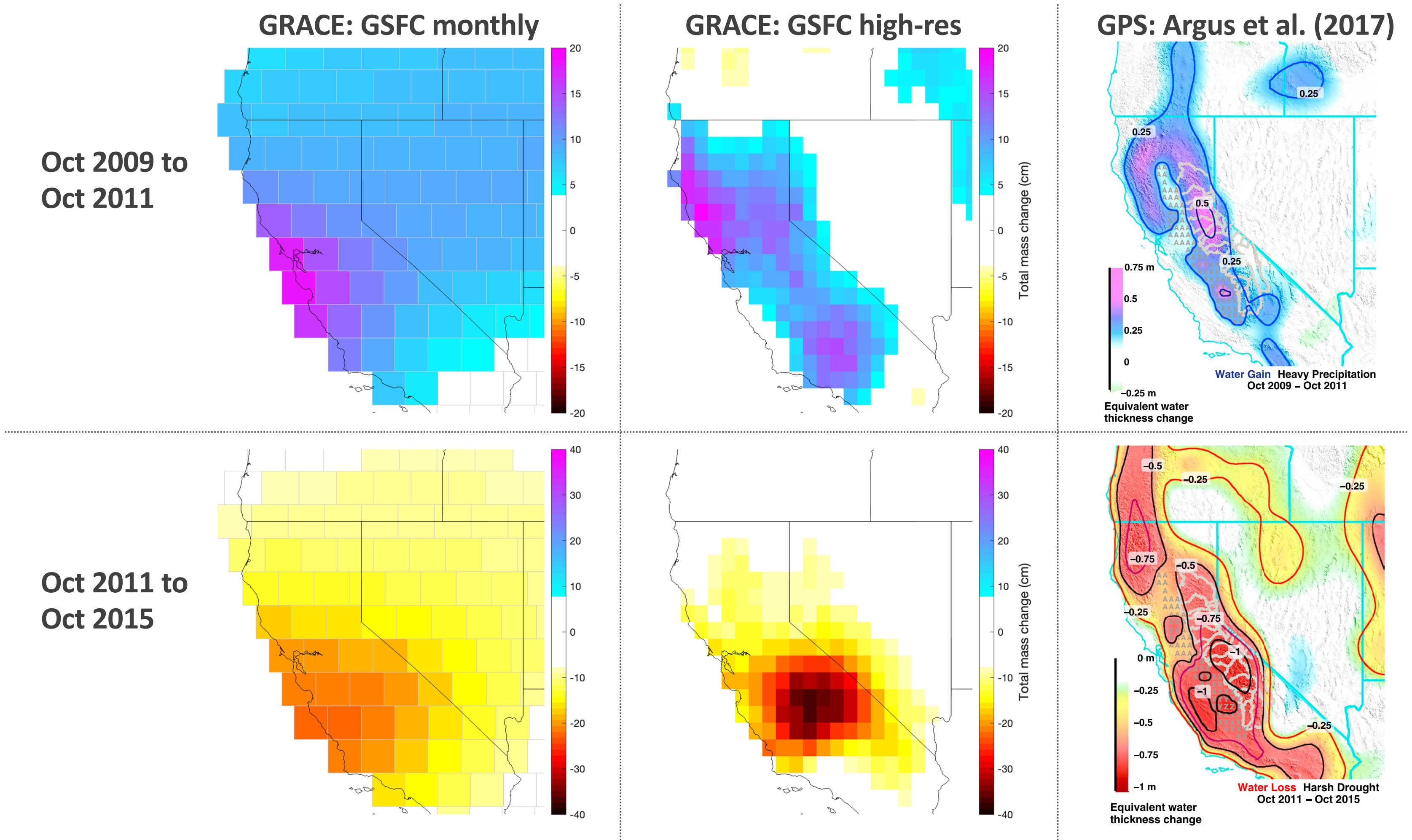
Loomis et al., (2019b), *Frontiers in Earth Science*, https://doi.org/10.3389/feart.2019.00235



Loomis et al., (2021), *JGR: Solid Earth*, https://doi.org/10.1029/2021JB023024

Background

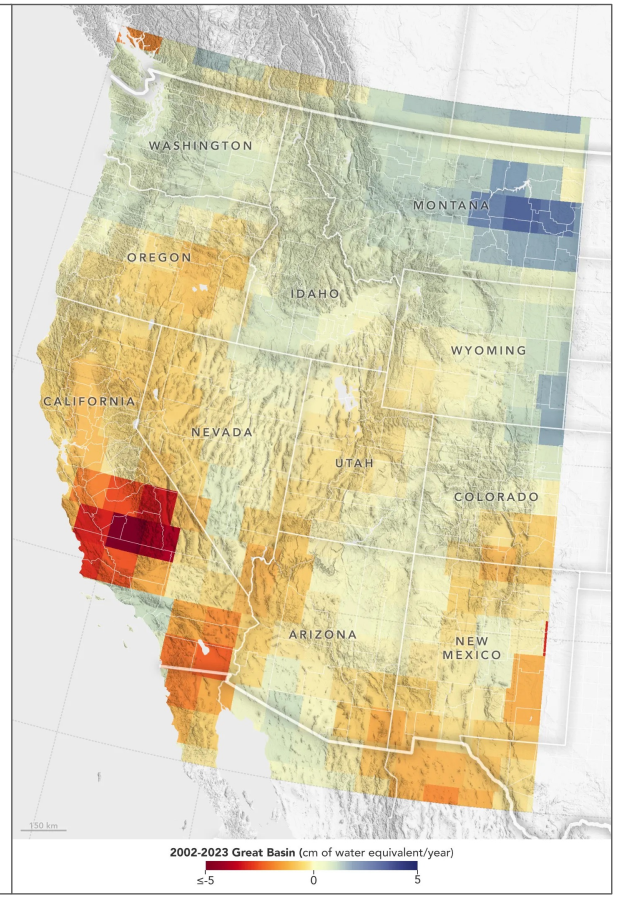
- Next logical step: Apply the same approach within the GRACE/GRACE-FO record to get both higher spatial resolution & temporal information



Loomis et al., (2022), *AGU Fall Meeting*

Recent Science Application: Hall et al., 2024

NASA Satellites Find Snow Didn't Offset Southwest US Groundwater Loss



Despite some years with significant snowfalls, long-term drought conditions in the Great Basin region of Nevada, California, Arizona, and Utah, along with increasing water demands, have strained water reserves in the western U.S. As a result, inland bodies of water, including the Great Salt Lake pictured here, have shrunk dramatically, exposing lakebeds that may release toxic dust when dried.
Dorothy Hall/University of Maryland

Study summary:

- Data sets
  - Terrestrial water storage (GRACE/-FO)
  - Snow mass (SWE, days snow cover, snow depth)
  - Land surface temperature (MODIS)
- The 2002–2023 terrestrial water storage decline in the Great Basin (GB) is more pronounced in the western GB than in the eastern GB.
- Even in notable snow years like 2010–2011, 2016–2017, 2018–2019, and 2022–2023, mass losses observed by GRACE/-FO remain consistent, due to the downward trend of groundwater storage.

NASA press release: https://www.nasa.gov/science-research/earth-science/nasa-satellites-find-snow-didnt-offset-southwest-us-groundwater-loss/

Hall, D. K., Loomis, B. D., DiGirolamo, N. E., & Forman, B. A. (2024). Snowfall replenishes groundwater loss in the Great Basin of the western United States, but cannot compensate for increasing aridification. *Geophysical Research Letters*, 51, e2023GL107913. https://doi.org/10.1029/2023GL107913

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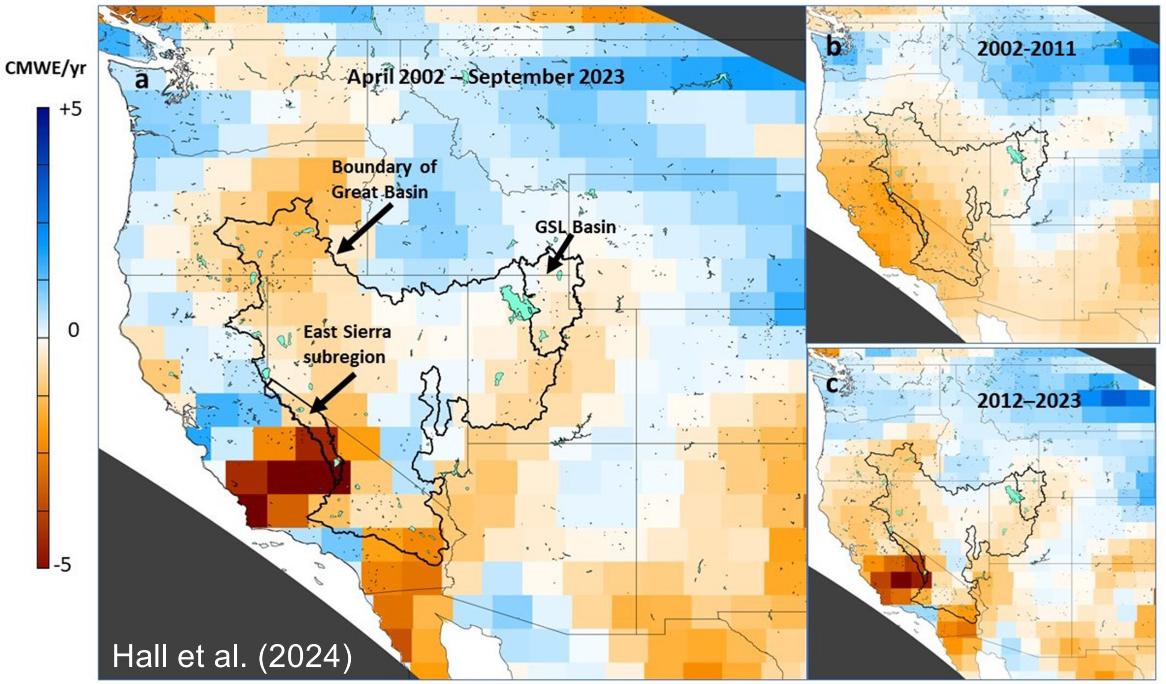


	Monthly trends		High-res trends	
	Rate (Gt/yr)	Total (Gt)	Rate (Gt/yr)	Total (Gt)
Oct 2002 – Sep 2011	-0.1	-0.6	-3.4	-30.5
Oct 2011 – Sep 2023	-4.4	-52.8	-3.2	-38.6
Sum of 1st two rows	--	-53.4	--	-69.1
Oct 2002 – Sep 2023	-4.0	-85.7	-3.4	-74.1

- Use of high-res reveals consistent trends in the GB across 2002-2011 and 2011-2023, which is the *opposite conclusion* from the monthly mascons

- Use of high-res trends *reduces* the estimated GB trend magnitude to due the *mitigation of leakage* from the Central Valley

- Disconnected trend estimates can lead to large differences in estimates of total mass change

Challenges with Multi-span Regression

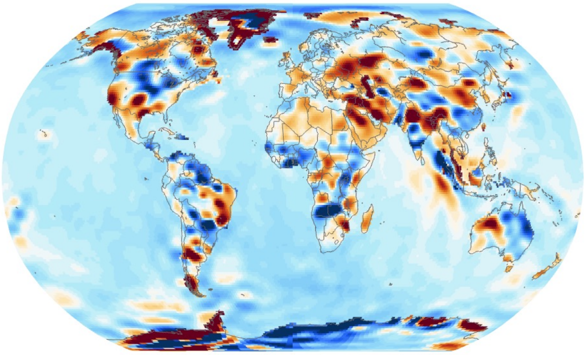
Challenge 1:

a) How to select the regularization parameter, λ? (challenge common to all regularized estimation)

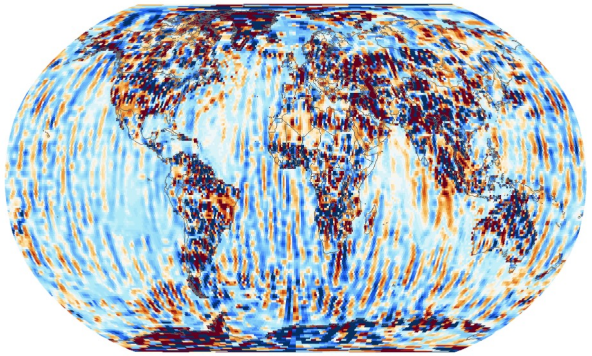
b) Should different temporal spans optimize λ separately, or use a common λ?

$$\hat{\boldsymbol{m}}_i = \left(\boldsymbol{A}^T\boldsymbol{W}\boldsymbol{A} + \lambda_i\boldsymbol{P}\right)^{-1}\boldsymbol{A}^T\boldsymbol{W}\boldsymbol{d}$$

Overdamped solution: λ is too large

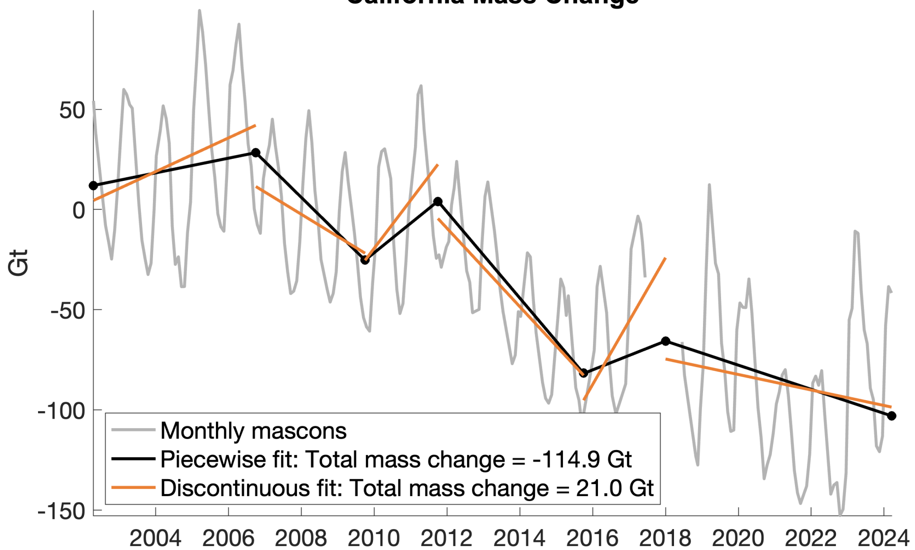


Underdamped solution: λ is too small



Challenge 2:

a) To recover total mass change over multiple spans, the regression model should enforce continuity (EIGEN RL04 does this to degree/order 90)



Challenges with Multi-span Regression

Challenge 1:

a) How to select the regularization parameter, λ? (challenge common to all regularized estimation)

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Challenge 2:

a) To recover total mass change over multiple spans, the regression model should enforce continuity (EIGEN RL04 does this to degree/order 90)

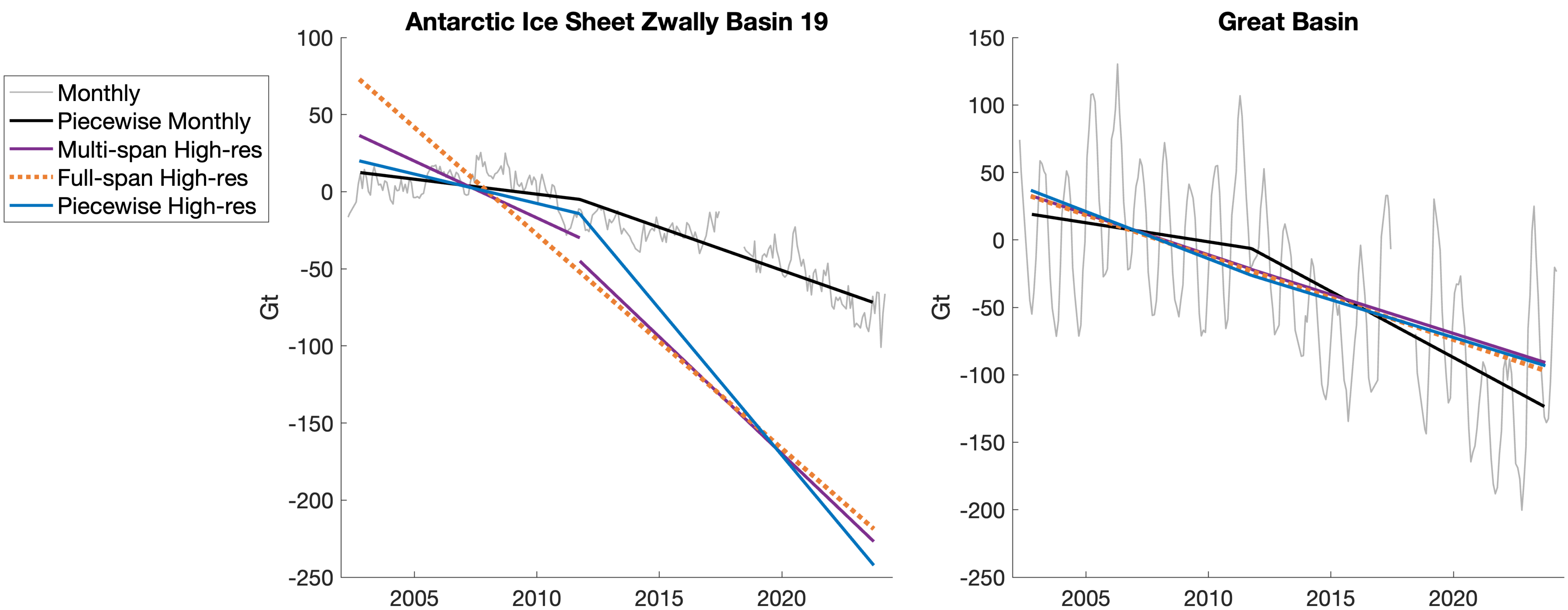
Challenges 1b and 2a are both addressed by modifying our regression model approach.

Original: Independent stacked regression solutions over multiple time intervals

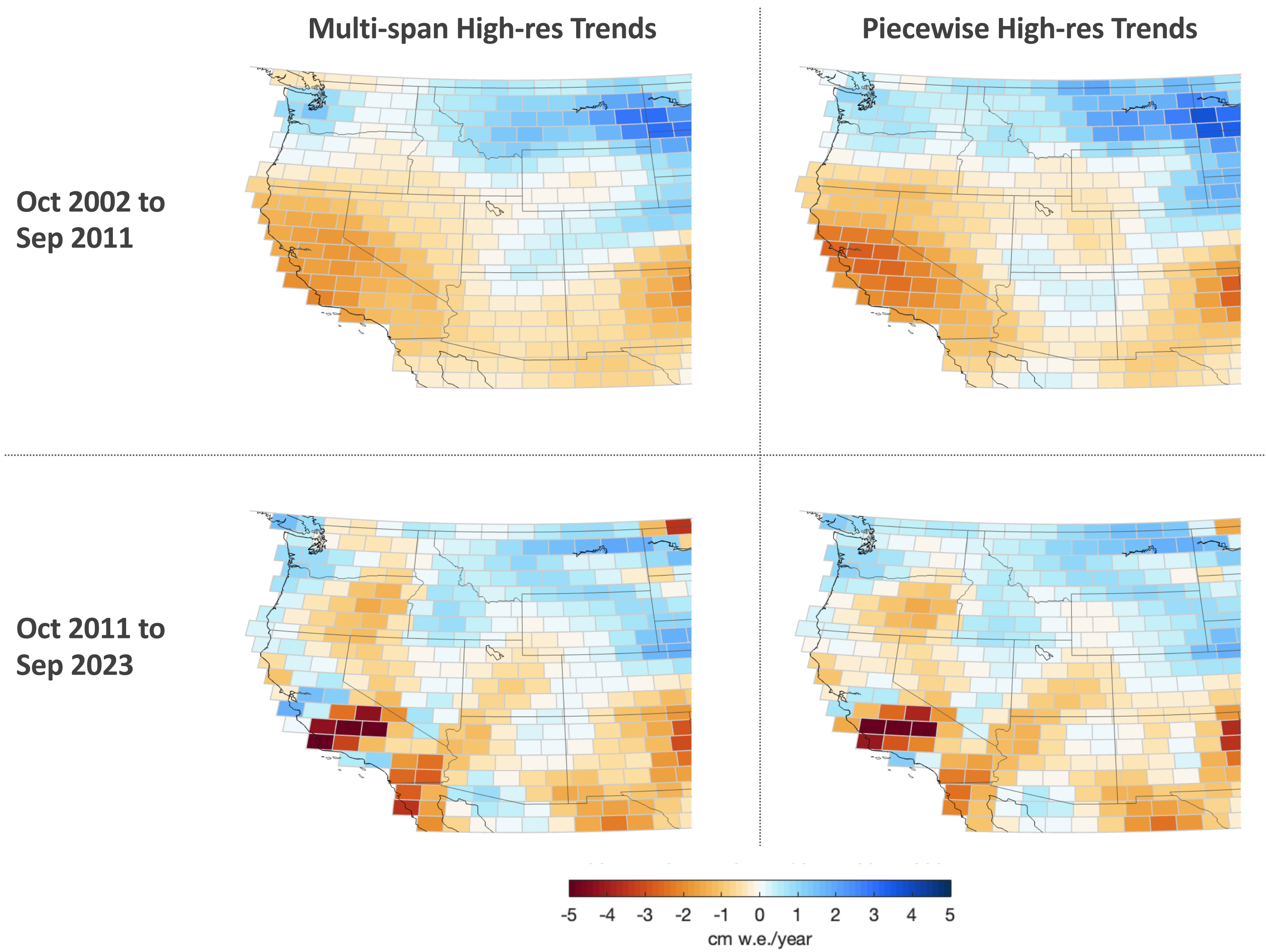
New: Spline regression model over entire span

- Proof of concept – 3 parameters consisting of 1 bias and 2 trends over the same two time intervals as the Great Basin study
- Future work – Spline parameters can be expanded to include more than two time intervals and additional parameters, e.g., annual, semi-annual, x^2, x^3, stochastic

First Results: Spline Regression Model



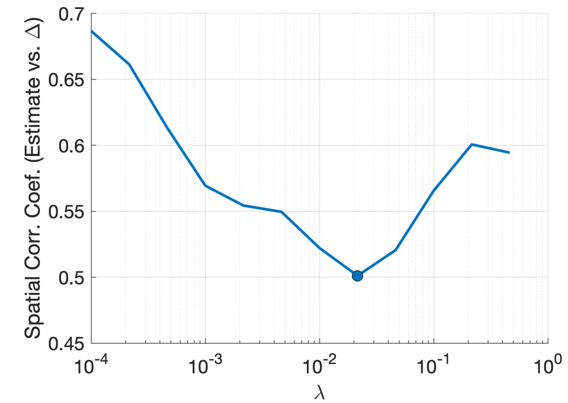
First Results: Spline Regression Model



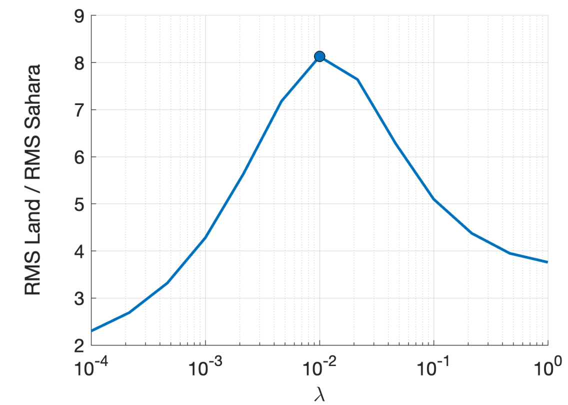
Challenge 1a: How to select λ?

Previous work for selecting λ:

- Monthly mascons use an approach that minimizes the spatial correlation between the estimate and change in the estimate due to increasing λ (Croteau et al., 2021)
- High-resolution trend estimation (Loomis et al., 2021) used the same approach as the monthly



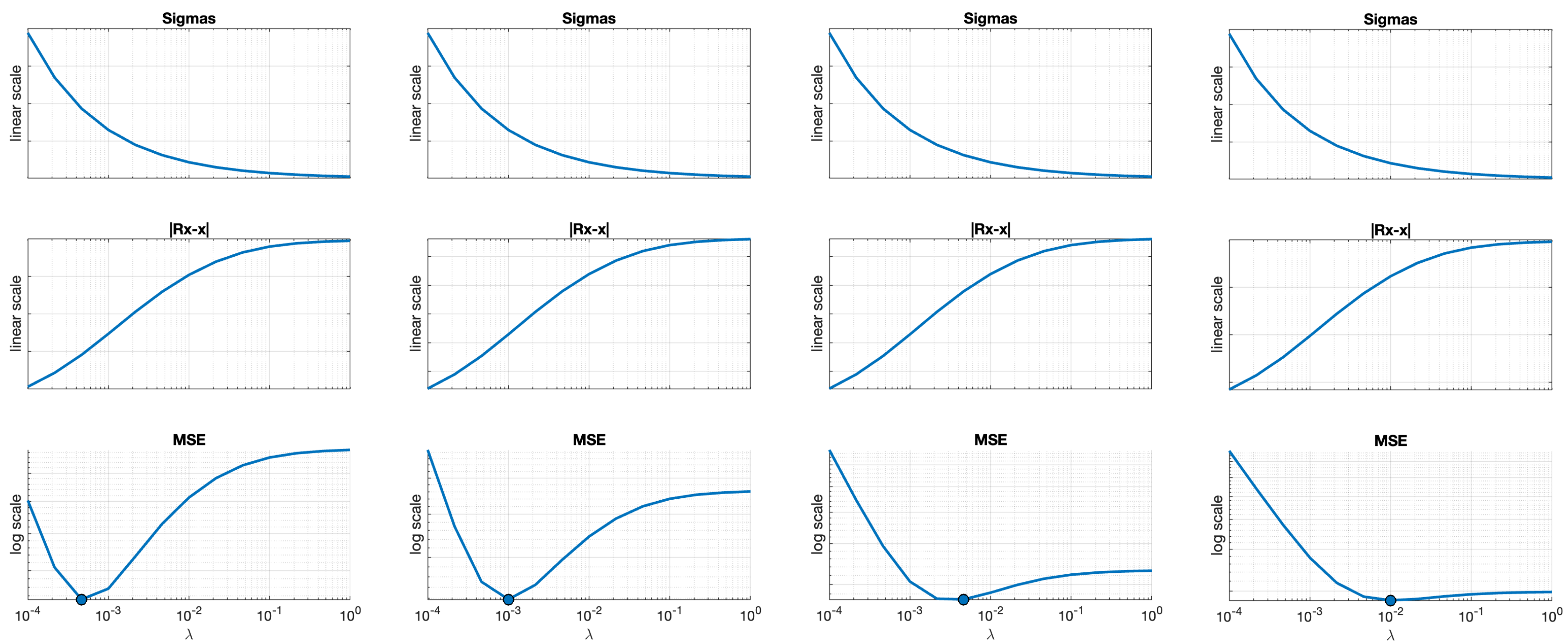
- Hall et al. (2024) optimized Signal-to-Noise, for full span, and used the same λ for subintervals (where Signal ≡ Land RMS, Noise ≡ Sahara RMS)



Challenge 1a: How to select λ?

Other potential approaches to select λ:

- L-curve criterion – Previously explored and tends to provide overdamped solutions (Save et al., 2012)
- Mean Squared Error (MSE) = Sum of covariance and bias, where bias is a measure of smoothing/ leakage (Loomis et al., 2019a) – Difficult to interpret due to unknown truth vector, x



← Increasing magnitude of x vector

Summary

Background:

- Stacking normal equations is very successful at enhancing spatial resolution and signal recovery
- Stacked regression mascons improve signal-to-noise via regularization
- We have successfully applied this method to specific science questions (e.g., Hall et al., 2024)

Today's presentation:

- We have demonstrated a new regularized regression spline estimator to further leverage this technique to maximize spatial resolution while also recovering valuable temporal information
- Selecting the regularization parameter, λ, remains a bit of an art form; current methods seem viable

Future work:

- Expand spline parameters and test for specific science questions – Please reach out if interested!
- Improved uncertainty quantification (previous studies have used differences to GOCO-06S over common time span)